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Finite Element Micro-Modeling of Infilled Frames

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ABSTRACT: In this paper, a realistic criterion is proposed to describe the frame-infill separation, in order to better simulate the complicated behavior of infilled frames under lateral loads. The basic characteristic of this analysis is that the infill/frame contact lengths and the contact stresses are estimated as an integral part of the solution, and are not assumed in an ad-hoc way. In order to implement the method, a specific computer program for the analysis of infilled plane frames, under lateral loads, has been developed. Using this method, the response of a single-bay single-story masonry infilled R.C frame, under a lateral load in the beam level, has been investigated. The large magnitude of the variation of the contact lengths between the infill and the different frame members is clearly shown.

KEYWORDS: Infilled Frame, contact stress, masonry frame

1 INTRODUCTION

In many countries, situated in seismic regions, reinforced concrete frames are infilled by brick masonry panels. Although the infill panels significantly enhance both the stiffness and strength of the frame, their contribution is often not considered mainly because of the lack of knowledge of the composite behavior of the frame and the infill. However, extensive experimental (Smith 1966; Smith and Carter 1969; Page et al 1985; Mehrabi et al 1996; Buonopane and White 1999; Santhi, Knight and Muthumani 2005), and semi-analytical investigations (Liauw and Kwan 1984; Dhanasekar and Page 1986; Saneinejad and Hobbs 1995; Asteris 2003; Moghaddam 2004) have been made. Recently, it has been shown that there is a strong interaction between the infill masonry wall and the surrounding frame, leading to:

- The behavior of the composite frame not only depending on the relative stiffness of the frame and the infill and the frame geometry, but also critically influenced by the strength properties of the masonry.
- Considerable increase of the overall stiffness and of the in plane moment of inertia of the composite frame, as well as an increase of dissipated energy.
- Redistribution of action-effects and, sometimes, unpredictable damages along the frame (it has been also found that present code formula overestimates the shear forces along the height of the frame since it does not consider the effect of infill panels (Santhi, Knight and Muthumani 2005).
- Considerable reduction of the probability of collapse, even in cases of defective infilled frames, when they are properly designed.

Approximately 80% of the cost of damages of structures from earthquakes is due to damage of the infill walls and to consequent damages of doors, windows, electrical and hydraulic installations (Tiedeman 1980). Inspite of its broad application and its economical significance, this structural system has resisted analytical modeling; the following reasons may explain this situation:

- Computational complexity: The particulate infill material and the ever-changing contact conditions along its interface to concrete constitute additional sources of analytical burden. The real composite behavior of an infilled frame is a complex statically indeterminate problem according to Smith (Smith 1966).
- Structural uncertainties: The mechanical properties of masonry, as well as its wedging conditions against the internal surface of the frame, depend strongly on local construction conditions.
- The non-linear behavior of infilled frames depends on the separation of masonry infill panel from the surrounding frame.

The current study aims to present a simple method of simulating the complicated behavior of infilled frames under lateral loads. The basic characteristic of this analysis is that the infill/frame contact lengths and the contact stresses are estimated as an integral part of the solution, and are not assumed in an ad-hoc way. Using this method, the response of one-story one-bay infilled frame under lateral static load in the beam level is studied. The method can be used to produce design aids/rules of such composite structural systems.

2 REVIEW OF INFILLED FRAMES NUMERICAL MODELS

Attempts at the analysis of infilled frames since the mid-1950s have yielded several analytical models. For a better understanding of the approach and capabilities of each model it may be convenient to classify them into macro- and micro- models based on their complexity, the detail by which they model an infill wall, and the information they provide to the analyst about the behavior of a structure. A basic characteristic of a macro- (or simplified) model is that they try to encompass the overall (global) behavior of a structural element without modeling all the possible modes of local failure. Micro- (or fundamental) models, on the other hand, model the behavior of a structural element with great detail trying to encompass all the possible modes of failure. The following sections constitute a brief review of the most representative macro- and micro-models.

2.1 *Macro-Models*

Since the first attempts to model the response of the composite infilled frames structures, experimental and conceptual observations have indicated that a diagonal strut with appropriate geometrical and mechanical characteristics could possibly provide a solution to the problem. In 1958, Polyakov (Polyakov 1960) suggested the possibility of considering the effect of the infilling in each panel as equivalent to diagonal bracing and this suggestion was later taken up by Holmes (Holmes 1961) who replaced the infill by an equivalent pin-jointed diagonal strut made of the same material and having the same thickness as the infill panel and a width equal to one third of the infill diagonal length (Figure 1). The ‘one-third’ rule was suggested as being applicable irrespective of the relative stiffnesses of the frame and the infill. Stafford Smith (Smith 1966) and Stafford Smith and Carter (Smith and Carter 1969) related the width of the equivalent diagonal strut to the infill/frame contact lengths using an analytical equation which has been adapted from the equation of the length of contact of a free beam on an elastic foundation subjected to a concentrated load (Hetenyi 1946). Based on the frame/infill contact length, alternative proposals for the evaluation of the equivalent strut width have been given by Mainstone (Mainstone 1971) and Kadir (Kadir 1974).

Stafford Smith and Carter (Smith and Carter 1969), and Mainstone (Mainstone 1971) used the equivalent strut approach to simulate infill wall in steel frames and study the behavior of infilled structures subjected to monotonic loading. They also developed equations by which the properties of these struts, such as initial stiffness and ultimate strength, were calculated. This approach proved to be the most popular over the years because of the ease with which it can be applied.

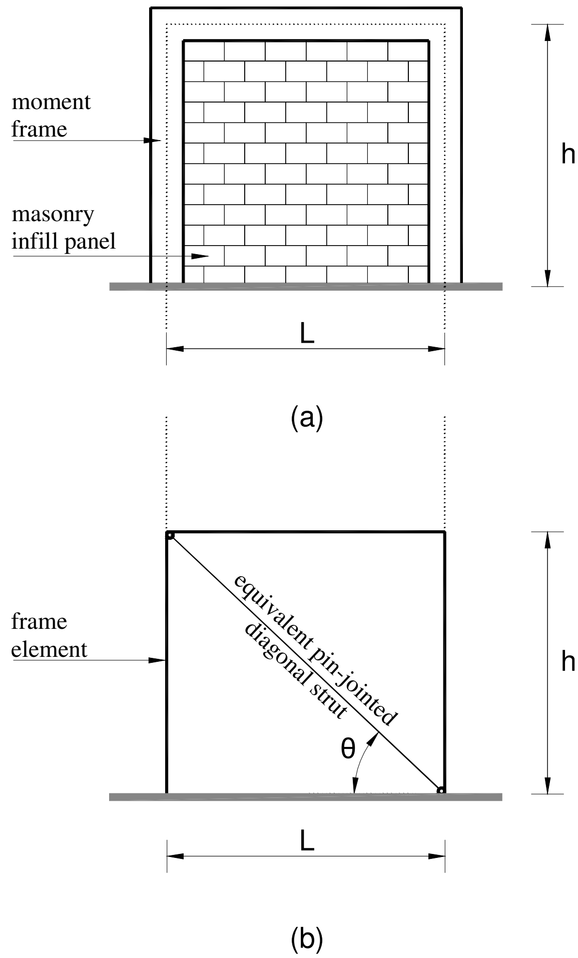

Figure 1. Equivalent strut model for masonry infill panel in frame structures: (a) Infilled frame structure; (b) Infilled frame model.

In the last two decades it became clear that one single strut element is unable to model the complex behavior of the infilled frames. As reported by many researchers (Reflak and Faijfar 1991; Saneinejad and Hobbs 1995; Buonopane and White 1999), the bending moments and shearing forces in the frame members cannot be replicated using a single diagonal strut connecting the two loaded corners. More complex macro-models were then proposed, but they were still usually based on a number of diagonal struts.

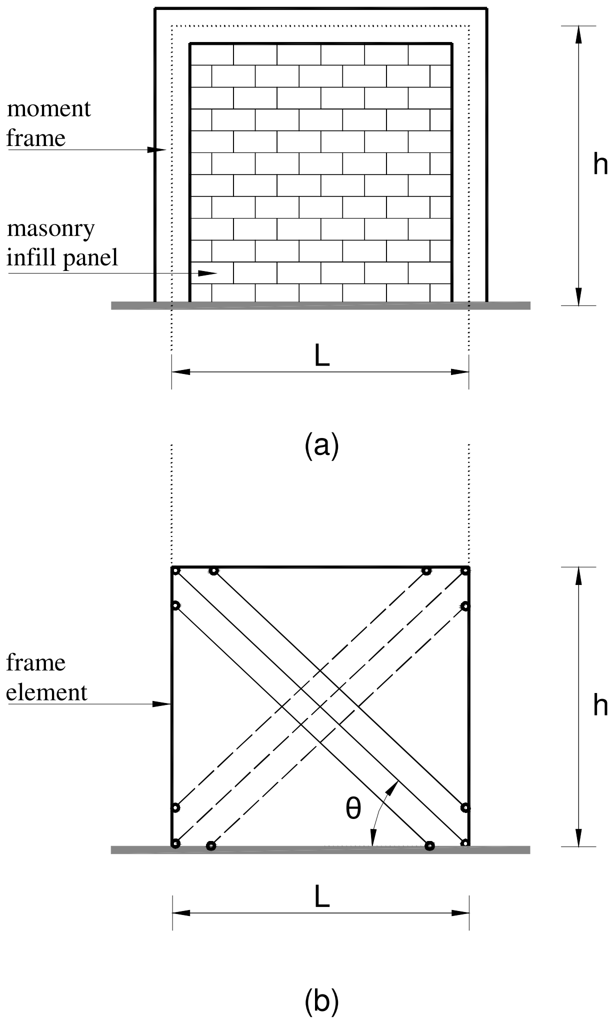

Figure 2. Six-strut model for masonry infill panel in frame structures: (a) Infilled frame structure; (b) Infilled frame model.

Chrysostomou (Chrysostomou 1991) had the objective of simulating the response of infilled frames under earthquake loading by taking into account stiffness and strength degradation of the infills. They proposed to model each infill panel by six compression-only inclined struts (Figure 2). Three parallel struts are used in each diagonal direction and the off-diagonal ones are positioned at critical locations along the frame members. At any point during the analysis of the non-linear response only three of the six struts are active, and the struts are switched to the opposite direction whenever their compressive force reduces to zero. The advantage of this strut configuration over the single diagonal strut is that it allows the modeling of the interaction between the infill and the surrounding frame.

2.2 *Micro-Models*

All models described in this section are based on the Finite Element Method, using three different kinds of elements to represent the behavior of infilled frames subjected to lateral loading. According to these models the frame is constituted by plane or beam element, the infill by plane elements, and the interface behavior by interface elements or by one-dimensional joint elements.

Mallick and Severn (Mallick and Severn 1967), and Mallick and Garg (Mallick and Garg 1971) suggested the first finite element approach to analyze infilled frames, addressing the problem of an appropriate representation of the interface conditions between frame and infill. The infill panels were simulated by means of linear elastic rectangular finite elements, with two degrees of freedom at each four nodes, and the frame was simulated by beam element ignoring axial deformation. This was a consequence of the assumption that the interaction forces between the frame and the infill along their interface consisted only of normal forces. In this model, the slip between the frame and the infill was also taken in account, considering frictional shear forces in the contact regions. Several single story rectangular infilled frames under static loading were analyzed and the results were in a good agreement with experimental results if the height to span ratio was not greater than two.

Liauw and Kwan (Liauw and Kwan 1984) used three different types of elements to study the behavior of infilled frames subjected to monotonic loading. The infill-frame interface was modeled by simple bar type elements capable of simulating both separation and slip. The infill panel was modeled by triangular plane stress elements. In tension, the material was idealized as a linear elastic brittle material. Before cracking, the material was assumed to be isotropic and after cracking was assumed to become anisotropic due to the presence of the crack. It was assumed that for an open crack the Young's modulus perpendicular to the crack and the shear modulus parallel to the crack were zero. When the crack was closed, the Young's modulus was restored, and the shear force is assumed to be taken over by friction. In compression, the panel was assumed to exhibit extensive nonlinearity in the stress-strain relationship. Although the material was subjected to bi-axial stress, it was assumed that the panel was under uni-axial stress based on experimental results, which show that one of the principal stresses is much smaller than the other. Using an iterative procedure with incremental displacement, several four-story one-bay model frames infilled with micro-concrete were analyzed. Close agreement between experimental and analytical results has been observed.

Dhanasekar and Page (Dhanasekar and Page 1986), using one-dimensional joint elements to model the mortar joint between the infill and the frame, have shown that the behavior of the composite frame not only depends on the relative stiffness of the frame and the infill and the frame geometry, but is also critically influenced by the strength properties of the masonry (in particular, the magnitude of the shear and tensile bond strengths relative to the compressive strength).

A simpler and much quicker finite element technique (Axley and Bertero 1979) consist in reducing, by condensation, the stiffness of the infill to the boundary degrees of freedom. It is assumed that the frame constrains the form (but not the degree) of deformation on the infill. Separate stiffnesses are formed. A constraint relation is assumed between the 12 frame degrees of freedom (DOF) and the boundary degrees of freedom. Thus, a congruent transformation of the separate systems to a composite approximate frame-infill system (with only 12 DOF) is possible.

The brief review of existing analytical models for the analysis of brick masonry infilled frames, which has been presented above, suggests that a computational procedure must be sought along the following directions:

- The infill/frame contact lengths and the contact stresses should be considered as an integral part of the solution, and are not assumed in an ad-hoc way.
- New finite element and associated computer code to model the anisotropic behavior of brick masonry infill panel must be developed, since none of the available ones appear to be able to take into account this anisotropic and composite behavior.

In the present paper, these improvements are incorporated in a method, which makes use of a new finite element technique.

3 PROPOSED METHOD OF ANALYSIS

To overcome the problem of the ever-changing contact conditions between the brick masonry infill and the surrounding frame, our analysis utilizes a new finite element technique for the modeling of infilled frames. The basic characteristic of the analysis is that the infill/frame contact lengths and the contact stresses are estimated as an integral part of the solution, and are not assumed in an ad-hoc way. Especially, the current study aims to calculate the infill/frame contact lengths for the case of unidirectional lateral loading and elastic response of the infill. This approach enhances the knowledge of the elastic response of composite structure, which is considered to be critical for a thorough understanding of its response under reversed cyclic loading. The adopted method is an extension of the Method of Contact Points, which has been previously developed (Asteris 1996 & 2003). In the present paper, this method is firmly put on a more rigorous quantitative basis, extended and calibrated by systematically comparing its results to experimental data. In order to implement the method, the following are required:

- A criterion for the separation of masonry panel from the surrounding frame.
- A finite element to model the in-plane anisotropic behavior of masonry infill panel.
- A finite element computer program to implement this procedure.

3.1 *Criterion for the Frame-Infill Separation*

In order to model the complicated behaviour of the infilled plane frames under lateral load, a realistic criterion, in terms of physical meaning, is used to describe the frame-infill separation. The main goal of this criterion is to describe the evolution of the natural response of these composite structures as a boundary condition problem. The objective of the present study is to find a valid geometrical equilibrium condition for the composite structure of the infilled frame under certain loading conditions, given that the real overall behaviour of an infilled frame is a complex, statically indeterminate problem. The analysis has been performed on a step-by-step basis based on the following:

The major "physical" boundary condition between the infill and the frame is that the infill panel cannot get into the surrounding frame; the only accepted "natural" conditions between the infill and the frame are either the contact or the separation condition.

The frame, while directly carrying some of the lateral loads, serves primarily to transfer and distribute the bulk of the loads to the infill. The stiffness response of the infill is influenced, to a considerable extent, by the way in which the frame distributes the load to it. Simultaneously, the frame's contribution to the overall stiffness is affected by the change in its mode of distortion, as a result of the reaction of the infill.

The proposed finite element procedure can be summarized as follows:

Step 1. Initially, the finite elements of the infill are considered to be linked to the elements of the surrounding frame, at two corner points (only), at the ends of the compressed diagonal of the infill. (When the load is applied, the infill and the frame are getting separated over a large part of the length of each side, and contact remains only adjacent to the corners at the ends of the compression diagonal).

Step 2. Compute the nodal forces, displacements, and the stresses at the Gauss points of the elements.
Step 3. Check whether the infill model points overlap the surrounding frame finite elements. If the answer is negative, step 5 of the procedure is followed.
Step 4. If the infill model points overlap the surrounding frame elements, the neighbouring to the previous linked points are linked, and the procedure is repeated from step 2.
Step 5. This final step is a further check on the acceptability of the derived deformed mesh. This check will determine if at any point of the derived contact area, tension is occurred. In particular, what is checked is whether normal stresses along to the x-axis (for the linked points on the vertical part of the interface), and along to the y-axis (for the linked points on the horizontal part of the interface), are tensile. If the answer is negative, the procedure is stopped. If the answer is positive, the linked points are considered to be unlinked and the procedure is repeated from step 2.

3.2 *The Finite Element Model*

The basic concepts of the finite element method are well documented and will not be repeated in this paper. Only the essential features will be presented. For the analysis, a four-node isoparametric rectangular finite element model with 8 degrees of freedom (DOF) has been used (Figure 3). The major assumption of modeling the masonry behavior under plane stress is that the material is homogeneous and anisotropic. Especially, the material shows a different modulus of elasticity (Ex) in the x direction (direction parallel to the bed joints of brick masonry) and a different modulus of elasticity (Ey) in the y direction (perpendicular to the bed joints). In the case of plane stress the elasticity matrix is defined by

$$D=\begin{bmatrix} \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \dfrac{E_y\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \quad (1)$$

in which Ex and Ey are the moduli of elasticity in the x and y direction respectively; ν_{xy}, ν_{yx} are the Poisson's ratios in the xy and yx plane respectively; and G_{xy} is the shear modulus in the xy plane. It is worth noticing that in the case of plane stress in an anisotropic material the following equation holds

$$E_x\nu_{yx}=E_y\nu_{xy} \quad (2)$$

Displacement functions: Figure 3 shows the four-node isoparametric rectangular finite element model, with nodes 1, 2, 3, 4 numbered in an anticlockwise order.

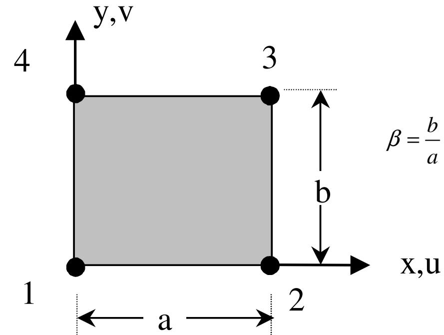

Figure 3. Finite element dimensions.

The displacements of a node have two components, namely

$$\delta_i=\begin{bmatrix} u_i \\ v_i \end{bmatrix} \quad (3)$$

and the eight components of element displacements are listed as a vector

$$\delta^e=\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} \quad (4)$$

The displacements within an element have to be uniquely defined by these eight values. The simplest representation is given by two linear polynomials, namely

$$\begin{aligned} u(x,y)&=\alpha_1+\alpha_2 x+\alpha_3 xy+\alpha_4 y \\ v(x,y)&=\alpha_5+\alpha_6 x+\alpha_7 xy+\alpha_8 y \end{aligned} \quad (5)$$

The eight constants α_i (where i=1,2,…,8) can be evaluated easily by solving the two sets of four simultaneous equations, which will arise if the nodal coordinates are inserted and the displacements equated to the appropriate nodal displacements.

Strain (total): The total strain at any point within the element can be defined by its three components which contribute to the internal work. Thus

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u(x,y)}{\partial x} \\ \frac{\partial v(x,y)}{\partial y} \\ \frac{\partial u(x,y)}{\partial y} + \frac{\partial v(x,y)}{\partial x} \end{bmatrix} \quad (6)$$

With displacements known at all points within the element the strains at any point can be determined. These will always result in a relationship that can be written in matrix notation as

$$\varepsilon = B\delta^e \quad (7)$$

where B is a suitable linear operator.

The stiffness matrix: The stiffness matrix of the finite element is defined by the general equation

$$K^e = \int_{V^e} B^T D B \quad d(vol) \quad (8)$$

The analytical form of the anisotropic finite element stiffness matrix is defined by integration over the area of the element (see Appendix).

Extensive and in-depth research works on the modeling of brick masonry behavior using finite elements could be found in Samarasinghe PhD thesis (Samarasinghe 1980) that focused on the problem of the analysis of anisotropic non-linear masonry. More recently macro-models that represent the behavior of the infill panel as a whole have been given (Lourenco and Rots 1997; Lourenco, Rots and Blaauwendraad 1998; Zucchini and Lourenco 2002).

3.3 *The Finite Element Computer Program*

In order to implement the method, a specific computer program for a 2D linear elastic analysis of infilled plane frames under lateral static loads has been developed. The computer program is divided into four parts: the first part consists of the routines for the control and data input modulus, the second part consists of the routines for the macro-solution and output modules, the third part consists of the solution of the equation systems, and the fourth part consists of the graphics routines. This code has been developed using the FORTRAN programming language and has the capability of automatic mesh generation. For the solution of the system of stiffness equations, the LEQ1PB routine of the IMSL has been used. According to this routine, the global stiffness matrix of the infilled frame, which is stored in a band symmetric storage mode with only those of its coefficients below the principal diagonal, is decomposed into LL^T using the Cholesky algorithm (IMSL routine LUDAPB). Using this computer program, several cases of single- or multi-story brickwork infilled frames have been investigated. The Auto-Cad plot software package has been integrated to the computer program by means of the macro-language, using as input data the generated plot data files.

4 CONSTITUTIVE RULES FOR MASONRY MATERIAL

For a successful application of the proposed method special constitutive rules for the masonry material are required. Especially, a cubic tensor polynomial for the failure criterion of the masonry material was proposed. Masonry is a material that exhibits distinct directional properties because the mortar joints act as planes of weakness. To define failure under biaxial stress, the derivation of a 3D surface in terms of the two normal stresses and shear stress (or the two principal stresses and their orientation to the bed joints) is required. A failure surface of this form has been recently derived by Syrmakezis and Asteris (Syrmakezis and Asteris 2001). The authors present a method to define a general anisotropic failure surface of masonry under biaxial stress, using a cubic tensor polynomial. In particular, the authors propose an analytical methodology in order to describe the masonry failure surface under plane stress via a regular surface, that is, a surface defined by a single equation of the form $f(\sigma) = 0$ (Koiter 1953). The failure surface (Figure 4) for the masonry is described by the equation

$$\begin{aligned} &2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 + \\ &-0.30\sigma_x\sigma_y + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + \quad (9) \\ &+0.28398\sigma_x\tau^2 + 0.4689\sigma_y\tau^2 = 1 \end{aligned}$$

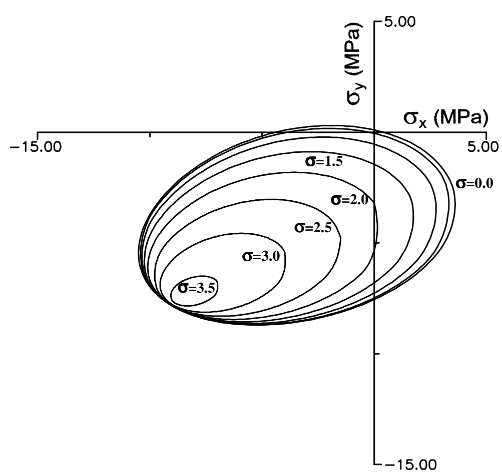

Figure 4. Failure surface of masonry in normal stress terms (Syrmakezis and Asteris 2001).

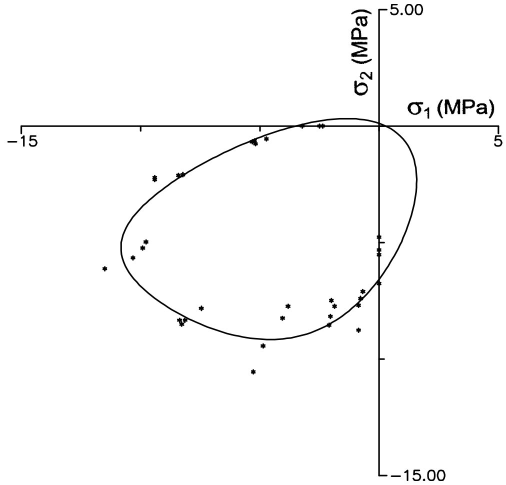

Figure 5. Failure curve of masonry in principal stress terms (θ=22.5°) (Syrmakezis and Asteris 2001).

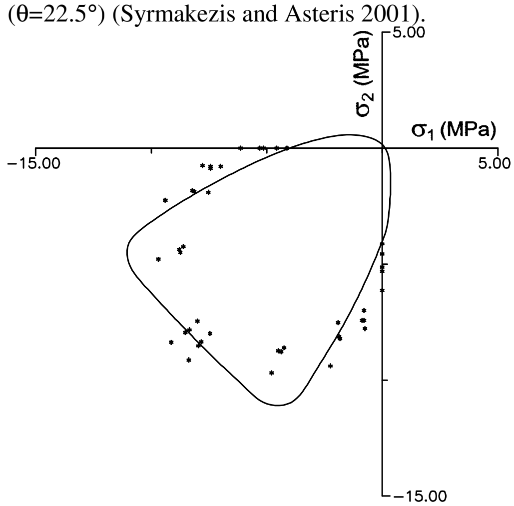

Figure 6. Failure curve of masonry in principal stress terms (θ=45°) (Syrmakezis and Asteris 2001).

The validity of the method is demonstrated by comparing the derived analytical failure surface of (9) with the existing experimental results (Page 1981). More than 100 experimental data points have been depicted in Figures 5 and 6. In the same figures, analytical curves are also depicted for the failure surface of (9). The good agreement of the experimental and analytical data is apparent for this general failure surface with a non-symmetric curve.

5 APPLICATION

In the example presented, the proposed finite element technique has been applied. The response of a single-bay single-story masonry infilled R.C. frame (Figure 7) under a lateral static load in the beam level is studied. The frame is constructed with reinforced concrete 30/40 cm sections for both columns and beams. The mechanical characteristics for both the reinforced concrete and the infill masonry walls are the ones shown on Table 1.

Table 1. Material's elastic properties

Material	Moduli of elasticity		Poison's ratio	
	E_x(kN/m^2)	E_y(kN/m^2)	ν_{xy}	ν_{yx}
Concrete	2.9×10^7	2.9×10^7	0.20	0.20
Masonry*	4.5×10^6	7.5×10^6	0.19	0.32**

* The values of the masonry material have been estimated experimentally by Page (Page 1981).

** $\nu_{yx} = \frac{E_y}{E_x}\nu_{xy}$

Figure 7b depicts the mesh according to the above-proposed method. In particular, following the first step of the method, the infill finite element models are initially considered to be linked to the surrounding frame finite element models at two corner points A and B (only), at the ends of the compressed diagonal of the infill.

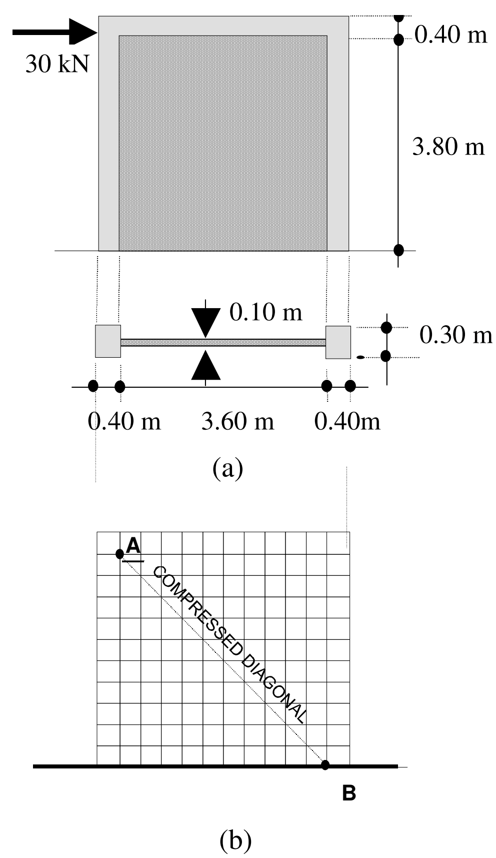

Figure 7. One-story one-bay brick masonry infilled frame: (a) Geometry and loading; (b) Mesh

Figure 8 shows the successive deformed meshes of the studied one-story one-bay infilled frame generated by the proposed method of contact points. In particular, Figure 8a depicts the deformed mesh based on the assumption that infill and frame are linked only at the two points A and B. According to this deformed mesh, two neighbouring points of B and one neighbouring point of A of the infill model points overlap the surrounding frame finite elements. Thus, according to the fourth step, these three neighbouring points (to the previous linked) are linked and the procedure continues. The process is iterated (Figures 8b to 8h), until a final equilibrium condition is reached (Figure 8h).

According to the derived deformed mesh (Figure 8h), different contact lengths between infill wall and surrounding frame members are observed, as is expected. In particular, the infill/frame contact lengths are varied between windward column and infill, beam and infill, and between infill and rigid base, thus demonstrating how unrealistic and inadequate is the modeling of the infill panel by a number of parallel compression inclined struts.

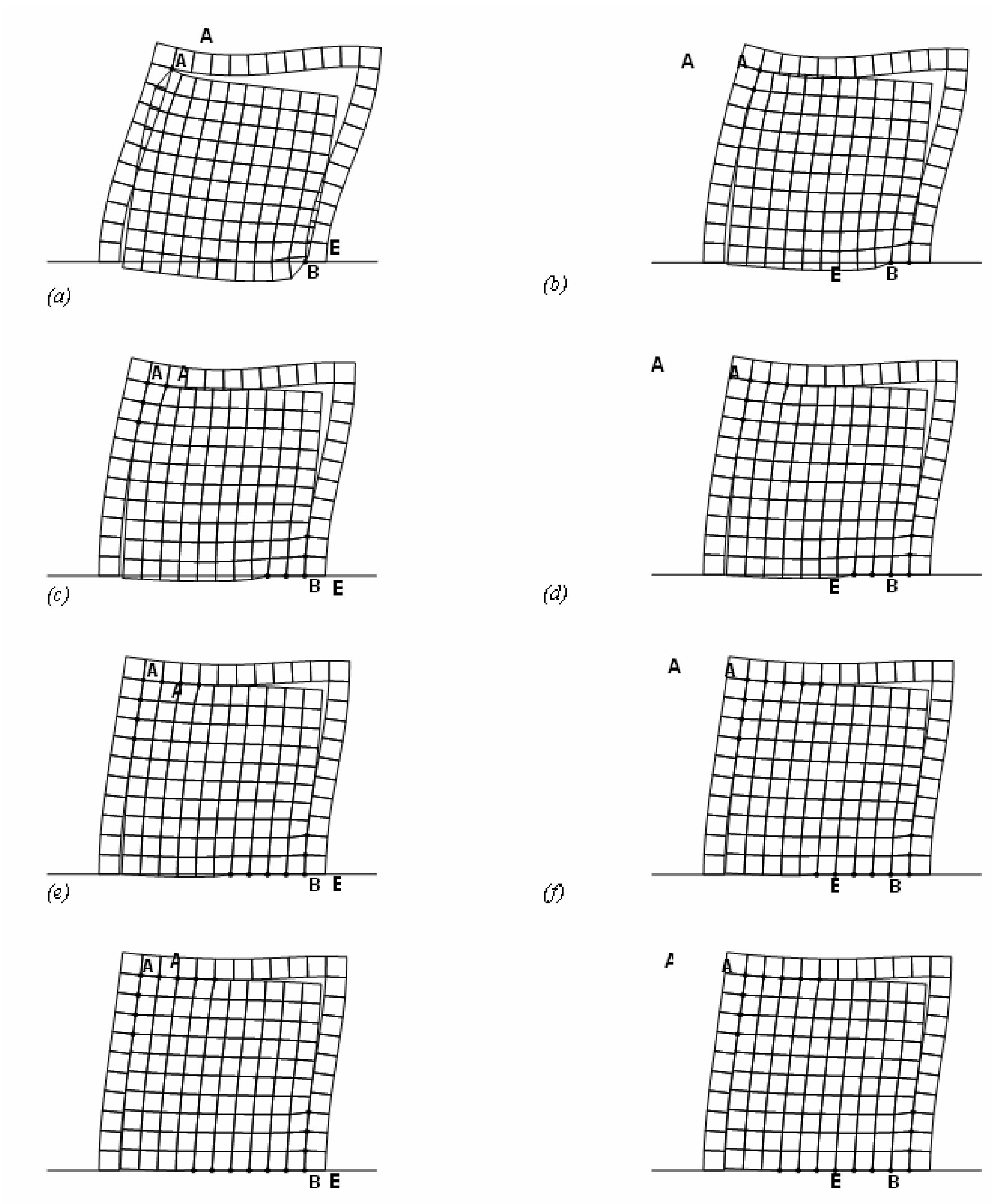

Figure 8. Successive deformed meshes of one-story one-bay infilled frame.

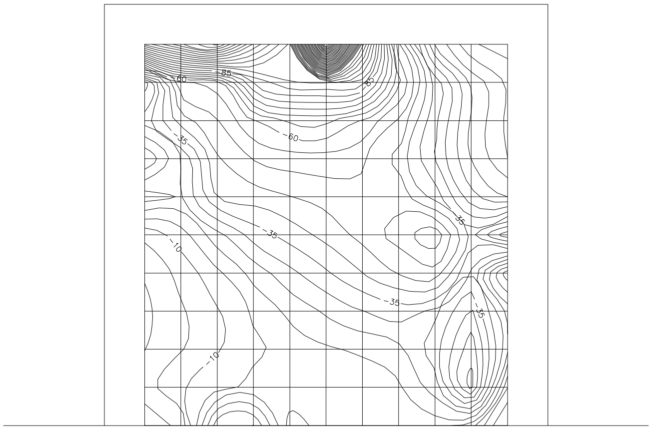

Figure 9. Contours of normal stress σ_x in the brick masonry infill plane.

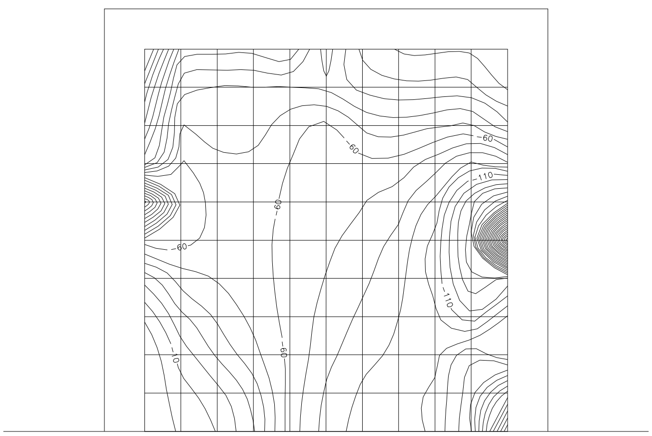

Figure 10. Contours of normal stress σ_y in the brick masonry infill plane.

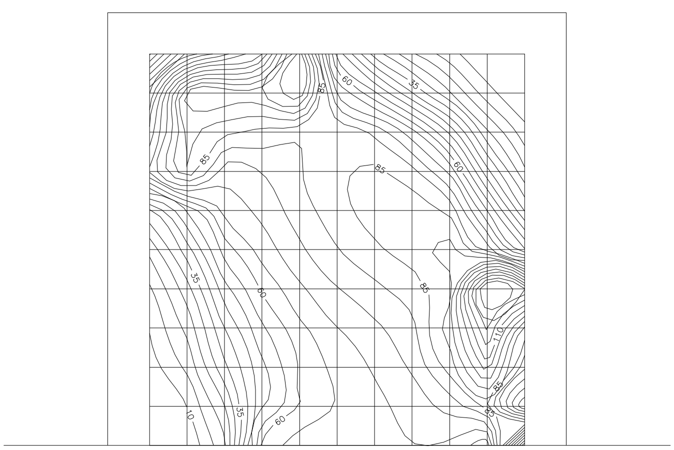

Figure 11. Contours of shear stress σ_{xy} in the brick masonry infill plane.

In Figures 9 to 11, the contours of normal and shear stresses are plotted respectively. They seem to be in good agreement with previous experimental (Smith 1966) and analytical results (Galanti, Scarpas and Vrouwenvelder 1998, Ghosh and Amde 2002, Moghaddam 2004). As is expected, the higher values of stresses are spread in a zone "parallel" to the compression diagonal as well as at the loaded corners. This could explain the two modes of infill failure, which were observed experimentally by Smith (Smith 1966). According to Smith, two modes of infill failure are observed. The first failure, developed as a crack extending from the center of the infill along the diagonal towards the loaded corners. The second failure mode occurs at one of the loaded corners, and the crushed region takes the shape of a quadrant bounded by the lengths of the contact as radii.

6 CONCLUSIONS

This paper presents a simple analytical method of micro-modeling the complicated behavior of infilled frames under lateral loads. Using this technique, the behavior of single-story single-bay masonry infilled frames under lateral loads has been investigated. Based on the present study, the following conclusions can be inferred:

The proposed analytical method calculates the infill/frame contact lengths as an integral part of the solution and not assumed in an ad-hoc way. Especially this technique calculates the infill/frame contact lengths for the case of unidirectional lateral loading and elastic response of the infill. Authors argue that the knowledge of the elastic response of composite structure will be very critical for a thorough understanding of its response under reversed cyclic loading. For that reason, the research focus of our paper concentrates on the elastic domain of the analysis.

The proposed technique is easier and more practical to apply, and requires much less computational time than micro-modeling techniques based on discretizing the infill panel as a series of plane stress elements interconnected by a series of springs or contact elements.

APPENDIX: ANISOTROPIC FINITE ELEMENT STIFFNESS MATRIX

The analytical form of the anisotropic finite element 8×8 stiffness matrix is defined by integration over the area of the element by the following, symmetric to the principal diagonal $\left(K_{ij} = K_{ji}\right)$ coefficients:

Stiffness Matrix Coefficients K_{1j} *(j=1):*

$$K_{11}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{\beta}{3E_yG_{xy}}+\frac{(1-\nu_{xy}\nu_{yx})}{3\beta E_xE_y}\right]$$

Stiffness Matrix Coefficients K_{2j} *(j=1, 2):*

$$K_{21}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{\lambda}{4}+\frac{(1-\nu_{xy}\nu_{yx})}{4E_xE_y}\right]$$

$$K_{22}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{1}{3\beta E_xG_{xy}}+\frac{\beta(1-\nu_{xy}\nu_{yx})}{3E_xE_y}\right]$$

Stiffness Matrix Coefficients K_{3j} *(j=1, 2, 3):*

$$K_{31}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{-\beta}{3E_yG_{xy}}+\frac{(1-\nu_{xy}\nu_{yx})}{6\beta E_xE_y}\right]$$

$$K_{32}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{-\lambda}{4}+\frac{(1-\nu_{xy}\nu_{yx})}{4E_xE_y}\right]$$

$$K_{33}=K_{11}$$

Stiffness Matrix Coefficients K_{4j} *(j=1, 2, …, 4):*

$$K_{41}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{\lambda}{4}-\frac{(1-\nu_{xy}\nu_{yx})}{4E_xE_y}\right]$$

$$K_{42}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{1}{6\beta E_xG_{xy}}-\frac{\beta(1-\nu_{xy}\nu_{yx})}{3E_xE_y}\right]$$

$$K_{43}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{-\lambda}{4}-\frac{(1-\nu_{xy}\nu_{yx})}{4E_xE_y}\right]$$

$$K_{44}=K_{22}$$

Stiffness Matrix Coefficients K_{5j} *(j=1, 2, …, 5):*

$$K_{51}=-\frac{1}{2}K_{11}\ ,\ K_{52}=-K_{21}$$

$$K_{53}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{\beta}{6E_yG_{xy}}-\frac{(1-\nu_{xy}\nu_{yx})}{3\beta E_xE_y}\right]$$

$$K_{54}=K_{32}\ ,\ K_{55}=K_{11}$$

Stiffness Matrix Coefficients K_{6j} *(j=1, 2, …, 6):*

$$K_{61}=K_{43}\ ,\ K_{62}=-\frac{1}{2}K_{22}\ ,\ K_{63}=-K_{32}$$

$$K_{64}=\frac{E_xE_yG_{xy}t}{1-\nu_{xy}\nu_{yx}}\left[\frac{-1}{3\beta E_xG_{xy}}+\frac{\beta(1-\nu_{xy}\nu_{yx})}{6E_xE_y}\right]$$

$$K_{65}=K_{21}\ ,\ K_{66}=K_{22}$$

Stiffness Matrix Coefficients K_{7j} *(j=1, 2, …, 7):*

$$K_{71}=K_{53}\ ,\ K_{72}=K_{41}\ ,\ K_{73}=-\frac{1}{2}K_{11}$$

$$K_{74}=K_{21}\ ,\ K_{75}=K_{31}\ ,\ K_{76}=K_{32}\ ,\ K_{77}=K_{11}$$

Stiffness Matrix Coefficients K_{8j} *(j=1, 2, …, 8):*

$$K_{81}=K_{32}\ ,\ K_{82}=K_{64}\ ,\ K_{83}=K_{21}\ ,\ K_{84}=-\frac{1}{2}K_{22}$$

$$K_{85}=K_{41}\ ,\ K_{86}=K_{42}\ ,\ K_{87}=K_{43}\ ,\ K_{88}=K_{22}$$

where

t is the thickness of the finite element,

$\beta=\frac{b}{a}$ (see Fig. 3), and

$\lambda=\frac{\nu_{yx}}{E_yG_{xy}}=\frac{\nu_{xy}}{E_xG_{xy}}$ according to (2).

NOTATION

E_x, E_y	:	moduli of elasticity in the x and y direction respectively
G_{xy}	:	shear modulus in the xy plane
t	:	thickness of the brick masonry infill
h	:	height of the frame (on center-lines of beams)
h′	:	height of the brick masonry infill
w	:	effective width of infill considered as a single diagonal strut
α	:	infill/frame contact length
β	:	ratio of finite element dimensions
ν_{xy}, ν_{yx}	:	Poison’s ratios in the xy and yx plane respectively
σ_x,σ_y	:	normal plane stresses along x and y axes, respectively
τ	:	shear stress measured in the x-y plane

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