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Behaviour of retrofitted reinforced concrete beams under combined bending and torsion : A numerical study

R.Santhakumar

Assistant Professor, National Institute of Technical Teachers Training and Research, Chennai.
E-mail: rscrescent@yahoo.co.in

R.Dhanaraj
Professor, Madras Institute of Technology, Chennai.

E.Chandrasekaran
Professor, Crescent Engineering College, Chennai.

ABSTRACT: This paper presents the numerical study on unretrofitted and retrofitted reinforced concrete beams subjected to combined bending and torsion. Different ratios between twisting moment and bending moment are considered. The finite elements adopted by ANSYS are used for this study. For the purpose of validation of the finite element model developed, the numerical study is first carried out on the unretrofitted reinforced concrete beams that were experimentally tested and reported in the literature. Then the study has been extended for the same reinforced concrete beams retrofitted with carbon fiber reinforced plastic composites with $\pm 45^{o}$ and $0/90^{o}$ fiber orientations. The present study reveals that the CFRP composites with $\pm 45^{o}$ fiber orientations are more effective in retrofitting the RC beams subjected to combined bending and torsion for higher torque to moment ratios.

1. INTRODUCTION

The reinforced concrete (RC) structural elements such as the peripheral beams in each floor of multistoried buildings, ring beams at the bottom of circular tanks, edge beams of shell roofs, the beams supporting canopy slabs and the helicoidal staircases are subjected to significant torsional loading in addition to flexure and shear. Strengthening or upgrading becomes necessary when these structural elements cease to provide satisfactory strength and serviceability. Fiber Reinforced Plastic (FRP) composites can be effectively used as an external reinforcement for upgrading such structurally deficient reinforced concrete structures.

One major application of composites to structural retrofit is to increase the flexure and shear capacity of the beams. Strengthening of RC flexural and shear beams with external bonded FRP laminates and fabric has been studied by several investigators (Saadatmanesh 1990, Ghazi 1994, Sharif 1994, Norris 1997, Thanasis 2000, Amir 2002). However study on the torsional strengthening of structural elements using FRP has received less attention. Ghobarah, *et al*.(2002) investigated the effectiveness of FRP strengthening of RC beams subjected to pure torsion and presented the most effective wrapping material and a pattern for upgrading the torsional resistance.

Only recently, researchers have attempted to simulate the behavior of reinforced concrete strengthened with FRP composites using finite element method. Arduini, *et al*.(1997) used finite element method to simulate the behaviour and failure mechanisms of RC beams strengthened with FRP plates. The FRP plates were modeled using two dimensional plate elements. However the crack patterns were not predicted in that study. Tedesco, *et al.*(1999) modeled an entire FRP strengthened reinforced concrete bridge by finite element analysis. In their study truss elements were used to model the FRP composites. Kachlakev, *et al*.(2001) used the finite elements adopted by ANSYS to model the uncracked RC beams strengthened for flexure and shear with FRP composites. Solid 46 elements were used to model the FRP composites. Comparisons between the experimental data and the results from finite element models showed good agreement.

In this paper, using finite element method an attempt has been made to study the behaviour of retrofitted and unretrofitted reinforced concrete beams subjected to combined bending and torsion. The finite elements adopted by ANSYS were used for this study. The numerical study on the behaviour of unretrofitted RC beams that were experimentally tested and reported by Gesund, *et al*.(1964) was first carried out to validate the finite element model developed in this study. This study was further extended for the same RC beams retrofitted using carbon fiber reinforced plastic (CFRP) composites. CFRP composites with ±45° and 0/90° fiber orientations were considered. The study was carried out for different ratios between twisting moment and bending moment such as 0, 0.25, 0.5, and 1.0.

2. GEOMETRY AND MATERIAL PROPERTIES

2.1 Reinforced concrete beam

The geometry and the material properties as reported by Gesund, *et al.*(1964) were used for this study. An overall view of the beam under load is presented in Figure 1. The cross section of the beam was 203 mm (8 in) width and 203 mm (8 in) depth, and the length of the test section was taken as 1625 mm (64 in). Different twisting to bending moment ratios could be achieved by changing the length of the moment arms.

All the beams were reinforced with three 12.7 mm diameter (#4) bars as tension reinforcement, two 12.7 mm diameter (#4) bars as compression reinforcement. The yield strength of these longitudinal reinforcements was reported as 352 MPa (51000 psi). The 9.5 mm diameter (#3) closed stirrups with a yield strength of 345 MPa (50000 psi) were placed at 50.8 mm (2 in) center to center along the length of the beam. The elastic modulus and Poison's ratio for all reinforcements were considered as 200 kN/mm^2 (29000 ksi) and 0.3 respectively. The concrete cover for the reinforcements at top, bottom and sides was taken as 38 mm (1.5 in).

The compressive strength of concrete was considered the same as reported by Gesund, *et al.*(1964). The Poison's ratio for concrete was assumed as 0.2. The elastic modulus and tensile strength of the concrete were calculated from the established empirical relations given in ACI 318 (1999). Table 1 summarizes the properties of concrete for all beams.

Table 1. Properties of concrete

Beam designation	Compressive strength MPa (psi)	Tensile strength MPa (psi)	Elastic modulus MPa (ksi)
C25, R25# and R25+	39.5 (5740)	3.92 (568.22)	29780 (4318.4)
C50, R50# and R50+	32.27 (4680)	3.54 (513.07)	26890 (3899.4)
C100, R100# and R100+	36.54 (5300)	3.76 (546)	28616 (4149.7)
C0, R0# and R0+	36.54* (5300)*	3.76 (546)	28616 (4149.7)

*assumed value

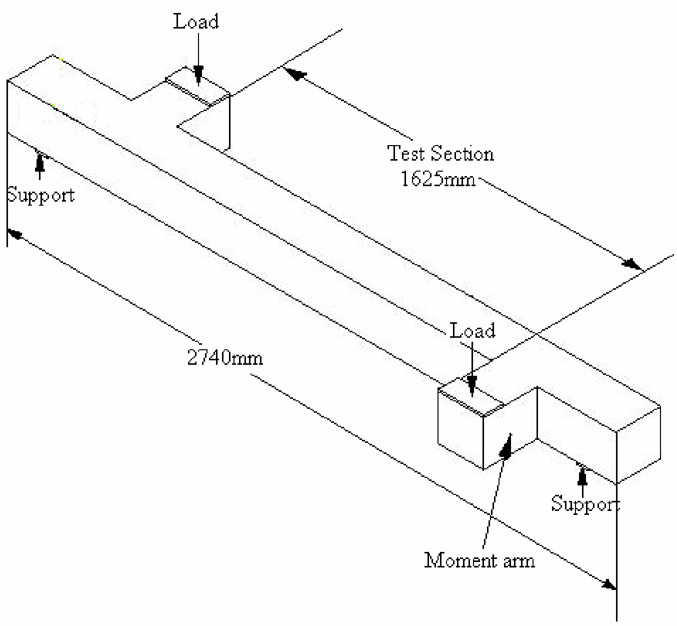


Figure 1. View of the beam under load

Each beam was designated in a way to reflect the design variables involved in that beam. The letters C and R are used to designate the control (unretrofitted) beams and retrofitted beams respectively. These letters are followed by numbers 0, 25, 50, 100 indicating the percentage of twisting to bending moment ratios. The symbols # and + indicate ±45° and 0/90° fiber orientations of CFRP composites, respectively.

2.2 CFRP composites

The CFRP composites and their material properties used by Norris, *et al.*(1997) were considered for this study. Two layers of CFRP laminate with 1 mm (0.043 in.) thickness in each layer were used to make the composites. The thickness of CFRP composites was obtained from the theoretical moment of resistance (Andre 1995). The composites with ±45° and 0/90° fiber orientations were used for strengthening the RC beams. The longitudinal modulus (E_x), transverse modulus (E_y), shear modulus (E_s) and Poison's ratio (μ_{xy}) were taken as 34.1 GPa (4900 ksi), 4.6 GPa (600 ksi) and 6.3 GPa(900 ksi) and 0.36 respectively (Norris 1997).

3. NUMERICAL STUDY

3.1 Finite element modeling

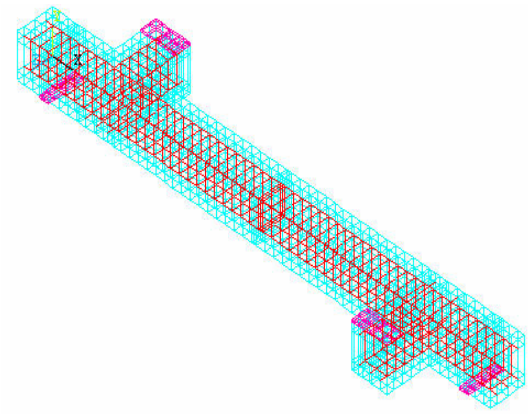
Solid 65, a 3-D structural reinforced concrete solid element was used to model the concrete. This element is capable of cracking in tension and crushing in compression. It is defined by eight nodes having three translational degrees of freedom at each node. The important aspect of this element is the treatment of nonlinear material properties. Though Solid 65 is a reinforced concrete element, the reinforcement capability of this element was not considered in this study. All the reinforcements were modeled separately using Link 8, a 3-D spar element which is an uniaxial tension-compression element defined by two nodes with three translational degrees of freedom at each node. This Link 8 element is also capable of plastic deformation. Solid 45, a 3-D structural solid element was used to model the steel plates at the support and under the load. A layered solid element, Solid 46 was used to model the CFRP composites.

In addition to the material properties discussed earlier, shear transfer coefficient (β_t) for open and closed cracks in concrete was required for the analysis. The value of β_t used in many studies varied between 0.05 and 0.25 (Bangash 1989, Barzegar 1997, Hemmaty 1998). A number of preliminary analyses were attempted in this study with various values for β_t within this range to avoid convergence problems. The shear transfer coefficient of 0.1 for open crack was found to be suitable for analyzing the beams subjected to combined bending and torsion. Slightly higher value of 0.12 was used as β_t for closed crack. For beams under pure bending the value of β_t was taken as 0.2 for open crack and 0.22 for closed crack (Kachlakev 2001). The uniaxial compressive stress – strain curve for concrete was constructed following the empirical relations and used in this study (Desayi 1964, Gere 1997).

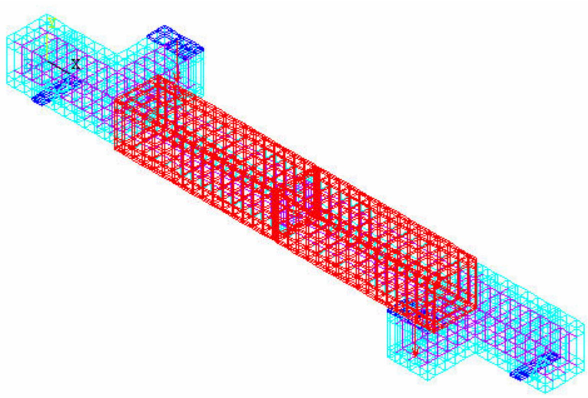
The bond between steel reinforcement and concrete was assumed to be perfect and no loss of bond between them was considered in this study (Kachlakev 2001, Fanning 2001). The Link 8, 3-D spar element for the steel reinforcement was connected between nodes of each adjacent concrete Solid 65 elements so that the two materials share the same nodes. The same approach was adopted for the CFRP composites to simulate the perfect bonding. The thickness of the Solid 46 element was modified due to geometric constraints from the other concrete elements in the model. However the equivalent overall stiffness of the Solid 46 element was maintained by making changes in the elastic and shear moduli (Kachlakev 2001). Figures 2(a) and 2(b) show the finite element models of the control and retrofitted beams respectively.

3.2 Non-linear solution and failure criteria

In this study the total load applied was divided in to a series of load increments (or) load steps. Newton – Raphson equilibrium iterations provide convergence at the end of each load increment within tolerance limits. The automatic time stepping in the ANSYS program predicts and controls load step sizes for which the maximum and minimum load step sizes are required. After attempting many trials the number of load steps, minimum and maximum step sizes was determined. During concrete cracking, steel yielding and ultimate stage in which large numbers of cracks occur, the loads were applied gradually with smaller load increments. Failure for each model was identified when the solution for 0.0045 kN (0.001 kips) load increment was not converging.



(a). Control beam



(b). Retrofitted beam

Figure 2. Finite element models

4. RESULTS AND DISCUSSIONS

4.1 Comparison with experimental results

The failure bending and twisting moments for the control beams obtained from the numerical study were compared with the experimental results reported by Gesund *et. al*.(1964), and are presented in Table 2. From the Table 2, it is seen that the results show good agreement except for the beam No.4 in which the values obtained from the numerical analysis are higher by 10 % when compared with the experimental results. This may be due to the assumption of uniform shear transfer coefficient for beams subjected to all non-zero twisting to bending moment ratios.

Table 2. Comparison of bending and twisting moments at failure

Beam identification		Twisting / Bending (Ø)	Mending moment at failure kN-mm (kips-in)		Twisting moment at failure kN-mm (kips-in)	
Numerical	Experimental		Numerical	Experimental	Numerical	Experimental
C100	2	1	11517 (101.93)	11524 (102)	11517 (101.93)	11524 (102)
C50	4	0.5	16727 (148.05)	15140 (134)	8363 (74.025)	7570 (67)
C25	8	0.25	20269 (179.39)	19885 (176)	5067 (44.85)	4971 (44)

The experimental moment – strain curve reported by Gesund *et. al*.(1964), for the beams 2 and 4 were compared with moment – strain curve of the corresponding C100 and C50 beams obtained from the numerical study and is shown in Figure 3.

The moment –strain curves obtained from the numerical study closely follows the experimental curves. However, significant deviations are seen between the experimental and numerical curves before cracking of concrete and at ultimate stage. It is presumed that during the actual testing there may be relaxation of constituent materials, whereas this type of relaxation will not occur in a pure numerical solution. The sharp increases in the strains of C50 and C100 beams indicate the cracking of concrete and the sudden transfer of stresses from concrete to steel which has not been reported in the experimental study. Once the concrete cracks, there exists an excellent conformity between the numerical and experimental behaviour of these beams which is important since the retrofitting of concrete beams gains significance once the concrete begins to crack. This validates the application of the present finite element modeling for further analysis

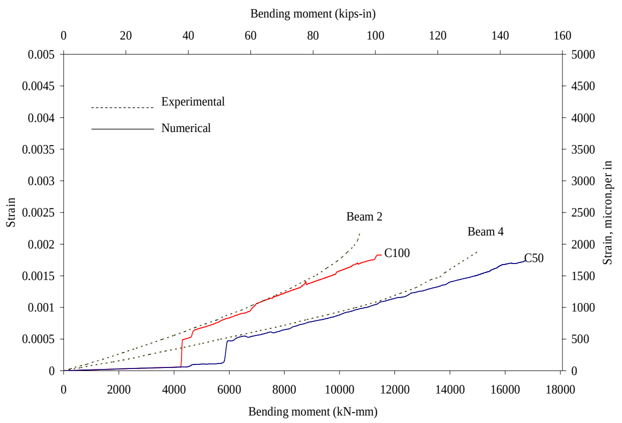


Fig 3. Strain in center bar of longitudinal tension reinforcement

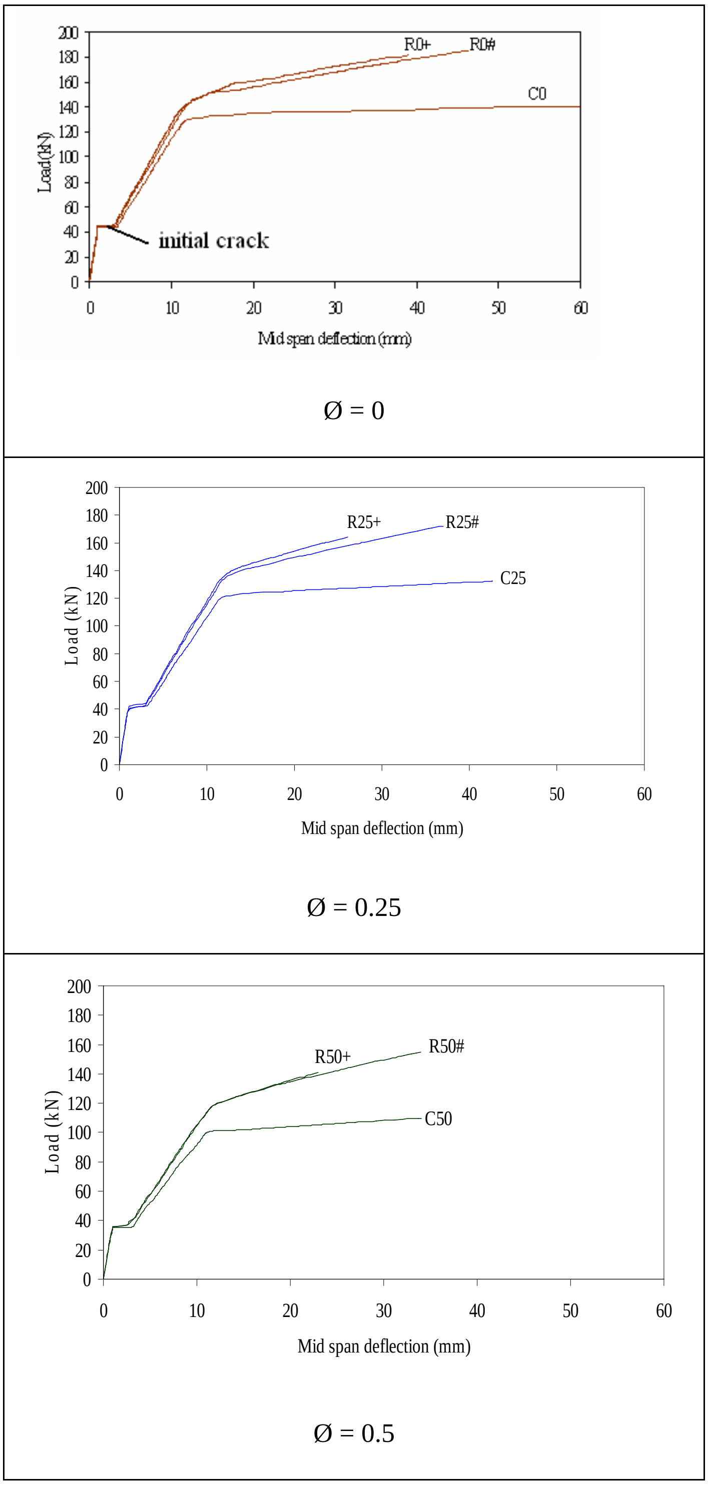
4.2 Behaviour of retrofitted beams

The numerical study is extended to the reinforced concrete beams strengthened for combined bending and torsion and tested for different twisting moment to bending moment ratios (Ø). The results obtained are presented and discussed.

The flexural stiffness of strengthened reinforced concrete beams is compared with that of the corresponding control beams. This comparison is done through the load versus deflection curves. Figure 4 shows load – deflection diagram for different Ø values of 0, 0.25, 0.5 and 1. Control beams are represented as C0, C25, C50 and C100. The beams retrofitted with CFRP composites having 0/90^o fiber orientations are indicated as R0+, R25+, R50+ and R100+ whereas the notations R0#, R25#, R50# and R100# represent the retrofitted beams with ±45^o fiber orientations

From the load deflection curves of all twisting to bending moment ratios, it is seen that wrapping of CFRP composites around the beams does not result in increased initial stiffness. The stiffness of the control and strengthened beams remain unaltered in the initial stages of loading when the cracks are not developed. This observation suggests that in the case of strengthened beams, the addition of FRP laminates has no significant effect on the initial stiffness of the RC beams.

The load versus deflection curves for all values of Ø show that there is a progressive increase in the stiffness of the strengthened beams when compared with the control beam from the state of first cracking of concrete till the ultimate stage. This shows that any strengthening of the RC beams with FRP composites will be effective after the initial cracking of concrete. This is an interesting observation since such additional strengthening of the RC elements are required only after the beams have developed cracks and are to be rehabilitated.



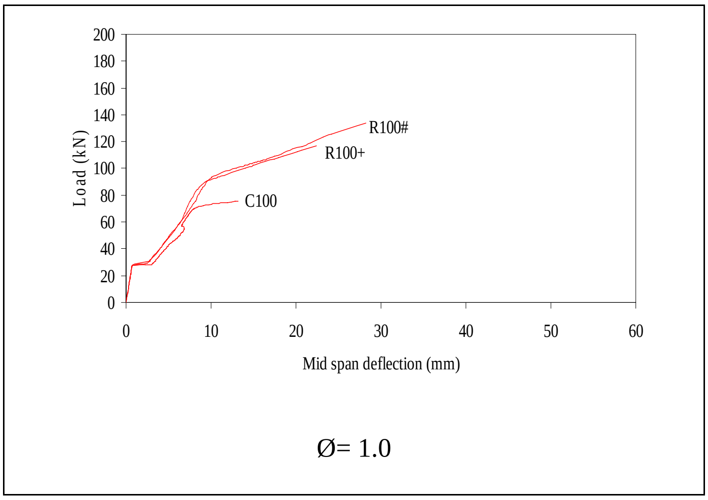


Figure 4. Load versus deflection diagram

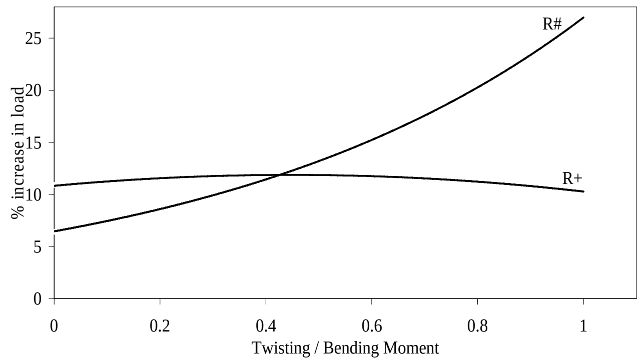


Figure 5. Role of fiber orientations

4.3 Role of fiber orientations

The percentage increase in service load of retrofitted beams with respect to control beam is plotted against the different values of Ø and is shown in Figure 5. Service load represents the load corresponding to the deflection of 0.004 times of span (BIS 456-2000). From Figure 5, it is readily seen that the CFRP laminates wrapped around the beams are found to be more effective in increasing the load capacity for higher values of Ø. The beams strengthened by the CFRP laminates with $0/90^o$ fiber orientations are effective for Ø less than 0.43 (approximately). However, for Ø greater than 0.43, there is an exponential increase in the load capacity of the beams retrofitted by CFRP laminate with $\pm 45^o$ when compared with $0/90^o$ fiber orientations. The predominant effect of shear at higher twisting to bending moment ratios is effectively taken care by $\pm 45^o$ fiber orientations.

4.4 Flexural Strength of the beam under combined bending and torsion

The reduction in the flexural strength of the control and strengthened beams are discussed through the ratio between the flexural strength of the beams in combined bending and torsion ($M_{u,bt}$) and the flexural strength in bending ($M_{u,b}$).

The ratio of the flexural strengths ($M_{u,bt} / M_{u,b}$) of the control beam (C) and beams strengthened by CFRP laminates with 0/90^o and ±45^o fiber orientations (R+ and R#) are plotted for different Ø values as shown in the Figure 6.

From the Figure 6, it is seen that the flexural strength of the beam under combined bending and torsion decreases as Ø increases. The percentage reduction in the flexural strength of the beams C, R+ and R# are found to be 43.2, 28.96 and 22.4 respectively. More rapid decrease in strength is observed in R+ beams when compared with R# beams. This shows that wrapping of beams with CFRP laminates having ±45^o fiber orientations are more effective in strengthening the beams under combined bending and torsion.

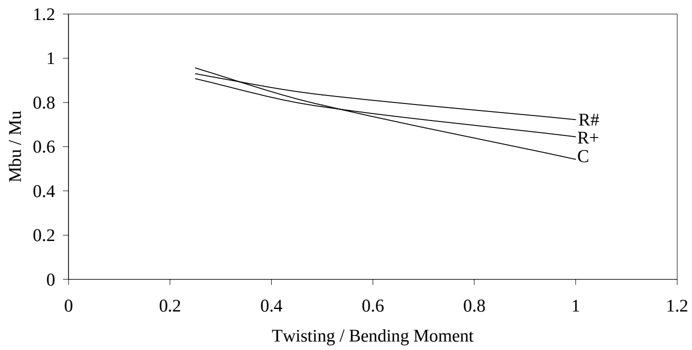


Figure 6. Variation in flexural strength of the beams under combined bending and torsion

5. CONCLUSION

A finite element analysis has been carried out to study the flexural behaviour with respect to the stiffness and strength of RC beams strengthened for combined bending and torsion. Based on the results obtained from the numerical study, the major conclusions drawn are summarized below.

- In strengthened beams, the addition of FRP laminate has no significant effect on the initial stiffness of beams.
- Strengthening of the RC beams with FRP is found to be effective only after the initial cracking of concrete.
- The FRP composites wrapped around the beams are effectively utilized in improving the load capacity with increase in the twisting moment to bending moment ratio.
- The laminates with ±45^o fiber orientations are found to be more effective for higher values of twisting to bending moment ratios.

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