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Numerical studies of a steel beam in a frame sub-assembly at elevated temperatures

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ABSTRACT: The analysis of structural members in a compartment in a steel framed building subjected to fire attack is often difficult owing to the combined complexity of the material degradation and of the actions induced by thermal strains. Although such analyses can be carried out using advanced finite element packages such as ABAQUS and ANSYS, they do not permit an 'overall feeling' of the underlying structural mechanics to be fully articulated or understood. It is important that the response of structures under fire conditions, and the parameters that influence this response, are fully understood so that safe and economical performance-based fire designs can be achieved, and that codified treatments can be prescribed. This paper presents numerical studies of a steel beam subjected to fire attack in a steel framed compartment. The analysis is based on an analytical procedure that allows for a solution for the actions and deformations to be stated in closed form prior to the attainment of first yield of the steel at elevated temperature. It is found that the response of members under fire attack is governed by the slenderness ratio of the member subjected to elevated temperature the restraint afforded by the adjacent, cooler elements in the building frame and the applied thermal regimes.

Keywords: Elastic restraint; fire; nonlinear; numerical study; steel structures.

1 INTRODUCTION

This paper concerns the response of a member in a compartment in a steel framed building under fire loading. Owing to the complexity of the design and analysis of structures under conditions of fire, the compartment concept has been favoured in many contemporary research investigations [1-4] because if offers considerable promise as a methodology that leads to prescriptive codified design rules. Amongst these instigations are an analytical method of catenary action in steel beams at large deflections under fire loading reported by Yin et al. [5,6], who validated their results in the form of a comparison between predictions using their proposed method and simulations using ABAQUS [7], owing to the scarcity of relevant experimental results. Bradford [3] presented a generic nonlinear analysis of a member in a frame sub-assembly in the elastic range of structural response using the principle of virtual displacements. It was shown that under thermal loading, certain thermal regimes exist (combinations of uniform temperature and temperature gradient) that can result in tensile forces developing in a beam, even in the elastic range of structural response. This study was extended by Bradford et al [4], in which the effect of non-uniform degradation of the mechanical properties of the steel over the beam section was considered; this degradation of being caused by the non-uniform temperature distribution in the steel beam that results from a heat sink effect provided by the cooler concrete floor slab above, which may or may not induce composite action with the steel beam.

The results of a numerical study of a steel beam in a frame sub-assembly at elevated temperatures are presented in this paper. A comprehensive parametric study which is based on a generic nonlinear modelling technique [4] is presented herein, and it is shown how the deflection and compressive axial force are dependent on thermal profile and degradation of the mechanical properties of the steel material at elevated temperatures.

2 THEORY

The numerical studies presented in this paper are based on a generic nonlinear analytical procedure for steel members in a frame sub-assembly under fire attack by Bradford *et al.* [4]. Figure 1 shows a steel member of length $2L$ within a steel frame sub-assembly, in which the end transverse displacements are taken to be $v(z = \pm L/2) = 0$ and which is re-

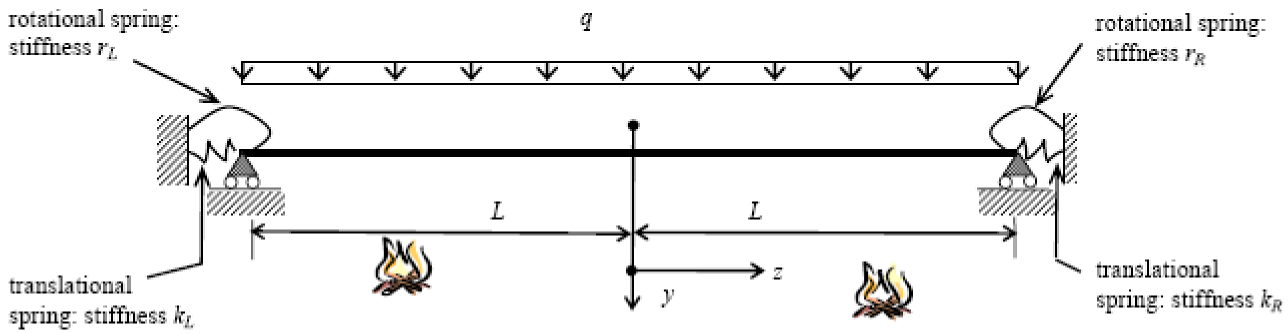

Fig. 1 Generic flexural member.

strained elastically at both ends viz translational springs (k_L, k_R) and rotational springs (r_L, r_R), respectively. The member is subjected to a uniformly distributed load q and a thermal regime $\Im$, which is defined by temperatures at the extreme fibres of the cross-section, relative to the ambient state, of T_1 and T_2 that allows for the mean thermal strain and the linear thermal gradient to be defined as

$$\varepsilon_C = \alpha T_C; \; \nabla_T = \alpha\left(T_2 - T_1\right)/h \tag{1a,b}$$

where $T_C = (T_1 + T_2)/2$ is the mean temperature, ∇_T is the thermal gradient (assumed linear), α is the coefficient of thermal expansion that is assumed constant and h is the depth of the cross-section.

A full derivation of the analytical model has been presented by Bradford *et al.* [4], and only pertinent components of the derivation are given here. The non-linear formulation of the strain-displacement relationship is:

$$\varepsilon = w' + \tfrac{1}{2}v'^2 - yv'', \tag{2}$$

here w is the deformation in z direction and y is an arbitrary distance from the reference level (Fig. 1). The mechanical strain ε_e and stress σ can be found by decomposing ε into its mechanical and thermal components, so that

$$\varepsilon_e = w' + \tfrac{1}{2}v'^2 - \varepsilon_C - y\left(v'' + \nabla_T\right) = \varepsilon_0 - y\left(v'' + \nabla_T\right) \text{ and } \sigma = E(y)\cdot\varepsilon_e, \tag{3}$$

where ε_0 is the membrane strain and $E(y)$ is the modulus of elasticity, which is calculated at an appropriate temperature T at the location y.

By applying principle of virtual displacements, substituting the appropriate components of Eq. 2 and enforcing the reference axis as being at the elastic centroid of the member's cross-section, the familiar equation for the deformation of a beam-column or tie-beam [8] results that takes the form

$$\overline{EI}v^{iv} + Nv'' = q \tag{4}$$

where N is the axial force developed in the member, which is determined from

$$N = \varepsilon_0 \overline{EA} \tag{5}$$

in which

$$\overline{EA} = \int_A E\left(y\right)\mathrm{d}A; \quad \overline{EI} = \int_A y^2 E\left(y\right)\mathrm{d}A \tag{6}$$

are cross-sectional properties.

The closed-form solution to the differential equation Eq. 4 can be found in the form

$$v = A_{1c}\cos\mu z + A_{2c}\sin\mu z + A_{3c} + A_{4c}z + \frac{qz^2}{2N} \tag{7}$$

if the member is in compression, or

$$v = A_{1t}\cosh\mu z + A_{2t}\sinh\mu z + A_{3t} + A_{4t}z - \frac{qz^2}{2N} \tag{8}$$

if it is in tension, where A_1, , A_4 are four constants that can be determined from the boundary static and kinematic conditions; and where μ is the axial force parameter given by

$$\mu = \left(\frac{|N|}{\overline{EI}}\right)^{\frac{1}{2}}. \tag{9}$$

The axial force parameter μ in Eq.8 can be determined from the equilibrium equation of the member, given by

$$\frac{\varphi^2}{\overline{\lambda}^2} + \varepsilon_c + \varphi^2\overline{k} - f\left(\varphi\right) = 0 \tag{10}$$

Where

$$f\left(\varphi\right) = \tfrac{1}{4}\int_{-L}^{L}\left(-\mu A_{1c}\sin\mu z + \mu A_{2c}\cos\mu z + A_{4c} + \frac{qz}{\mu^2\overline{EI}}\right)^2 \mathrm{d}z \quad \text{with} \quad \phi = \mu L \tag{11}$$

if the load is compressive,

$$f(\varphi)=\tfrac{1}{4}\int_{-L}^{L}\left(\mu A_{1t}\sinh\mu z+\mu A_{2t}\cosh\mu z+A_{4t}-\frac{qz}{\mu^2\overline{EI}}\right)^2 \mathrm{d}z \quad \text{with} \quad \phi=\mu L \tag{12}$$

if the load is tensile, and where

$$\overline{\lambda}=\frac{L}{\overline{r}}, \quad \overline{r}^2=\frac{\overline{EI}}{\overline{EA}} \quad \text{and} \quad \overline{k}=\frac{\overline{EI}}{2L^2}\left(\frac{1}{k_L}+\frac{1}{k_R}\right). \tag{13}$$

This analytical procedure, which considers a varying modulus of elasticity $E(y)$ over the member cross-section, can particularly be useful in the analysis of composite beam members, in which the temperature of the top of the steel beam in contact with the concrete slab can be much lower than that of the lower steel flange exposed to the fire. It also permits an 'overall feeling' of the underlying structural mechanics and allows for the important parameters that govern the behaviour of a steel flexural member under thermal loading to be determined.

3 NUMERICAL STUDIES

The aim of the numerical studies is to establish the important parameters that influence the response of members in fire conditions. These parameters include the slenderness ratio, restraint stiffness and variations of the thermal regime under to which the member is subjected. In each case, only the parameter under consideration was varied while the other two parameters were kept constant. The effects of these parameters on the response of the member are demonstrated in sections 3.3.

3.1 *Set up of the numerical problem*

Numerical studies were conducted on an Australian 460UB82.1 section, which is subjected to thermal regimes defined by a temperature rise TC at the geometric centroid of the section and a linear thermal gradient ρ over the cross-section depth. The member is also subjected to a total downwards uniformly distributed load of q = 5N/mm, which was chosen to simulate the sustained service load that the member is carrying. End restraints are applied to the member via elastic springs, in which their stiffness can be written as

$$r_L=r_R=r=\beta\frac{E_{20}I}{2L}; \quad k_L=k_R=k=\beta\frac{E_{20}A}{2L}, \tag{14}$$

where E_{20} is the modulus elasticity of the steel at room temperature and taken as E_{20} = 200×10^3 N/mm^2; I and A are the cross-section second moment of area and area respectively; L is the member half length (Figure 1) and β is the dimensionless relative stiffness coefficient, which allows for the spring stiffness to be easily varied.

Variations of the steel modulus of elasticity E_T and yield stress f_{yT} are taken from an empirical relationship given in the Australian Standard for steel structures AS 4100 [9] as

$$\frac{E_T}{E_{20}}=1.0+T/\left[2000\ln(T/1100)\right] \qquad (T\in[0^\circ\mathrm{C},600^\circ\mathrm{C}]) \tag{15}$$

$$\frac{f_{yT}}{f_{y20}}=1.0 \qquad (T\in[0^\circ\mathrm{C},215^\circ\mathrm{C}])$$

and (16a)

$$\frac{f_{yT}}{f_{y20}}=(905-T)/690 \qquad (T\in[215^\circ\mathrm{C},905^\circ\mathrm{C}]) \tag{16b}$$

where E_T and f_{yT} are modulus of elasticity and yield stress of the steel at a temperature T and f_{y20} is the yield stress of steel at room temperature. The elastic limiting temperature is the temperature at the geometric centroid of the section at which first yield is attained.

3.2 *Verification of numerical results*

The numerical results obtained using the current model were validated against an ABAQUS model of a steel beam under the same thermal regimes and loading conditions. Comparisons between the results of the present study and those obtained from the ABAQUS model show fairly good agreement.

3.3 *Numerical results*

Numerical studies are directed towards considering the effects of (i) the slenderness ratio; (ii) the end restraint stiffness; and (iii) thermal gradient on the behaviour of the steel beam under fire attack. In this study, the members are subjected to increasing temperatures until the yield stress is exceeded and stresses that develop are checked at the mid-span and at the supports to ensure elasticity is not exceeded. The axial force and mid-span deflection were then recorded at intermediate temperatures.

Effects of slenderness ratio

In this case, a thermal gradient ρ = 0.25ºCmm-1, a total downwards uniformly distributed load q =

5N/mm and the relative stiffness coefficient β = 1.0 were kept constant for members with different lengths 2L. The slenderness ratio is defined in this study as the ratio of the length of the member and the cross-section radius of gyration about the strong axis (for the Australian 460UB82.1 section, r = 188mm). As can be seen in Figures 2 and 3, the slenderness ratio imposes quite profound effects on the axial force in the member and the deflection which the member experiences. The larger the slenderness ratio, the larger the axial force and the larger deflection the member experiences.

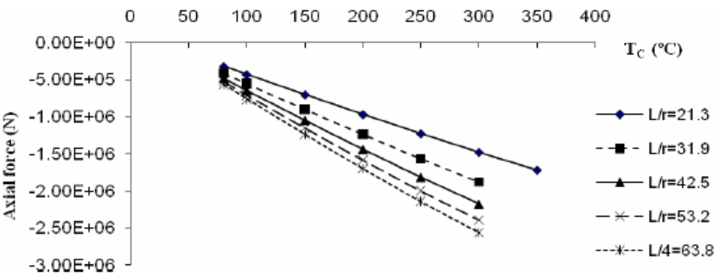

Fig. 2 Effects of slenderness ratio on axial force developed in the member.

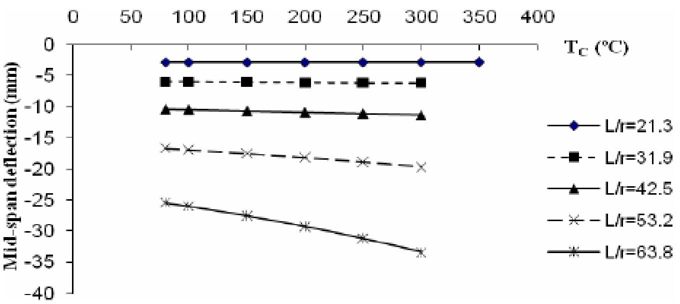

Fig. 3 Effects of slenderness ratio on mid-span deflection of the member.

For instance, when members which have the same end restraint conditions are subjected to a centroidal temperature of T_C = 300ºC, members with slenderness ratios of 63.8, 53.2, 42.5 and 31.9 experience 73.4%, 62%, 47% and 27.1% increases in their axial force compared to the axial force in the stockiest member, for which the slenderness ratio is 21.3. These longer members also appear to deflect more than stockier members. The obtained numerical results of the present study are in very good agreement with the predictions of the axial force and mid-span deflection of the ABAQUS model. While the comparison of the axial force results shows an almost perfect agreement (Figure 4), the ABAQUS model predictd slightly smaller mid-span deflectiond of the member than the generic nonlinear analytical approach (Figure 5).

Effects of end restraint

It is widely known that within a compartment fire the cooler columns provide considerable restraint to the hot steel beam, that the usually semi-rigid connections provide rotational restraint to this steel beam and that the cooler concrete floor produces a temperature gradient through the depth of the steel beam. Heated members with high level of restraint against *thermal expansion* can experience substantial compression forces while a temperature gradient will result against a curvature (also known as the *thermal bowing effect*) and subsequently induce additional end moments and tensile axial actions owing to the 'contraction' effects of the member on the supports. In general, members that are subject to high restraint will experience significant thermal actions. For the case of ρ = 0.25°Cmm^{-1}, the total-downwards uniformly distributed load q = 5N/mm and length $2L$ = 6000mm, the relative stiffness coefficient was varied to simulate different end restraint conditions from a very loosely restrained member (β = 0.1) to a highly restrained member (β = 0.1). As illustrated in Figure 6, the degrees of member end

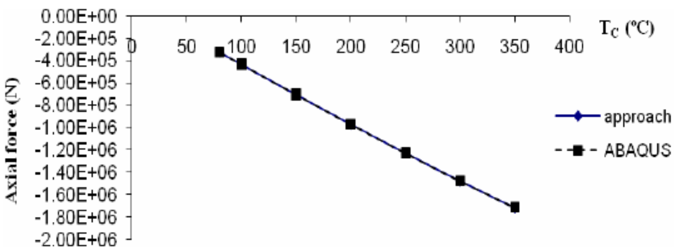

Fig. 4 Comparison of axial force predictions between the numerical study and those of the ABAQUS model for a member of slenderness ratio 21.3.

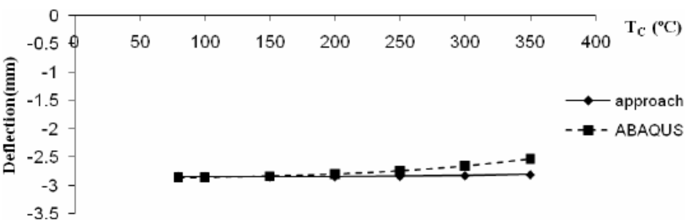

Fig. 5 Comparison of mid-span deflection predictions between the numerical study and those of the ABAQUS model for a member of slenderness ratio 21.3.

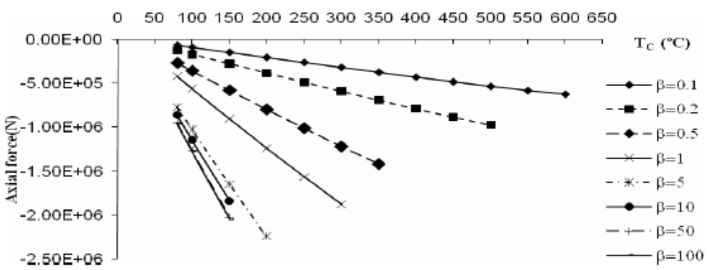

Fig. 6 Effects of degrees of end restraint on axial force developed in the member.

restraint have significant effects on the axial force developed in the member. As expected, the higher the degree of restraint, the larger the axial force the members appeared to experience. For instance, when members of the same length are subjected to a centroidal temperature T_C = 300ºC, the members which are more highly-restrained (relative restraint stiffness coefficients β = 0.2, 0.5 and 1.0), experienced 85.7%, 282.3% and 490.8% increases in the axial force with respect to the most loosely restrained member (with a relative restraint stiffness coefficient β = 0.1).

High levels of end restraint, in contrast to its effect on increasing the axial force in the member, reduces the deflection. As can be seen in Figure 7, mid-span deflection decreases dramatically with increasing levels of end restraint. The combined effect, in which the axial force increases while the deflection is restrained, results in large increases in compressive stress with increasing temperature, causing member to develop inelastic behaviour rapidly.

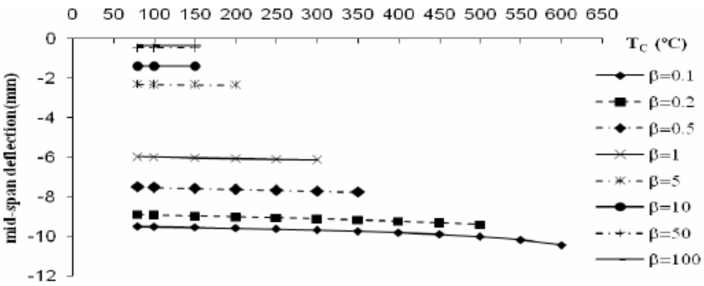

Fig. 7 Effects of amount of end restraint on mid-span deflection the member.

Comparison of the obtained numerical results with the ABAQUS beam model again shows excellent agreement in the axial force predictions while the ABAQUS model predicts slightly lesser deflection when the temperature at the geometric centroid of the cross-section exceeds T_C= 400ºC.

Effects of thermal gradient

The interaction between thermal expansion and thermal bowing effects because of thermal gradient in a steel beam under fire attack is often complex. *Thermal expansion* induces compressive actions in members with thermal restraint while *thermal bowing* can result in additional end moments owing to thermal-induced curvature. At the same time, this thermal-induced curvature develops 'contraction' effects, resulting in tensile actions in the member, which offsets some of the thermal-induced compressive action. In a member that is subjected heating, initially thermal expansion will govern and the response of the beam is compressive because the deflection at this stage is small, and so the 'contraction' strain is negligible. As the temperature and temperature gradient increases, the 'contraction' effects induce tensile actions and they can become significant and if the tensile action is large enough exceed the thermal-induced compressive actions, thermal bowing will govern and the beam may develop a 'cantenary' action as a result, even in the elastic range of structural response.

In this study, the total downwards uniformly distributed load q = 5N/mm, length $2L$ = 6000mm and end restraint relative stiffness coefficient β = 0.1 were kept constant while thermal regimes $\Im$ (T_C, ρ) were varied. As can be seen in Figure 10, when the thermal gradient is varied between ρ = 0.1°Cmm^{-1} and 0.25°Cmm^{-1}, the axial force developed in the member only changes slightly. This is because the tensile membrane action in the elastic range is negligibly small in comparison with the compressive actions induced by thermal expansion. This indicates that for thermal regimes under which the response of the member is elastic, the behaviour of the member is governed by thermal expansion. However, the effects of thermal gradient are quite evident on the deflection of the member because the member subjected to larger thermal gradient undergoes larger thermal-induced curvature due to bowing effects. The ABAQUS model shows a similar response of the member in the axial force developed in the member and the mid-span deflection which the member experiences (Figures 12 and 13).

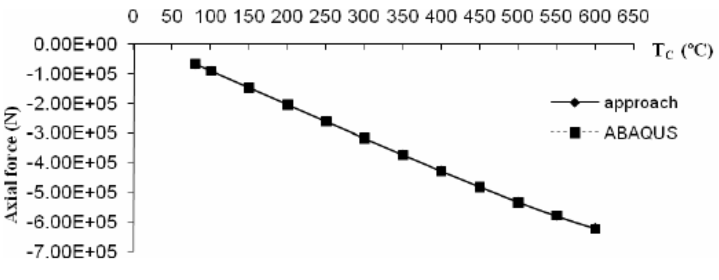

Fig. 8 Comparison of axial force predictions between the numerical study and those of ABAQUS model for a member which have end restraint coefficient β=0.1.

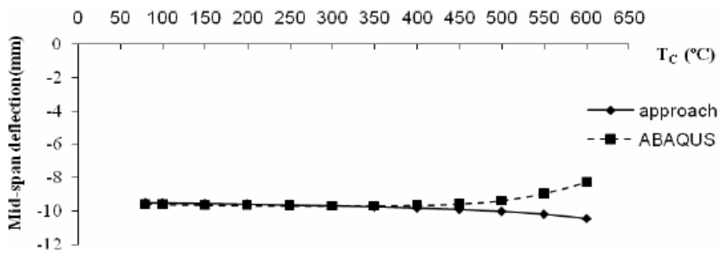

Fig. 9 Comparison of mid-span deflection predictions between the numerical study and those of the ABAQUS model for a member which have end restraint coefficient β=0.1.

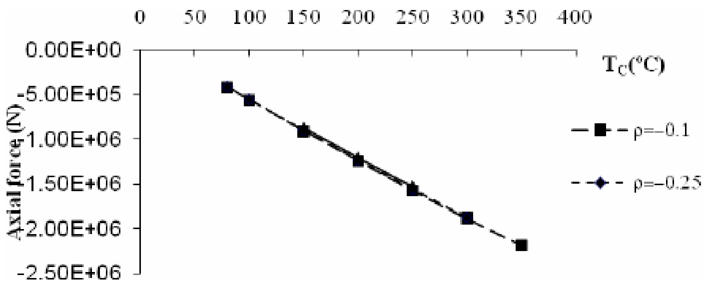

Fig. 10 Effects of thermal gradient on the axial force developed in the member.

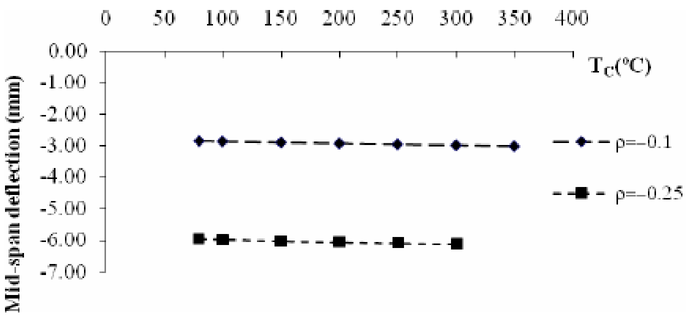

Fig. 11 Effects of thermal gradient on mid-span deflection of the member.

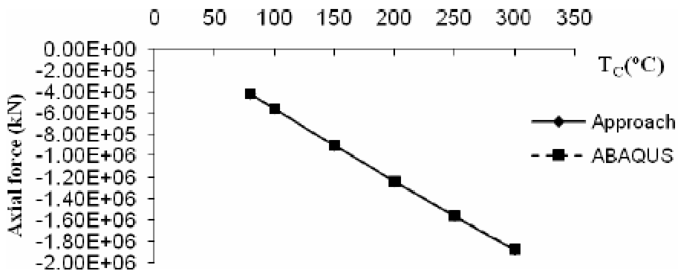

Fig. 12 Comparison of axial force predictions between numerical study and those of the ABAQUS model for a member subjected to a thermal gradient ρ = 0.25 ° Cmm^{-1}

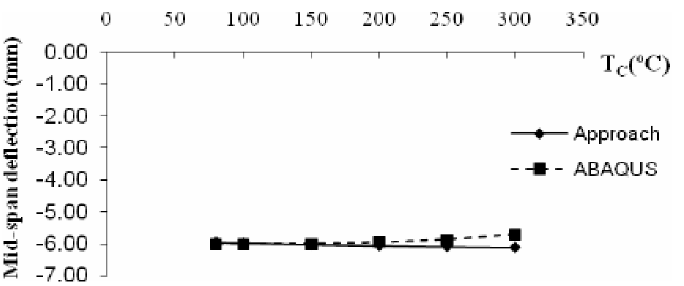

Fig. 13 Comparison of mid-span deflection predictions between the numerical study and those of the ABAQUS model for a member subjected to a thermal gradient ρ = 0.25 ° Cmm^{-1}

4 CONCLUDING REMARKS

This paper has presented the results of a numerical study of the behaviour in the elastic range of a steel beam member within a frame sub-assembly in a steel framed building under fire attack using a generic nonlinear analytical approach, rather than a finite element package as is often chosen in other studies reported in the literature. This approach allows a complete transparency of the underlying structural mechanics and the important parameters that influence the structural behaviour. In this study, the three most important parameters which govern the response of the steel beam member under thermal loading were identified and investigated. It was found the slenderness ratio of the member subjected to elevated temperatures, the restraint provided by the adjacent cooler elements in the building frame and the applied thermal regimes can significantly influence the behaviour of a steel beam subjected to thermal loading. A steel beam which is highly restrained against thermal expansion and rotation will experience substantial compressive actions and subsequent yielding at relatively low temperatures. Thermal-induced axial force and deflection are more significant in a long beam than in a stocky member, if both the long and the stocky members have the same end restraint and under the same thermal loading conditions. Although the axial force developed in the member at initial stages of the fire is governed by thermal expansion, the thermal gradients over the depth of the member cross-section can have profound effects on the member's deflection. It is important that these effects are considered so that efficient and safe fire designs can be achieved.

5 ACKNOWLEDGEMENT

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