

Flexural fatigue strength prediction of steel fibre reinforced concrete beams

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ABSTRACT: Results of an investigation conducted to study the fatigue strength of steel fibre reinforced concrete (SFRC) containing fibres of mixed aspect ratio are presented. Approximately eighty one beam specimens of size 500 mm x 100 mm x 100 mm were tested under four-point flexural fatigue loading in order to obtain the fatigue lives of SFRC at different stress levels. About thirty six static flexural tests were also carried out to determine the static flexural strength of SFRC prior to fatigue testing. The specimens incorporated 1.0, 1.5 and 2.0% volume fraction of corrugated steel fibres. Each volume fraction incorporated fibres of two different sizes i.e. 2.0 x 0.6 x 25 mm and 2.0 x 0.6 x 50 mm by weight of the longer and shorter fibres in the ratio of 50% - 50%. Fatigue life data obtained has been analysed in an attempt to determine the relationship among stress level, number of cycles to failure and probability of failure for SFRC. It was found that this relationship can be represented reasonably well graphically by a family of curves. The experimental coefficients of the fatigue equation have been obtained from the fatigue test data to represent the curves analytically.

KEYWORDS: Fibre reinforced concrete, Flexural strength, fatigue

1 INTRODUCTION

The increased application of steel fibre reinforced concrete as an engineering material demands an additional knowledge of its behaviour under several types of loading to which it is subjected. Such knowledge is necessary not only to provide safe, efficient, and economical designs for the present, but also to serve as a rational basis for extended future applications.

Since tests conducted by Feret (1906), many researchers have carried out laboratory fatigue experiments to investigate the fatigue behaviour of plain as well as steel fibre reinforced concrete. Most of the studies on steel fibre reinforced concrete were mainly confined to the determination of its flexural fatigue endurance limit for different type/volume fraction/aspect ratio of fibres (Batson et al. 1972, Ramakrishnan et al. 1987, Tartro 1987, Ramakrishnan 1989, Ramakrishnan 1989, Johnston & Zemp 1991). Some investigations focused on studying other aspects of fibre reinforced concrete in respect of fatigue. Yin et al. (1995) studied the fatigue behaviour of steel fibre reinforced concrete under uniaxial and biaxial compression and observed that the S-N curves can be approximated by two straight

lines connected by a curved knee instead of a single straight line. Wei et al. (1996) studied the effect of fibre volume fraction, amount of silica fume and their composite action on fatigue performance of SFRC. Ong et al. (1997) investigated the behaviour of steel fibre mortar overlaid concrete beams under cyclic loading whereas the behaviour of composite concrete sections reinforced with conventional steel bars and steel fibres, and subjected to flexural cyclic loading was analysed by Spadea & Bencardino (1997). Jun & Stang (1998) reported that the accumulated damage level in fibre reinforced concrete in fatigue loading was 1-2 order of magnitude higher than the level recorded in static testing of the same material. Effect of fly ash on fatigue properties of hardened cement mortar was studied by Taylor & Tait (1999) whereas, Daniel & Loukili (2002) investigated the behaviour of high strength fibre reinforced concrete under cyclic loading. SFRC with two types of hooked end steel fibres was tested by Cachim et al. (2002) in an experimental study to evaluate the performance of plain and fibre reinforced concrete under compressive fatigue loading. In a review paper, Lee & Barr (2004) provided a general overview of recent developments in the study of the fatigue behaviour of plain and fibre reinforced concrete.

2 RESEARCH SIGNIFICANCE

Steel fibre reinforced concrete containing mixed fibres can be used in many applications by combining fibres of varying length i.e. mixed aspect ratio in a matrix. Although, the fatigue test data of SFRC shows considerable variability, even at a given stress level, due to the random orientation of fibres in concrete, a little effort has been directed to develop relationships among stress level, fatigue life and probability of failure or survival probability. The research work of McCall (1958) provides good opportunity to generate a family of S-N-P_f curves for SFRC containing fibres of mixed aspect ratio. It is also proposed to develop a mathematical model for SFRC from the fatigue test data obtained in this investigation to represent the family of S-N-P_f curves analytically.

3 EXPERIMENTAL PROGRAMME

The concrete mix used for casting the test specimens, its 28 days compressive strength and static flexural strength is shown in Table 1. Ordinary Portland Cement, crushed stone coarse aggregates with maximum size 12.5 mm and river sand were used. The materials used conformed to relevant Indian Standard specifications. The specimen incorporated three different volume fractions i.e. 1.0, 1.5% and 2.0% of corrugated steel fibres, each volume fraction incorporating two different aspect ratios of fibres namely 20 (fibre size 2.0 x 0.6 x 25 mm) and 40 (fibre size 2.0 x 0.6 x 50 mm) by weight of the longer and shorter fibres in the mix proportions of 50% - 50%. The detail of the mixes along with 28 day compressive strength and static flexural strength are presented in Table 2. The specimen used for flexural fatigue tests as well as static flexure tests were fibre concrete beams of size 100 x 100 x 500 mm. Cube specimens of size 150 x 150 x 150 mm were used to determine the 28 days compressive strength of concrete.

The static flexural tests on a particular batch were conducted just before the fatigue testing of the same. All the static flexural tests were conducted on a 100 kN INSTRON Universal Testing Machine. The beams were simply supported on a span of 450 mm and loaded at third points. The fatigue tests were conducted on a 100 kN MTS Universal Testing Machine. The span/points of loading in the fatigue tests were kept the same as those for the static flexural tests. Fatigue tests were conducted at different stress levels 'S' ($S = f_{max}/f_r$, f_{max} = maximum fatigue stress, f_r = static flexural stress), ranging from 0.90 to 0.70.

The load applied to the specimen was sensed by the load cell attached to the cross head of the machine, and the load cell output was used as a feedback signal to control the load applied by the actuator. The number of cycles to failure of the specimen at a particular stress level were noted from the cycle counter of the machine and recorded as its fatigue-life.

4 ANALYSIS OF FATIGUE TEST RESULTS

The fatigue life data, arranged in ascending order, obtained for SFRC with 1.0, 1.5 and 2.0% volume fractions containing fibres of mixed aspect ratio is summarized in Tables 3, 4 and 5 respectively.

4.1 Development of Family of S - N - P_f Curves

In this analysis, the graphical method similar to the one employed by McCall (1958) is used with a slight modification. As might be expected, the different specimens tested at a given stress level failed at different number of cycles. The data is analyzed by ranking the specimens in the order of the number of cycles to failure and the probability of failure P_f is calculated by dividing the rank of each specimen 'm' by (n + 1), where 'n' equals the total number of specimens tested at a particular stress level. The calculated values of probability of failure P_f are shown in Tables- 3, 4 and 5 for SFRC with 1.0, 1.5 and 2.0% fibre content respectively. The reason for dividing by (n + 1), rather than 'n' is to avoid obtaining a probability of failure equal to 1.0 for the specimen having greatest fatigue life (Kennedy & Neville 1986).

Figure-1 presents a family of S-N-P_f curves for fatigue life data for SFRC with 1.0% fibre content. The probability of failure P_f is plotted against the fatigue life N for each stress level as a first step. This plot is shown in the lower left part of the Fig.-1. This may be termed as a family of N-P_f curves. The next step consists of generating a family of S-N curves. Using the N-P_f curves obtained in the previous step, a family of S-N curves are obtained. This plot is shown in the upper right part of Fig.-1. From the S-N curves, S-P_f curves are plotted and these are shown in the upper left part of Fig.-1. Figures 2 and 3 represent the family of S-N-P_f curves obtained in the similar manner for fatigue life data of SFRC with 1.5 and 2.0% fibre content respectively.

Table 1: Concrete mix proportions, compressive and static flexural strength of plain concrete.

Water/Cement Ratio	Sand/Cement Ratio	Coarse Aggregate/Cement Ratio	28 Days Compressive Strength (MPa)	Static Flexural Strength (MPa)
0.35	1.35	2.12	57.82*	5.35**

* Average of 10 tests; ** Average of 12 tests.

Table 2: Steel fibrous concrete mixes, compressive and static flexural strength.

Fibre Volume Fraction (%)	Fibre Mix Proportion by Weight (%)		28 Days Average Compressive Strength (MPa)	Average Static Flexural Strength (MPa)
	50 mm Long Fibres ⁺	25 mm Long Fibres ⁺⁺		
1.0	50	50	62.89	7.45
1.5	50	50	65.85	8.44
2.0	50	50	65.42	8.92

⁺ 50 mm long, 2 mm wide and 0.6 mm thick; ⁺⁺ 25 mm long, 2 mm wide and 0.6 mm thick.

Table 3: Fatigue life data for SFRC at different stress levels, $V_f = 1.0\%$,

S.No.	Stress Level 'S'			Probability of Failure $P_f = m/(n+1)$
	0.80	0.85	0.90	
1	46803	6281	929	0.1000
2	50810	9283	1306	0.2000
3	84871	9320	1458	0.3000
4	92033	9946	1867	0.4000
5	106169	16387	2577	0.5000
6	130139	21273	3295	0.6000
7	204223	24494	3564	0.7000
8	231996	27488	3657	0.8000
9	317929	37321	5450	0.9000

Table 4: Fatigue life data for SFRC at different stress levels, $V_f = 1.5\%$,

S.No.	Stress Level 'S'			Probability of Failure $P_f = m/(n+1)$
	0.70	0.80	0.85	
1	96180	3778	390	0.1000
2	170168	6409	531	0.2000
3	182933	7015	800	0.3000
4	297931	7499	900	0.4000
5	318835	9200	1037	0.5000
6	395384	11334	1503	0.6000
7	608535	14468	1956	0.7000
8	745099	24319	2697	0.8000
9	1154990	30028	3225	0.9000

Table 5: Fatigue life data for SFRC at different stress levels, $V_f = 2.0\%$,

S.No.	Stress Level 'S'			Probability of Failure $P_f = m/(n+1)$
	0.70	0.80	0.85	
1	36347	845	174	0.1000
2	58703	937	203	0.2000
3	83573	1258	345	0.3000
4	92561	1596	403	0.4000
5	123188	2080	494	0.5000
6	146041	2361	624	0.6000
7	206339	4395	781	0.7000
8	236496	5103	832	0.8000
9	430098	6977	1439	0.9000

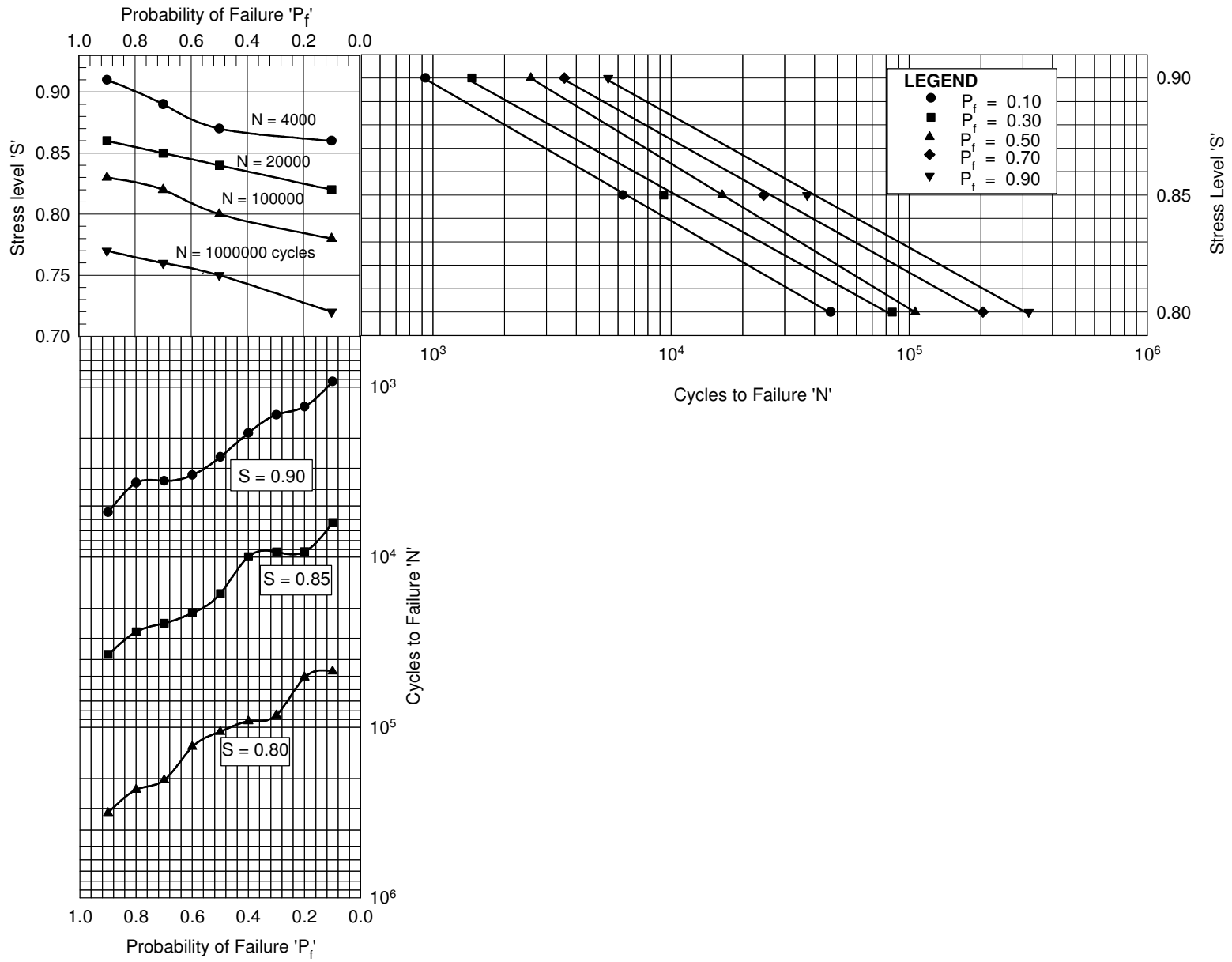


Figure 1. A-N-P_f diagram for SFRC under flexural loading, V_f= 1.0%

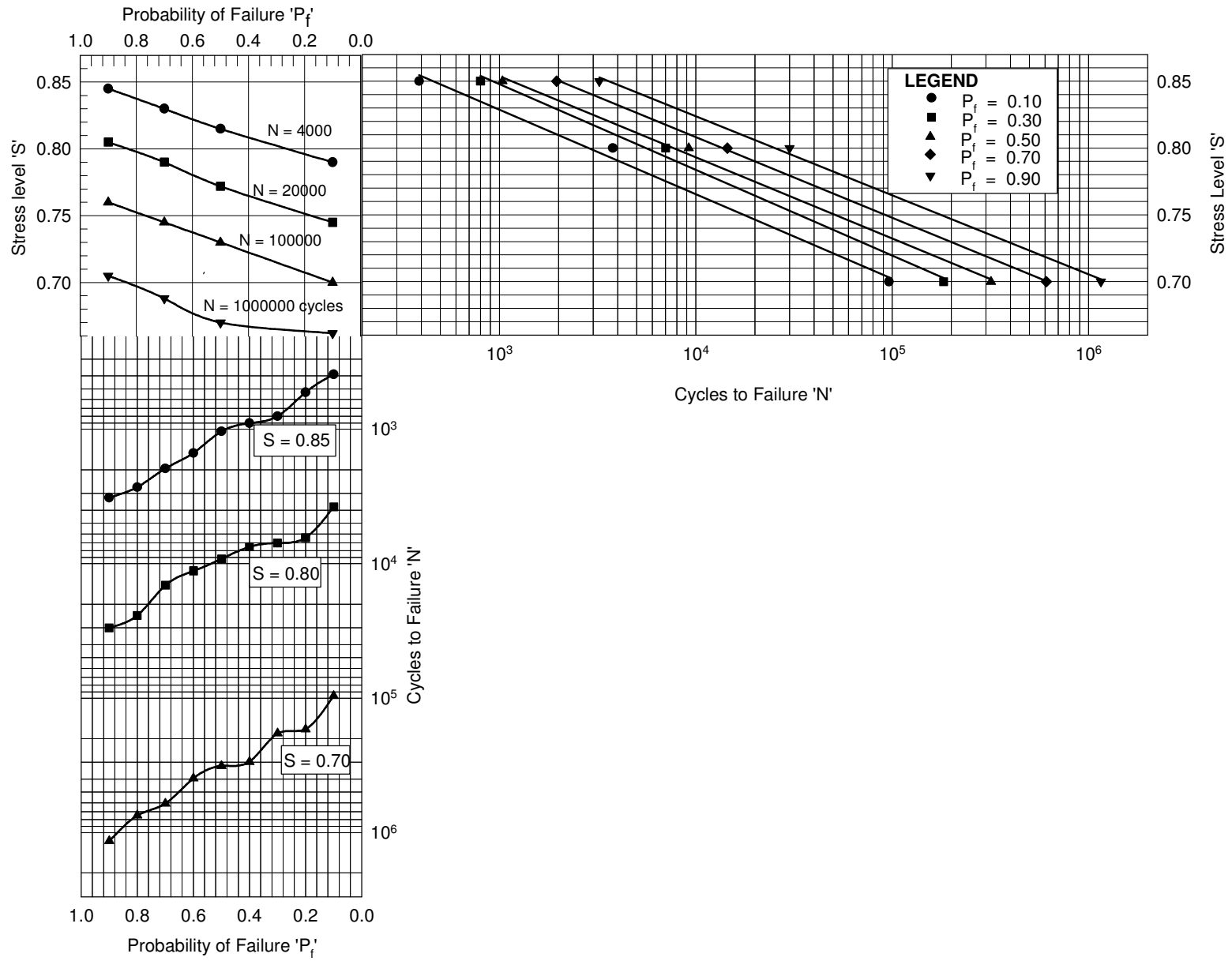


Figure 2. S-N-P_f diagram for SFRC under flexural loading, V_f = 1.5%

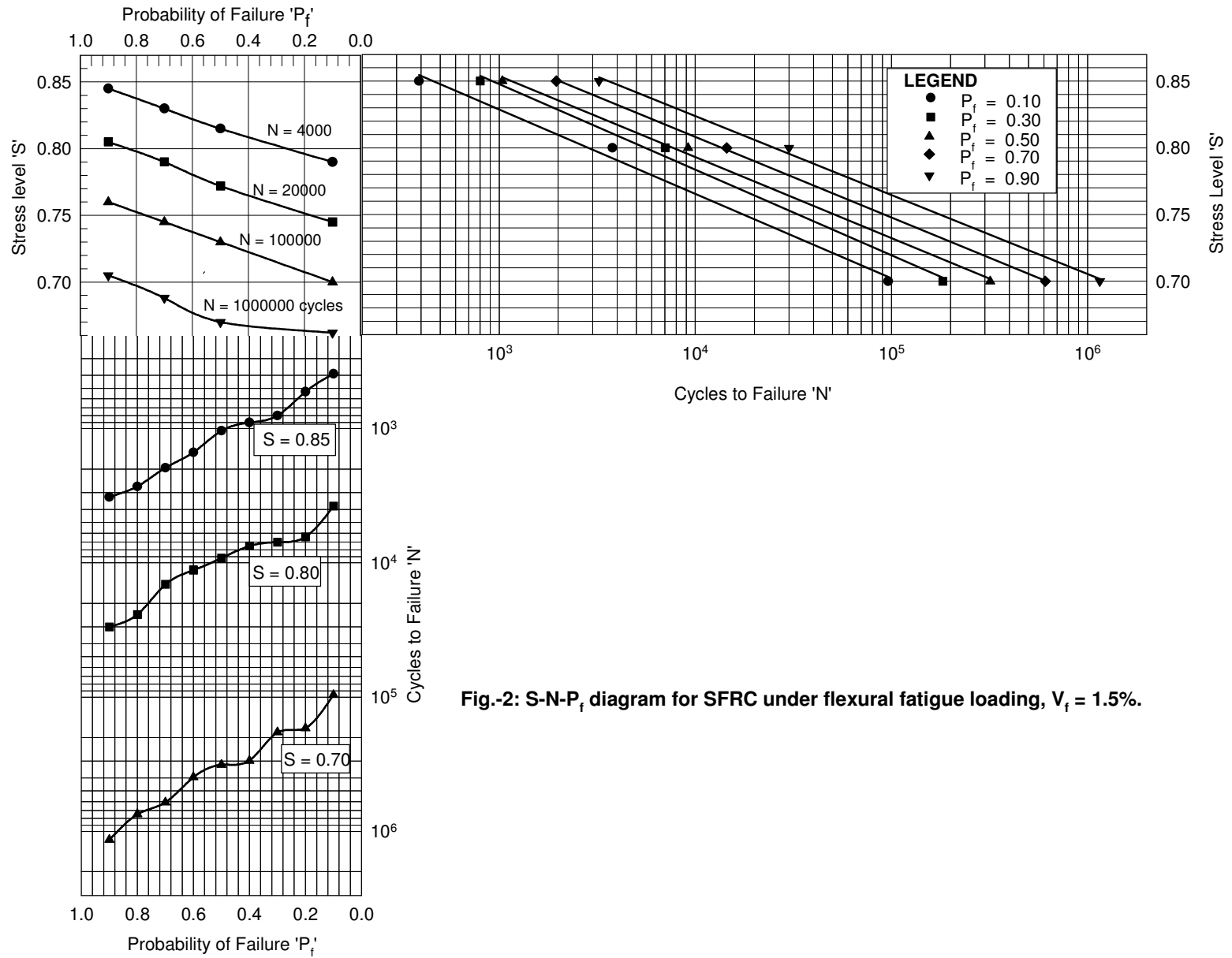


Fig.-2: S-N-P_f diagram for SFRC under flexural fatigue loading, V_f = 1.5%.

Figure 3. S-N-P_f diagram for SFRC under flexural loading, V_f = 1.5%

4.2 Development of Fatigue Equations

Another way to describe the S-N- P_f relationship is by using a mathematical relation. Following function used by McCall (1958) to describe the S-N- P_f relationship for plain concrete is adopted here to represent S-N- P_f relationships for steel fibre reinforced concrete:

$$L = (10)^{-a(S)^b (\log N)^c} \quad (1)$$

where a, b and c are the experimental constants, S is stress level and L is the survival probability. Survival probability L can be taken as equal to $1 - P_f$, where P_f is probability of failure.

To develop the equations for the proposed S-N- P_f relationships for SFRC containing different volume fractions of steel fibres, the experimental constants a, b and c are evaluated using fatigue life data obtained in this investigation. In this way, three different relationships i.e. one each for SFRC containing 1.0, 1.5 and 2.0% volume fraction of mixed fibres representing S-N- P_f curves will be obtained. These relationships are developed as explained in the following paragraphs:

Taking logarithms twice of the both sides of Eq.(1), one gets

$$\log(-\log L) = \log(a) + b \log(S) + c \log(\log N)$$

The following equation can be obtained after changing variables:

$$Y = A + bX + cZ \quad (2)$$

where $Y = \log(-\log L)$, $A = \log(a)$, $X = \log(S)$ and $Z = \log(\log N)$

Since it is required to determine Z from X and Y i.e. to determine fatigue life for a given stress level and certain survival probability, the Eq.(2) can be modified as follows:

$$Z = A' + B'X + C'Y \quad (3)$$

$$\text{in which } A' = -\frac{A}{c}, B' = -\frac{b}{c} \text{ and } C' = \frac{1}{c}$$

As it is convenient to work with the variables measured from their sample means than the variables themselves², the following relationship can be derived:

$$\sum Z = \sum A' + B' \sum X + C' \sum Y$$

$$\frac{1}{n} \sum Z = A' + B' \frac{\sum X}{n} + C' \frac{\sum Y}{n}$$

$$\bar{Z} = A' + B' \bar{X} + C' \bar{Y} \quad (4)$$

Subtracting Eq.(4) from Eq.(3), the following expression is obtained:

$$Z - \bar{Z} = B'(X - \bar{X}) + C'(Y - \bar{Y})$$

or

$$z = b'x + c'y \quad (5)$$

in which $z = Z - \bar{Z}$, $x = X - \bar{X}$ and $y = Y - \bar{Y}$

From the fatigue life data as obtained in this investigation for SFRC containing fibres of mixed aspect ratio, the following statistics are calculated:

For SFRC containing 1.0% volume fractions of fibres of mixed aspect ratio:-

$$\sum x^2 = 0.011778 \quad \sum y^2 = 4.393307$$

$$\sum z^2 = 0.162055$$

$$\sum xy = 0.000000 \quad \sum yz = 0.283854$$

$$\sum xz = -0.040920$$

$$\bar{X} = -0.0710 \quad \bar{Y} = -0.5750 \quad \bar{Z} = 0.6170$$

By means of least square normal equations, the following equations are obtained:

$$b' \sum x^2 + c' \sum xy = \sum xz \quad (6)$$

$$b' \sum xy + c' \sum y^2 = \sum yz \quad (7)$$

Using the statistics calculated above, the constants b' and c' in Eqs.(6) and (7) are determined for fatigue life of SFRC for and the following equation is obtained by substituting these in Eq.(5):

$$z = -3.4743x + 0.0646y$$

or in the other form

$$Z = 0.4075 - 3.4743X + 0.0646Y$$

The final equation for SFRC with 1.0% fibre content can be written as follows:

$$L = (10)^{-4.71 \times 10^{-7} (S)^{53.78} (\log N)^{15.48}} \quad (8)$$

Similarly, the fatigue life data of SFRC for other volume fractions of fibres i.e. 1.5 and 2.0% has been analysed and the corresponding equations which have been developed are summarized as follows:

For SFRC with 1.5% fibre content

$$L = (10)^{-4.70 \times 10^{-5} (S)^{37.60} (\log N)^{12.77}} \quad (9)$$

for SFRC with 2.0% fibre content

$$L = (10)^{-1.70 \times 10^{-3} (S)^{36.08} (\log N)^{10.97}} \quad (10)$$

The detailed calculations are not given here, the same can be obtained from the authors if required. Equations (8), (9) and (10) represents the family of S-N-P_f curves of SFRC for 1.0, 1.5 and 2.0% fibre content respectively and can be used to predict the flexural fatigue strength of steel fibre reinforced concrete for the desired level of survival probability or probability of failure.

5 CONCLUSION

Flexural fatigue life data obtained in this investigation for SFRC with 1.0, 1.5 and 2.0% fibre content, each containing fibres of mixed aspect ratio i.e. 50% 50 mm + 50% 25 mm long fibres has been analyzed in an attempt to determine the relationship among stress level S, fatigue life N and probability of failure P_f or survival probability L. A family of S-N-P_f curves has been generated for SFRC containing mixed fibres in different proportions. The equations have been developed to represent the S-N-P_f relationships mathematically and can be useful for predicting the flexural fatigue strength of steel fibre reinforced concrete for desired level of survival probability.

5.1 ACKNOWLEDGEMENT

The financial assistance provided to the second author by Indian Council for Cultural Relations, Government of India, New Delhi is gratefully acknowledged.

6 NOTATIONS

- S = stress level = f_{max}/f_r
 f_r = static flexural stress
 f_{max} = maximum fatigue stress
 f_{min} = minimum fatigue stress
L = survival probability
P_f = probability of Failure
N = fatigue-life or number of cycles to failure

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