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Theoretical Evaluation on Effects of Opening on Ultimate Load-carrying Capacity of Square Slabs

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ABSTRACT: Reinforced concrete slabs are the most common structural elements found in building construction. Despite the large number of slabs designed and built, the details of the elastic and plastic behaviour of slabs are not always appreciated or taken into account. Studies on slabs with openings are also scarce in the research on reinforced concrete elements. Therefore, a study on simply-supported and fixed-end, square slabs with opening at ultimate limit state using the yield line method was carried out and the results are presented herein. For simply-supported slabs, the analytical study on the ultimate load capacity of the slab shows that the ultimate total load decreases with the size of the opening. However, when the ultimate total load is converted to ultimate area load, the results show otherwise. In the study of fixed-end slabs, the results show that the opening has insignificant effect on the ultimate area load capacity for a small opening size of up to 0.3 times the slab dimension. For opening size of more than 0.5 times the slab dimension, the ultimate area load capacity increases drastically. The ultimate total load of a fixed-end slab with opening up to 0.3 times the slab dimension is also not affected by the opening. However, the ultimate total load increases drastically for opening size of 0.5 times or more of the slab dimension. All the results in this study are plotted in nomographs form and can be used as guidelines for the design of slabs with opening particularly in determining the ultimate load capacity.

KEYWORDS: Ultimate load, slab, slab opening, capacity

1 INTRODUCTION

Reinforced concrete slabs are the most common elements amongst structural elements in the construction of buildings. In these buildings, many pipes and ducts are necessary to accommodate essential services such as electricity, telephone, computer network, water supply, sewerage and air-conditioning. Due to the need for installing these pipes and ducts, slabs of buildings may need to provide openings for them to be interconnected.

Mansur & Tan (1999) had proposed analysis and design procedure for beams with circular and rectangular openings. The analytical model proposed is able to handle combined bending, shear and torsion in beams with openings, and subsequently design the reinforcements required for this combined action. However, the proposed analysis and design procedure are not applicable to reinforced concrete slabs.

Park & Gamble (2000) conducted a review on analysis of reinforced concrete slabs with openings and reported that an opening in a simply-supported square slab with dimension of 0.2 to 0.3 times of the slab dimension would cause a reduction of 11% in terms of ultimate load per unit area. Larger opening with dimension of 0.5 or more times the slab dimension would not result in reduction of ultimate load per unit area.

El-Salakawy et al. (1999) tested six full-scale reinforced concrete slabs, of which five were slabs with various arrangements of openings in the vicinity of the column. The openings in the prototypes were square with the sides parallel to the sides of the column; and there were two opening sizes, one which is the same size as the column and the other is 60% of the column size. It was reported that the larger and smaller opening sizes led to reduction in ultimate strengths of 30% and 12% respectively. Another full-scale testing on reinforced concrete slabs with openings was carried out by Teng et al. (2004). In this study, 20 slabs with openings supported on columns of various sizes were tested. This study distinguishes itself from the study carried out by El-Salakawy et al. (1999) by the arrangements of columns and openings. The slabs tested by Teng et al. (2004) had column support in the middle of the slabs whereas slabs tested by El-Salakawy et al. (1999) were having column support in the middle of the longer edges of the slab. It was reported by Teng et al. (2004) that openings reduce punching shear strength of slabs considerably, and the recommended locations for openings in slabs are along the longer side of a column.

In a separate study carried out by Tan & Zhao (2004), six prototype one-way reinforced concrete slabs with openings were tested. Five slabs were strengthened with externally bonded carbon fibre-reinforced polymer (CFRP) systems and the results were compared to the slab with opening without strengthening. It was found that the CFRP systems are effective in enhancing the load-carrying capacity and stiffness of the reinforced concrete slabs with openings.

From the studies as discussed above, opening in a slab reduces the load-carrying capacity. Therefore, a theoretical study on square slabs with opening was carried out to investigate the effect of openings on the ultimate load-carrying capacity. The analytical considerations and results are shown in the impending discussion.

2 YIELD LINE ANALYSIS

2.1 *Simply-supported slab*

Figure 1(a) shows a simply-supported square slab with an opening at the centre. The slab is assumed to have similar amount of reinforcement in both the longitudinal and transverse directions.

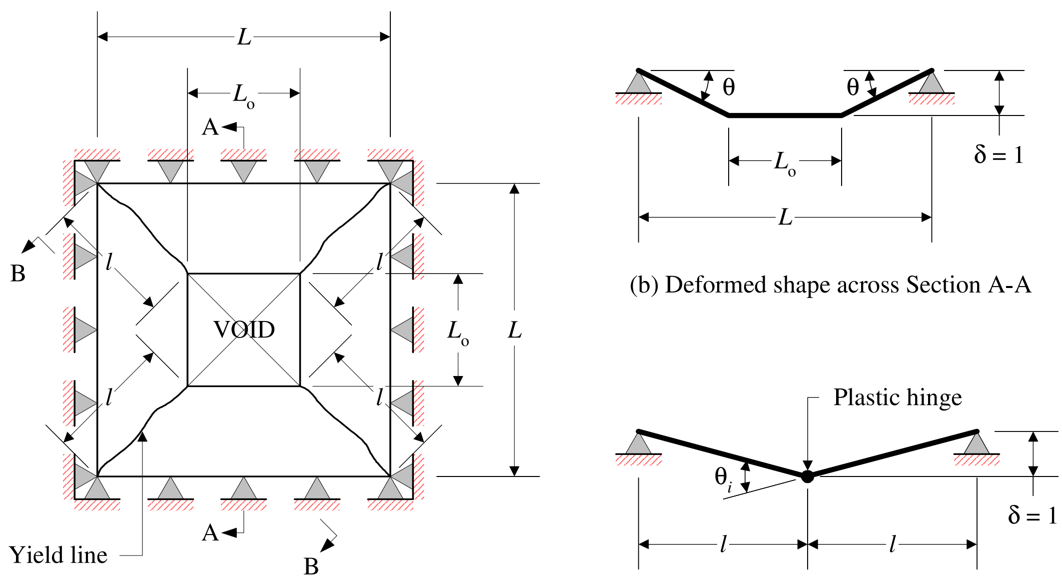


Figure 1. Yield lines and deformations of a simply-supported square slab with an opening at the centre.

At ultimate limit state, each yield line was assumed to stretch from the corner of the opening to the corner of the slab. When a virtual displacement defined by a unit value under uniform area load of w_o is applied to the corner of the opening as shown in Figure 1(b), the external work done by the load is

$$
\begin{aligned}
W_e &= 4\left\{ \begin{array}{l} \left[w_o \times L_o \times \dfrac{(L-L_o)}{2} \times \dfrac{1}{2} \right] \\ + \left[w_o \times \dfrac{1}{2} \times (L-L_o) \times \dfrac{(L-L_o)}{2} \times \dfrac{1}{3} \right] \end{array} \right\} \\
&= w_o \left[L_o(L-L_o) + \frac{(L-L_o)^2}{3} \right] \\
&= w_o \left[\frac{(L-L_o)(L+2L_o)}{3} \right]
\end{aligned}
\tag{1}
$$

in which L = dimension of the square slab; and L_o = dimension of the opening.

Defining moment m as the moment of resistance per unit length along the yield lines, the total internal work done by the moment of resistance along the yield lines is

$$W_i = 4ml\theta_i \tag{2}$$

where l = length of the yield lines; and θ_i = rotation along the yield lines given as [see Figure 1(c)]

$$\theta_i = \frac{1}{l} + \frac{1}{l} = \frac{2}{l} \tag{3}$$

Substituting Equation (3) into Equation (2) and equating the external work done in Equation (1) to the internal work done in Equation (2),

$$w_o = \frac{8m}{\left[\dfrac{(L-L_o)(L+2L_o)}{3}\right]} = \frac{24m}{(L-L_o)(L+2L_o)} \tag{4}$$

For a slab which has similar amount of reinforcement provided in both x-axis (longitudinal) and y-axis (transverse) direction, the moment of resistance per unit length along the x-axis m_x is the same as the moment of resistance per unit length along the y-axis m_y. This type of slab is known as isotropically reinforced slab (Nilson 1997) in which the unit moment m along the yield lines is the same as m_x or m_y.

Defining w_u as the ultimate area load for a slab without an opening, that is, $L_o = 0$, Equation (4) gives

$$w_u = \frac{24m}{L^2} \tag{5}$$

Hence, the ratio of ultimate area load for a slab with an opening to the ultimate area load for a slab without an opening can be obtained by dividing Equation (4) by Equation (5), that is,

$$\frac{w_o}{w_u} = \frac{L^2}{(L-L_o)(L+2L_o)} \tag{6}$$

For the study of total load, P_o is defined as the total ultimate load on the slab with an opening given as

$$P_o = w_o \times (L^2 - L_o^2) = \frac{24m(L^2-L_o^2)}{(L-L_o)(L+2L_o)} = \frac{24m(L+L_o)}{(L+2L_o)} \tag{7}$$

Similarly, for the total ultimate load on the slab without an opening P_u, it is given by

$$P_u = w_u \times L^2 = 24m \tag{8}$$

Hence, the ratio of total ultimate load for a slab with an opening to the total ultimate load for a slab without an opening can be obtained by dividing Equation (7) by Equation (8), that is,

$$\frac{P_o}{P_u} = \frac{L+L_o}{L+2L_o} \tag{9}$$

2.2 *Fixed-end slab*

Figure 2 shows a fixed-end slab with an opening at the centre. The external work done by a uniform area load of w_o is the same as the one in simply-supported slab given in Equation (1). Defining n as the ratio of the moment of resistance per unit length at the fixed end to the moment of resistance per unit length along the yield line extending from the corner of the slab to the corner of the opening, the total internal work done by the moment of resistance along the yield lines is (see Figure 2)

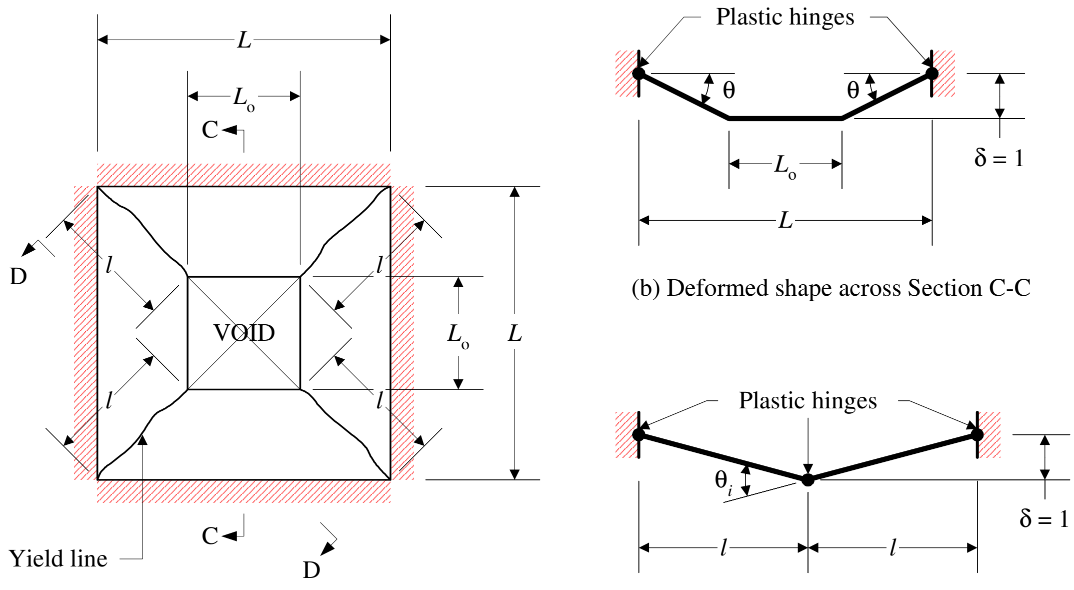
$$W_i = 4ml\theta_i + 4nmL\theta \tag{10}$$

in which l and θ_i are the same as in Equation (3); and θ = rotation at the fixed-end given as [see Figure 2(b)]

$$\theta = \frac{1}{\left(\dfrac{L-L_o}{2}\right)} = \frac{2}{L-L_o} \tag{11}$$

Substituting Equation (3) and Equation (11) into Equation (10) gives

$$W_i = 8m + 8nm\left(\frac{L}{L-L_o}\right) = 8m\left(\frac{L-L_o+nL}{L-L_o}\right) \tag{12}$$



(a) Yield lines

(b) Deformed shape across Section C-C

(c) Deformed shape across Section D-D

Figure 2. Yield lines and deformations of a fixed-end square slab with an opening at the centre.

Equating the external work done, W_e, in Equation (1) and the internal work done, W_i, in Equation (12), and solve for w_o,

$$w_o = \frac{24m(L - L_o + nL)}{(L - L_o)^2 (L + 2L_o)} \tag{13}$$

For a fixed-end slab without opening, that is, L_o = 0, the ultimate area load is given as

$$\begin{aligned} w_u &= \frac{24m(L + nL)}{L^3} \\ &= \frac{24m(1+n)}{L^2} \end{aligned} \tag{14}$$

Hence, the ratio of w_o to w_u is given as

$$\frac{w_o}{w_u} = \frac{L^2 (L - L_o + nL)}{(L - L_o)^2 (L + 2L_o)(1+n)} \tag{15}$$

It is noted that Equation (15) reduces to Equation (6) for the case of simply-supported slab, that is, when n = 0.

In order to obtain the ratio of P_o to P_u, similar procedure for the simply-supported slab can be used. However, it can also be conveniently assessed using Equation (15), which gives

$$\begin{aligned} \frac{P_o}{P_u} &= \frac{w_o \times (L^2 - L_o^2)}{w_u \times L^2} = \frac{L^2 (L - L_o + nL)(L - L_o)(L + L_o)}{L^2 (L - L_o)^2 (L + 2L_o)(1+n)} \\ &= \frac{(L + L_o)(L - L_o + nL)}{(L - L_o)(L + 2L_o)(1+n)} \end{aligned} \tag{16}$$

It is noted that Equation (16) reduces to Equation (9) for n = 0, that is, a simply-supported slab.

3 RESULTS AND DISCUSSION

3.1 *Simply-supported slab*

Figure 3 shows the results of the effect of size of opening on the ultimate area load capacity on the slabs. It is plotted using Equation (6) as derived in the preceding section. There are three slab sizes considered in the analysis, that is, L = 3 m, L = 4 m, and L = 5 m. It is however shown in Figure 3 that the slab size has no effect on the results obtained in which all three curves coincide with each other. Therefore, there is no size effect when the parameters are normalised and hence this effect is not discussed further in the impending sections.

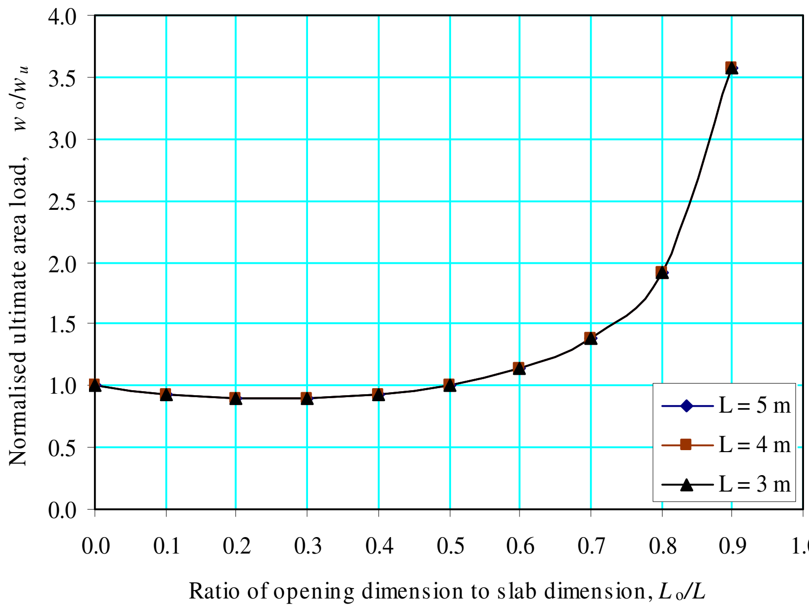


Figure 3. Effect of size of opening on ultimate area load of simply-supported square slabs.

It is indicated in Figure 3 that the ultimate area load capacity decreases with the size of opening for openings in between 0 to 0.5 times the slab dimension. The maximum decrease is at openings of 0.2 and 0.3 times the slab dimension, which is 11%, in agreement with the findings presented in the review by Park & Gamble (2000). When the opening size is increased beyond 0.5 times the slab dimension, there is an exponential increase in ultimate area load capacity in the slabs, with the maximum increase of 257% at an opening equal to 0.9 times the slab dimension.

The decrease in ultimate area load capacity with the increase in opening size up to 0.5 times the slab dimension is due to the fact that the load is still distributed near to the centre of the slab. Under this circumstance, the external work done by the load is still quite significant with one unit deflection imposed upon the slab at the edges of the opening. Therefore, a smaller area load is required to do the work in order to balance out the internal work done by the resisting moment along the yield lines. As the opening size increases beyond 0.5 times the slab dimension, the load is concentrated towards the supporting edges of the slab. Under this circumstance, the external work done by the load is quite minimal under a unit deflection imposed at the edges of the opening, resulting in more area load required to do the work in order to achieve equilibrium with the internal work done by the resisting moment along the yield lines.

Figure 4 shows the results of the effect of opening in simply-supported slab on the ultimate total load plotted using Equation (9). Similar to Figure 3, there is no effect of slab size as all three curves coincide with each other. Therefore, this effect is not discussed further.

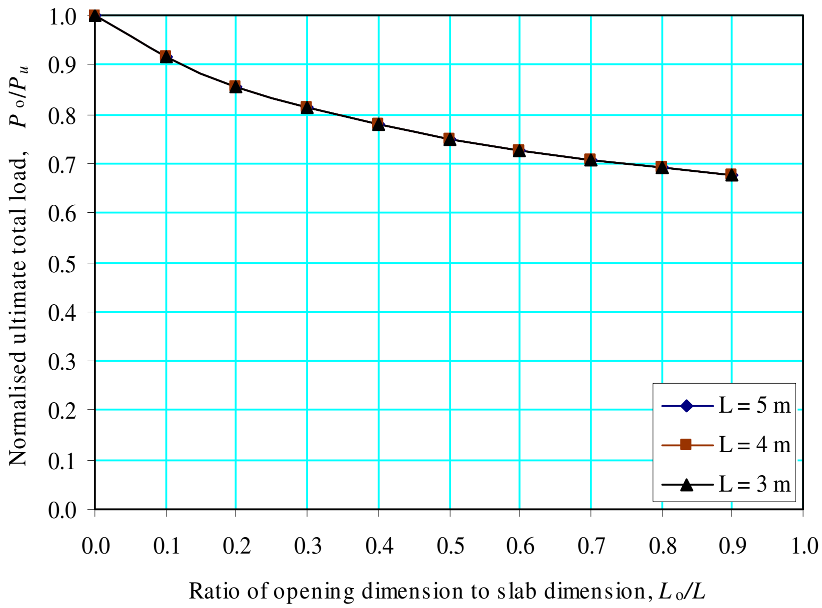


Figure 4. Effect of size of opening on ultimate total load of simply-supported square slabs.

It can be seen in Figure 4 that the ultimate total load capacity of the slab is decreasing with the increase in opening size. At an opening of 0.5 times the slab dimension, the reduction in ultimate total load is 25%. There is no increase in load carrying capacity as seen in Figure 3 for larger opening size, but instead, there is a continuous reduction in ultimate total load for larger opening with up to 32% reduction for an opening of 0.9 times the slab dimension.

It is noted that the opening of a slab in practice is usually 0.3 times the slab dimension or less. Therefore, most slabs with opening will be subjected to reduction in ultimate area load of up to 11% and a reduction in ultimate total load of 19%.

3.2 *Fixed-end slab*

Figure 5 shows the effect of opening size on the ultimate area load of fixed-end square slab. Each ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening, n, is plotted separately.

In Figure 5, it can be seen that when the opening size is less that 0.3 times the slab dimension for the case of n value of 0.5, there is a decrease in ultimate area load capacity with the maximum decrease of 4% occurring at the ratio of opening size to slab dimension, L_o/L, of 0.1. For slabs with n value of 1, the maximum decrease in ultimate area load capacity of 2% also occurs at L_o/L of 0.1, and there is no decrease in ultimate area load capacity when the L_o/L value is more than 0.2. For slabs with higher n values, there is a decrease in the ultimate area load capacity for L_o/L of less than 0.2, where the maximum decrease is about 1% at L_o/L of 0.1.

Beyond the demarcating points where the ultimate area load capacity decreases, that is, L_o/L of 0.3 for n value or 0.5 and L_o/L of 0.2 for higher n values, the ultimate area load capacity increases exponentially with the increase in ratio of opening size to slab dimension, L_o/L, particularly for L_o/L of more than 0.5. This is due to the fact that for larger ratio of opening size to slab dimension, the load is more concentrated at the area of the slab in which the deflection is smaller, therefore there is an increase in area load to create enough external work to balance the internal work done by the yield lines extending from the corner of slab to corner of the opening as well as those yield lines at the fixed ends.

There is a drastic increase in normalised ultimate area load, w_o/w_u, at L_o/L of 0.8 to 0.9, with the maximum recorded w_o/w_u of 25, for n of 2 and L_o/L of 0.9, showing an increase of 2400%. This large opening leaves a small area near the fixed ends for loading, leading to a very high stiffness of the slab. As a result, the load-carrying capacity of the slab becomes very high, even more so for higher n value.

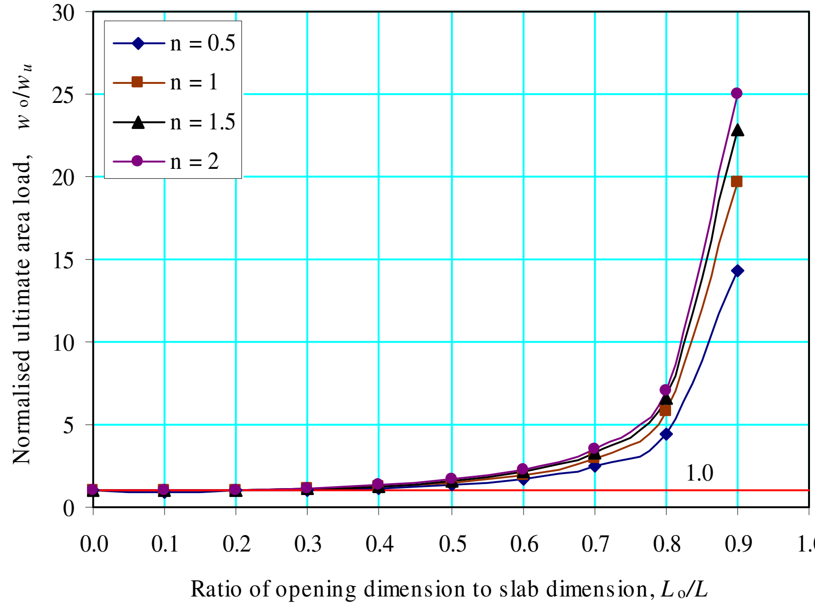


Figure 5. Effect of size of opening on ultimate area load of fixed-end square slabs.

Figure 6 shows the effect of opening size on the ultimate total load of fixed-end square slab with different ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening, n. For the slab with n value of 0.5, the normalised ultimate total load, P_o/P_u, decreases for opening size of 0.5 times of slab dimension or less, with the maximum decrease of 7% at L_o/L of 0.2 and 0.3. As for the slab with n value of 1, P_o/P_u decreases for L_o/L of 0.2 or less, with maximum decrease of 4% at L_o/L of 0.1. In the slabs of higher n values, there is no significant decrease in normalised ultimate total load for opening size of 0.3 times of slab dimension or less.

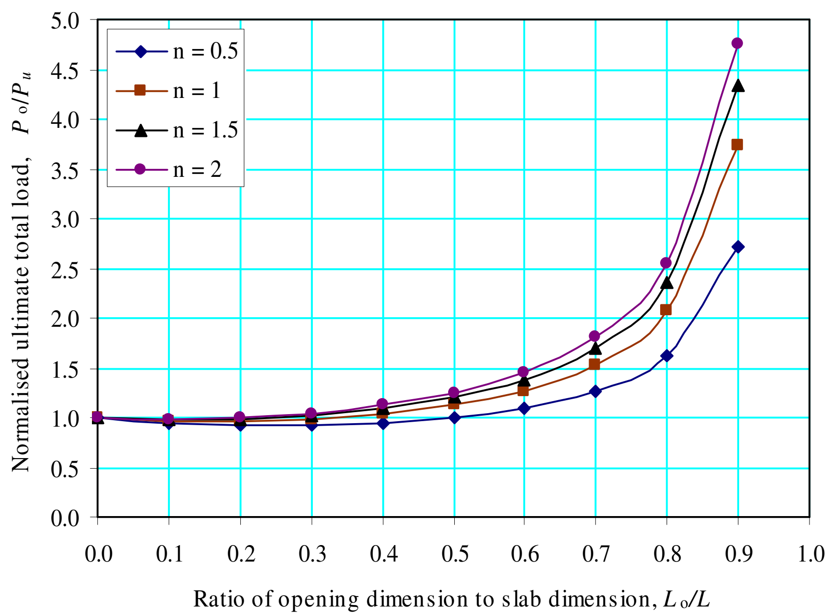


Figure 6. Effect of size of opening on ultimate total load of fixed-end square slabs.

Except for the slab with n value of 0.5, all the other slabs with higher n value show increase in ultimate total load over the slab without an opening for L_o/L of more than 0.4, that is, the P_o/P_u values are more than 1. The P_o/P_u values increase drastically for L_o/L of more than 0.6, with the maximum recorded P_o/P_u value of 4.75 for n of 2 and L_o/L of 0.9, showing an increase of 375%. This is then again due to the large opening resulting in load concentration near the support, which leads to very high stiffness of the slab.

On the note that most openings in slabs are generally small, that is, L_o/L is normally 0.3 or less, the results in Figures 5 and 6 may be summurised as that opening in a fixed-end square slab reduces the ultimate load capacity. The reduction in ultimate area load capacity is up to 4% and the reduction in ultimate total load capacity is up to 7%.

4 CONCLUSIONS

A study on the effect of opening on the load carrying capacity of simply-supported and fixed-end slabs was presented and discussed. In simply-supported slabs, it was found that the ultimate area load capacity decreases with the increase in size of opening up to 0.5 times the slab dimension. After which the ultimate area load capacity increases with the opening size and registered a maximum increase of 257% at an opening size of 0.9 times the slab dimension. However, when the ultimate area load capacity is converted to ultimate total load capacity, there is a continuous decrease in ultimate total load capacity with the increase in opening size. The maximum decrease is 32% with an opening size of 0.9 times the slab dimension in this study.

The results of the fixed-end slabs show that the opening reduces the ultimate area load capacity for a small opening of 0.1 times of the slab dimension, with the maximum reduction of 4% in the slab with the ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening of 0.5. When the opening size is more than 0.5 times the slab dimension, the ultimate area load capacity increases exponentially with the maximum recorded increase of 2400% at an opening size of 0.9 times the slab dimension for a slab where the ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening of 2. The ultimate total load of a fixed-end slab decreases with small opening size of up to 0.3 times the slab dimension. The maximum reduction is 7% at opening sizes of 0.2 and 0.3 times the slab dimension in the slab with the ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening of 0.5. However, the ultimate total load increases drastically for opening size of 0.5 times or more of the slab dimension, with the maximum recorded increase of 375% at an opening size of 0.9 times the slab dimension for a slab where the ratio of moment of resistance at fixed-end yield line to moment of resistance at yield line at corner of slab to corner of opening of 2.

Most slabs with opening have small opening size of up to 0.3 times the slab dimension. Therefore, a simply-supported slab may have a reduction in ultimate area load of up to 11% and a reduction of ultimate total load of up to 19%. A fixed-end slab has less significant reduction in both ultimate area load and ultimate total load capacities of 4% and 7%, respectively, for a small opening of up to 0.3 times the slab dimension.

The charts were presented in normalised load capacity and opening size. Therefore, they can be used as guidelines for predicting the load capacity of simply-supported and fixed-end slabs with openings.

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