Electronic Journal of Structural Engineering

Original Article

Cite this: DOI:10.56748/ejse.24620

Received Date:14 April 2024
Accepted Date: 17 July 2024

1443-9255
https://ejsei.com/ejse

Equation for the simplified calculation of the horizontal shear stress in composite beams of rectangular cross-section with considerable width

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Abstract

The theory of elasticity allows describing the law of variation of shear stresses τ_{zy} and τ_{zx} for a beam of rectangular cross-section, however, the expressions are extensive and complex. For beams with a width/depth ratio equal to 0.25, it is allowed to calculate approximately the maximum shear stress τ_{zy}, with the Collignon-Zhuravski equation. Depending on the width/depth ratio of the beam, it will depend on the percentage of error made. Unfortunately, for the shear stress τ_{zx} there is no approximate formula available to determine the value of the shear stress. Using a simplified procedure, a differential beam element was analyzed in the elastic field for different width/depth ratios and a very simple expression was obtained. The results of the approximate formula were compared with the values obtained from the application of the exact formula and the results of a linear numerical analysis using the finite element method. For cross-sections of homogeneous beams and composite beams of considerable width, it is essential to determine the horizontal shear stresses τ_{zx} in particular in the case of vertical planks, arranged side by side, connected solidly by horizontally arranged mechanical connectors, since the shear stresses τ_{zx} developed across the width of the cross-section may considerably exceed the vertical shear stresses τ_{zy} and furthermore govern the pattern, spacing and cross-section of the mechanical connectors. The validity of the approximate expression for calculating the maximum value of the shear stress τ_{zx} was limited to beams with a width/depth ratio between 0.25 and 10.0.

Keywords

Horizontal shear stress, Composite beam, Rectangular cross-section, Numerical analysis

1. Introduction

In beams of rectangular cross-section, subjected to loads on one of their inertia axes, as shown in Fig.1, a stress state occurs involving two shear stress components τ_{zy} and τ_{zx} and a normal stress component σ_{zz}, according to the direction of the "y-y", "x-x" and "z-z" axes respectively. For a beam with boundary conditions and loads according to Fig.2, the variation of stresses and the coordinates where their maximum values occur can be observed according to Fig. 3. For the exact determination of the values of the shear stresses τ_{zy} and τ_{zx}, Eq. (1) and Eq. (2) respectively, which have been deduced by [Reissner, 1946] through the theory of elasticity, are used.

The value of the stress τ_{zx} is usually neglected because, for the usual width/depth ratios used, its value is significantly lower than the dominant stress τ_{zy}, in addition to the fact that it constitutes a system in self-equilibrium according to [Fliess, 1974]. As for the stresses τ_{zy}, the application of the Collignon-Zhuravski equation given by Eq. (3) according to [Beer et al.,2006], is considered acceptable under the condition where the width/depth ratio is less than or equal to 0.25 before considerable differences with the exact elasticity formulae start to emerge.

$$\tau_{zy} = \frac{3P}{2A}\left\{1 - \frac{y^2}{a^2} + \frac{\nu}{1+\nu}\frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n \cos\frac{(2n+1)\pi y}{2a}}{(2n+1)^3}\left[\frac{(2n+1)\pi}{2}\frac{b}{a}\frac{\cosh\frac{(2n+1)\pi x}{2a}}{\sinh\frac{(2n+1)\pi b}{2a}} - 1\right]\right\} \quad (1)$$

$$\tau_{zx} = \frac{3P}{2A}\left\{\frac{\nu}{1+\nu}\frac{16b}{\pi^2 a}\sum_{0}^{\infty}\frac{(-1)^n \sin\frac{(2n+1)\pi y}{2a}}{(2n+1)^2}\left[\frac{x}{b} - \frac{\sinh\frac{(2n+1)\pi x}{2a}}{\sinh\frac{(2n+1)\pi b}{2a}}\right]\right\} \quad (2)$$

$$\tau_{zy} = \frac{QS^n}{I.2b} \quad (3)$$

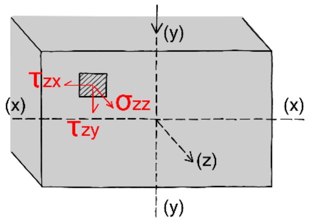

Fig. 1 The stress state of a generic cross-section, subjected to a load in the direction of one of its principal axes of inertia.

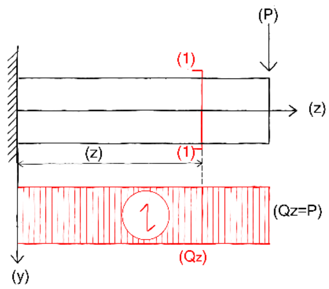

Fig. 2 Shear stress, for a cantilever beam, with a point load at its free end, without considering self-weight.

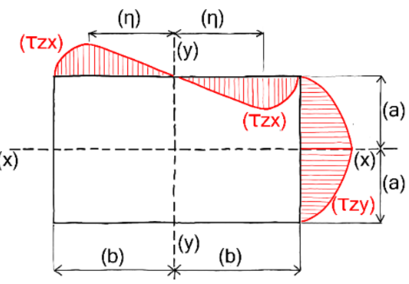

Fig. 3 Shear stress laws τ_{zx} and τ_{zy} respectively, for a generic cross-section.

The location of the maximum stress τ_{zx}, according to [Reissner,1946] is located at a distance "η" from the axis of symmetry of the cantilever cross-section, according to Eq. (4):

$$\frac{b-\eta}{2a} = \frac{1}{\pi} ln\left(\frac{4b}{\pi a}\right) \tag{4}$$

However, in the case of composite beams with connectors, there are two cases of interest to demonstrate the applications of Eq. (1) and Eq. (2). The most frequent case is that of beams with planks arranged horizontally one above the other and joined together by vertically arranged mechanical connectors, as shown in Fig. 4. In this case, to determine the cross-section, quantity, and spacing of the mechanical connectors, Eq. (1) could be used or the Collignon-Zhuravski formula given by Eq. (3) could be applied if the width/depth ratio condition of 0.25 is satisfied.

On the other hand, if the composite beam has its planks arranged side by side, linked solidly to horizontally arranged mechanical connectors, as shown in Fig. 5, it is no longer possible to use the Eq. (1) or Eq. (3) and it is essential to know the value of the shear stress τ_{zx} to calculate the cross-section, quantity, and spacing of the mechanical connectors. For this, as in the previous case, one option would be to use the exact equation given by Eq. (2). The other option would be to use an approximate formula, with a certain range of validity.

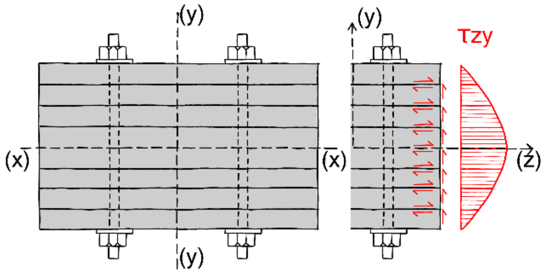

Fig. 4 Shear stresses τ_{zy} acting on the connectors of a beam composed of planks, arranged one on top of the other.

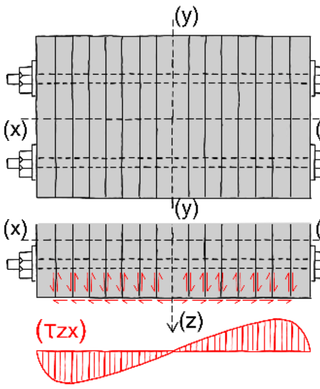

Fig. 5 Shear stresses τ_{zx} acting on the connectors of a beam composed of planks, arranged side by side.

Although there are calculation examples provided by [Aghayere, et al., 2007], [Desai, 2002] and even the recommendations of [NAHB, 1981], according to [DeStefano, et al., 1997], all the calculation cases are based on one of the two methods, the empirical or the rational method. Therefore, all of them dispense with the use of the horizontal shear stress τ_{zx} for the solution of the problem, which leads to conservative results. According to [Riola Parada, 2016], research on combining timber and steel in beams is relatively recent. Very few authors have attempted to systematize and organize all the existing information, some of which are based on geometry and reinforcement arrangement. In residential construction, composite beams, which are the subject of this research, according to [Coulbourne, 2017], are referred to as site fabricated beams, including the flitch and built-up beams. In the case of flitch-beams, according to [Wiesenfeld, 1989] there are different design practices, linked to practical experience and observation of structural behavior, which vary greatly from designer to designer in determining the section, pattern, and spacing of mechanical connectors. This situation is possibly attributable to the lack of a solid theoretical basis.

It is interesting to mention that there are no current references or discussions on theoretical attempts for the calculation of horizontal shear stresses τ_{zx} developed in cross-section beams of considerable width, whether homogeneous beams or composite beams, within the elastic field. Only a complex, extensive, and poorly disseminated exact solution developed (see Eq. (2)) and published by [Reissner, 1946], applicable in the elastic field, with a great theoretical value but of tortuous practical application, is available. However, after extensive literature review and publications, it is concluded that a simplified equation, equivalent in terms of the same degree of simplicity and practical applicability to the famous Collignon-Zhuravski equation, has never been developed.

For this reason, this work aims to obtain an alternative equation to that given by Eq. (2), of greater and notable simplicity than that developed by [Reissner,1946] through the theory of elasticity, to calculate the values of the horizontal shear stresses τ_{zx}. This new equation would allow the case of the composite beams in Fig. 5 to be solved in a relatively simple way. Subsequently, to validate the results, they will be contrasted with those obtained with the theory of Eq. (2) developed by [Reissner, 1946] and numerical simulation, through the implementation of the Finite Element Method.

2. Methodology

2.1 Analytical Modelling

To obtain the alternative equation of the stress shear τ_{zx}, the beam in Fig. 6 is considered, where a differential element is extracted for the analysis of its equilibrium. According to Fig.7, Eq. (5) is applied, and Eq. (7) is obtained, as a function of the shear stresses τ_{zy}, τ_{zx} and the bending stress σ_{zz}. It is essential to note that, to obtain a general expression that is valid for all boundary and load conditions, the variation of τ_{zx}, with respect to the variable z is initially considered to be non-zero. Subsequently, depending on the boundary and load case, the variation of τ_{zx} with respect to the variable z could be zero, which would simply indicate that the shear stress is constant along the length of the beam.

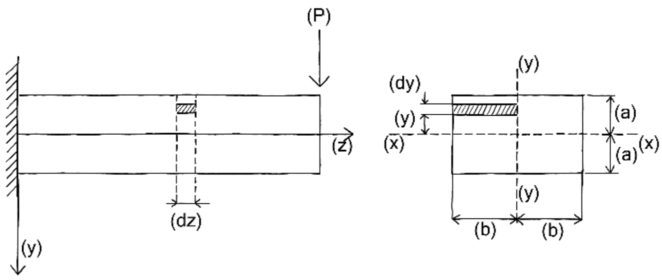

Fig. 6 Analysis of a differential element in a cantilever beam subjected to a point load at its free edge.

$$\sum M_{y-y} = 0 \tag{5}$$

$$\int_0^b \left(\frac{\partial \sigma_{zz}}{\partial z} dz\right) dxdy \left(\frac{b}{2}\right) - \int_0^b \left(\frac{\partial \tau_{zy}}{\partial y} dy\right) dxdz \left(\frac{b}{2}\right) - \int_0^b \left(\frac{\partial \tau_{zx}}{\partial z} dz\right) dxdy(dz) = 0 \tag{6}$$

$$\int_y^a \sigma_{zz}(bdy)\left(\frac{b}{2}\right) - \int_y^a \tau_{zy}(bdz)\left(\frac{b}{2}\right) - \int_y^a \tau_{zx} bdy(dz) = 0 \tag{7}$$

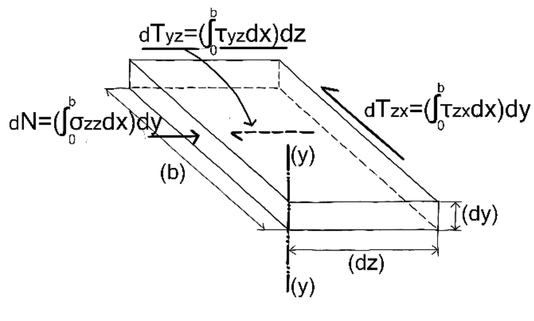

Fig. 7 Analysis of the equilibrium of moment around the y-axis of the differential beam element.

From Eq. (7) the value of the shear stress τ_{zx} can be made explicit. For this purpose, an energy equation was used, obtained from a differential element coming from the same beam subjected to bending. The hypothesis indicates that the internal work produced by the bending stress σ_{zz} and shear stress τ_{yz}, on the displacements Δ_σ and Δ_τ respectively, will be equal to the work produced by the bending moment, along the rotation of the section, along the width "b". In the first member of the equality given by Eq. (8), the contribution of the shear stress τ_{zx} has not been considered, because it is assumed that its value and distortion will be small relative to the shear stress τ_{zy} generated in the plane of loading.

$$\frac{1}{2}\int_y^a dT_\sigma + \frac{1}{2}\int_y^a dT_\tau \simeq \frac{1}{2}\int_y^a dT_M \,.\, \psi_1\psi_2 \tag{8}$$

$$\frac{1}{2}\int_y^a dN\Delta_\sigma + \frac{1}{2}\int_y^a dT_{yz}\Delta_\tau = \frac{1}{2}\,(M.d\varphi)\left(\frac{b}{2b}\right)\left(1 - \frac{y^2}{a^2}\right) \tag{9}$$

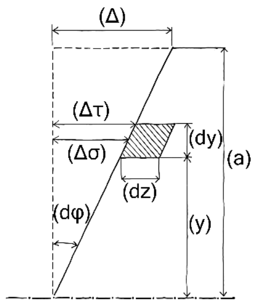

Fig. 8 Rotation of the section under the action of the bending moment and determination of the horizontal displacements.

In Eq. (9), the displacements Δ_σ and Δ_τ, are expressed by Eq. (10) and Eq. (11) respectively, deduced from the Fig. 8:

$$\Delta_\sigma = \left(y + \frac{dy}{2}\right)\left(\frac{\Delta}{a}\right) \tag{10}$$

$$\Delta_\tau = (y + dy)\left(\frac{\Delta}{a}\right) \tag{11}$$

Replacing Eq. (10) and Eq. (11) in Eq. (9), we obtain Eq. (12):

$$\int_y^a \sigma_{zz}(bdy) \int_y^a \left(y + \frac{dy}{2}\right) + \int_y^a \tau_{yz}\,(bdz) \int_y^a (y + dy) = \frac{M}{2} \tag{12}$$

By operating algebraically on Eq. (12), we obtain Eq. (13). From Fig. 9, it is shown that it is possible to replace Eq. (13) in Eq. (7) to obtain Eq. (14).

$$\tau_{\substack{yz \\ (a-y)}} = \frac{Q}{(b)\int_y^a (y+dy)}\left[\frac{1}{2} - \left(\frac{1}{I}\right)\left(\frac{S^n}{2}\right)\int_y^a \left(y + \frac{dy}{2}\right)\right] \tag{13}$$

$$\frac{1}{4a}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right)\left(y + \int_y^a dy\right) + \frac{1}{4a}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right)\left(y + \int_y^a \frac{dy}{2}\right) - \frac{1}{4}\frac{Q}{(2a)(2b)} = \frac{T_{zx}}{(2a)(2b)}\frac{1}{b}\left(y + \int_y^a dy\right) \tag{14}$$

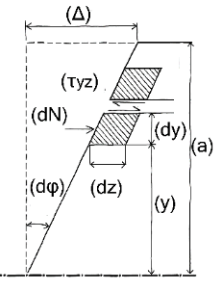

Fig. 9 At the boundary shared by both differential elements, the shear stress τ_{yz} is equalized.

Integrating the differentials in Eq. (14), we obtain Eq. (15):

$$\frac{1}{4}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right) + \frac{1}{4a}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right)\left(\frac{y+a}{2}\right) - \frac{1}{4}\frac{Q}{A} + C = \frac{T_{zx}}{A}\left(\frac{a}{b}\right) \tag{15}$$

In Eq. (15), for y=0, we obtain Eq. (16):

$$\left[\frac{1}{4}\left(\frac{1}{2}\frac{3Q}{2A}\right) + \frac{1}{8}\left(\frac{1}{2}\frac{3Q}{2A}\right) - \frac{Q}{4A} + C\right]\left(\frac{b}{a}\right) = \tau_{zx} \tag{16}$$

To meet the boundary condition, where the value of the shear stress τ_{zx} is zero, the value of the constant "C" from Eq. (17) was determined. This value is given by Eq. (18).

$$\tau_{zx} = \left[\frac{Q}{32A} + C\right]\left(\frac{b}{a}\right) \tag{17}$$

$$C = -\frac{Q}{32A} \tag{18}$$

In Eq. (15), for y=a, we obtain Eq. (19):

$$\tau_{zx} = -\frac{9}{32}\frac{Q}{A}\left(\frac{b}{a}\right) \tag{19}$$

Having determined the extreme values of the equation given by Eq. (15), it is necessary to verify whether for the remaining "y" coordinates, given by Eq. (20), all the values of the corresponding shear stresses have the same sign or whether there is a change of sign.

$$0 < \frac{y}{a} < 1.0 \tag{20}$$

Using Eq. (15) and assigning different values to the "y" coordinate according to Eq. (20) and for different values of width/ depth, we obtain the curves in Fig. 10, where we can see the crossing point where the shear stress is zero.

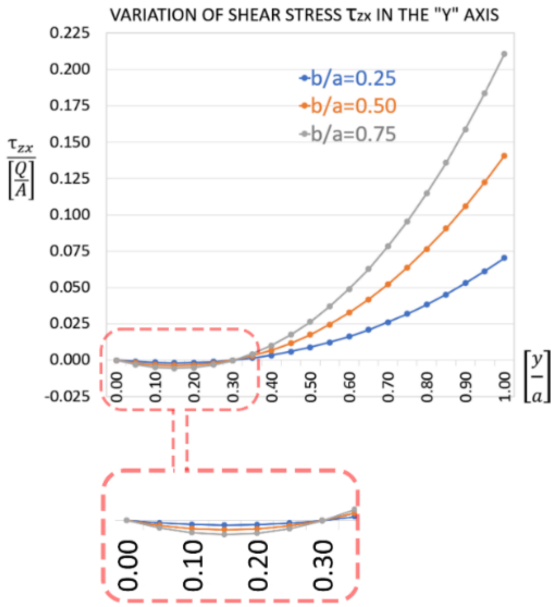

Fig. 10 Analysis of the shear stress τ_{zx} values along the depth "a" of the section, considering different width/depth ratios.

The value at which the condition of Eq. (21) is satisfied is always given for the "y" coordinate given by Eq. (22). This gives Eq. (23):

$$\tau_{zx} = 0 \tag{21}$$

$$\frac{y}{a} = 0.30277 \tag{22}$$

$$\frac{1}{4}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right) + \frac{1}{4a}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right)\left(\frac{y+a}{2}\right) - \frac{1}{4}\frac{Q}{A} + C = 0 \tag{23}$$

Since the shear stress τ_{zx} is a self-balancing system according to [Fliess,1974] (See Fig.3), a parabolic stress law is proposed by Eq. (24), where the maximum peak of shear stress is reached in the middle of the segment "b", according to Fig. 11. To determine the value of the peak of shear stress τ_{zx} in a parabolic distribution, the value previously found, having assumed a uniform distribution, is equated with the result of the integral of the parabolic curve, according to Eq. (25).

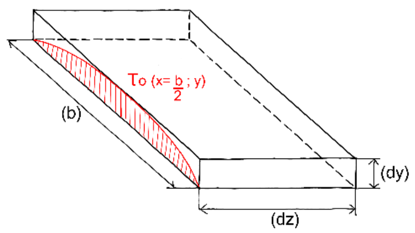

Fig. 11 Substitution of the shear stress distribution law τ_{zx}, by a parabolic distribution law.

$$\int_0^b 4\tau_o\left[\frac{x}{b} - \left(\frac{x}{b}\right)^2\right]dx = \tau_{zx} b \tag{24}$$

$$4\tau_o\left[\frac{x^2}{2b} - \frac{x^3}{3b^2}\right]_0^b = \tau_{zx} b \tag{25}$$

At the values of x=0 and x=b, the zero values of shear stress τ_{zx} occur. While at x=b/2, the maximum peak of shear stresses τ_{zx}, the value of which is given by Eq. (26):

$$\tau_o = \frac{3}{2}\tau_{zx} \tag{26}$$

To obtain the general expression of the peak shear stress τ_{zx} for different values of the y-coordinate, the expression given by Eq. (15) was substituted into Eq. (26) to obtain Eq. (27):

$$\tau_{zx(z;x=\frac{b}{2};y)} = \frac{3}{2}\left[\frac{1}{4}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right) + \frac{1}{4a}\left(\frac{Q}{I}\right)\left(\frac{S^n}{2}\right)\left(\frac{1}{2b}\right)\left(\frac{y+a}{2}\right) - \frac{1}{4}\frac{Q}{A} + C\right\}\left(\frac{b}{a}\right)\right] \tag{27}$$

When the value of the "y" coordinate is equal to the segment "a", Eq. (27) is greatly simplified, and Eq. (28) is obtained:

$$\tau_o = \frac{9}{32}\left(\frac{b}{a}\right)\tau_{\substack{Collignon \\ Zhuravski}} \tag{28}$$

Having obtained the simplified equation for the calculation of stresses τ_{zx}, it is necessary to proceed to the validation of the results. As a first option, the exact solution obtained from the theory of elasticity is proposed, as can be seen from the extensive and complex equation provided by Eq. (2). Finally, a second totally independent validation method is proposed, based on numerical simulation using the Finite Element Method.

2.2 Finite Element Modelling

According to the benefits of numerical simulations, highlighted by [Vaidotas, et al.,2012], the use of the finite element method has been considered as an alternative solution to compare and validate the results. For the comparison of the results, in the elastic field domain, the finite element software called "MEFI", version 1.2.3, has been used. For the discretization of a cantilever loaded at its free edge, as shown in Fig.12, 32-node hexahedral cubic serendipitous elements have been used. The number of elements has been varied in intervals differing by 2500 elements, starting from a less dense to a higher density discretization. For the selection of the analysis point, a point with the coordinates set by Eq. (29) has been considered. This situation is shown in Fig.3 and Fig.14, according to [Reissner,1946]:

$$\tau_{zx}(x = \eta; y = a) \tag{29}$$

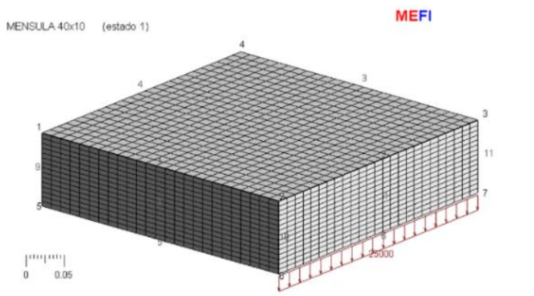

Fig. 12 Discretization of a cantilever beam, using the finite element method, subjected to a uniform load at its free end.

For the numerical simulation, a timber defined by [Porteous et al.,2007], as class C-14, is considered. The rest of the parameters are included in Table 1.

Table 1. Parameters considered for the numerical simulation of the model proposed by [Reissner, 1946].

	[Parameters]	[Characteristic Values]
Geometric	Cantilever width (2b)	0.40m
	Cantilever depth (2a)	0.10m
	Cantilever length (L)	0.40m
	Ratio (b/a)	4.0
	Element type (SCH)	Serendipitous Cubic Hexahedral (32 nodes)
Loads	Uniform edge load (q)	25000 N/m
Material	Wood	C-14
	Density (ρ)	290.0 N/m^3
	Modulus of elasticity (E)	$7.0x10^9$ N/m^2
	Poisson's Coefficient (ν)	0.30

3. Results

For a practical application case, the case of a cantilever beam subjected to a point load at its free end for a rectangular cross-section with a ratio (b/a=4.0) is analyzed. As mentioned above, the results obtained through the simplified equation given by Eq. (28) will be verified by comparison with the results obtained by Eq. (2), obtained by [Reissner, 1946], and finally through the numerical simulation implemented by the Finite Element Method.

3.1 Application of Simplified Equation

First, the simplified equation proposed by Eq. (28) is applied. The maximum value of stress τ_{zx} is obtained at the coordinate point (x=b/2; y=a), as indicated in Eq. (30):

$$\tau_{zx\,(x=\frac{b}{2};y=a)} \approx 4.22x10^5\,\frac{N}{m^2} \tag{30}$$

3.2 Application of the Exact Solution of Elasticity Theory

Next, the exact solution obtained from the theory of elasticity by [Reissner, 1946] given by Eq. (2) is applied to determine the stresses τ_{zx}. To apply Eq. (2), the value of Poisson's modulus ν=0.30, according to [Porteous et al.,2007], for a wood classified as C-14, is assumed. Replacing values in Eq. (2) gives Eq. (31). As for the location of the coordinate of the point where the maximum value of shear stresses τ_{zx} is reached, it is necessary to apply the expression provided by Eq. (4) and making η explicit gives Eq. (32).

$$\tau_{zx(x=\eta;y=a)} = 3.9285x10^5\,\frac{N}{m^2} \tag{31}$$

$$\eta \approx 0.14818\,m \tag{32}$$

3.3 Application of the Finite Element Method

Finally, the numerical simulation is carried out using the Finite Element Method. According to [Beer, et al. 2006], considering a point sufficiently far from the load application zone or from the support conditions of the cantilever beam, the results of the shear stress τ_{zx} can be considered reasonably acceptable, as can be seen in Fig. 13 and Fig. 14.

After analyzing different mesh densities, finally for a mesh composed of 10.000 elements, the value of the stress τ_{zx} given by Eq. (33) was obtained for the point located at the distance "η" by Eq. (34).

$$\tau_{zx(x=\eta;y=a)} \approx 4.1622x10^5\,\frac{N}{m^2} \tag{33}$$

$$\eta \approx 0.14818\,m \tag{34}$$

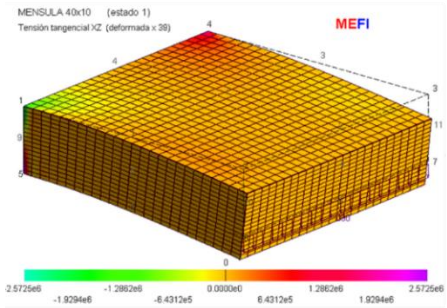

Fig. 13 Result of the shear stresses τ_{zx}, for a mesh with a density of 10.000 elements of the 32-node hexahedral cubic serendipity type.

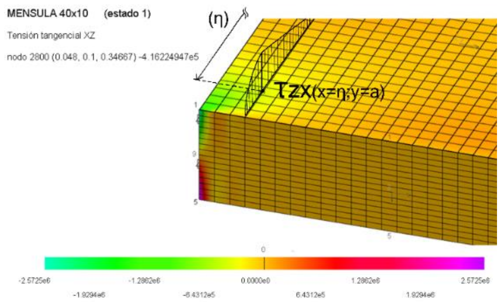

Fig. 14 Analysis of the maximum value of the shear stress τ_{zx} at a distance η, from the vertical axis of symmetry, at a certain distance from the embedment.

4. Comparison of results

By plotting the curves for different width/depth ratios of the beam on the abscissa axis and the ordinate axis, the ratio of the peak shear stress τ_{zx} at the y=a coordinate, and the Collignon-Zhuravski shear stress, one can compare the values of the elasticity theory tabulated by [Timoshenko, et al.,1951], with the values of the simplified equation given by Eq. (28). In Fig. 15 we see the graph of equation given by Eq. (35), for the values calculated by both theories. It is mentioned that the Poisson coefficient of 0.25 was used to plot the curve proposed by [Reissner, 1946].

$$\frac{\tau_{zx}=\tau_o}{\frac{3Q}{2A}} = \alpha \tag{35}$$

Finally, the curves obtained through the application of the three theories are plotted for the case study, as shown in Fig.16. Furthermore, the internal shear stress law τ_{zx} in the rectangular cross section, along depth "a", was plotted applying the approximate theory proposed in this work, given by Eq. (27) and the theory of elasticity according to the general expression given by Eq. (2) deduced by [Reissner,1946]. (See Fig. 17). In the approximate stress law τ_{zx} there is a reduced segment of "a", expressed by the Eq. (22). In this interval, shear stress values τ_{zx} of opposite sign and negligible magnitude occur. Interestingly, there is a remarkable improvement in the approximation of the point-to-point stress values when a linear stress law is considered, whose maximum value is given by Eq. (28). Additionally, a linear approximation, for a y-coordinate, is proposed, established by Eq. (36):

$$\tau_{zx\,(x=\frac{b}{2};y)}\left[\frac{9}{32}\left(\frac{b}{a}\right)\tau_{\substack{Collignon\\Zhuravski}}\right]\left(\frac{y}{a}\right) \qquad (36)$$

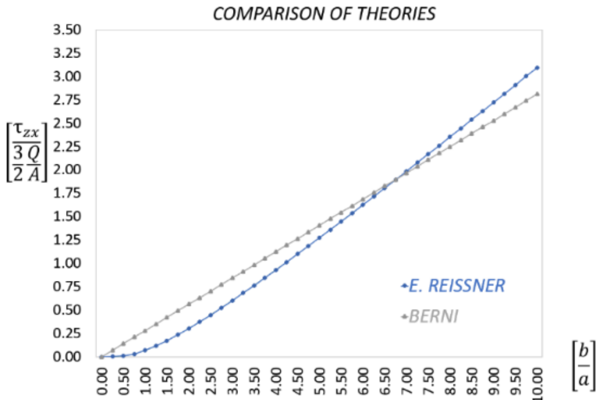

Fig. 15 Comparison of the maximum shear stresses τ_{zx}, obtained from the theory by [Reissner, 1946] and the proposed approximate method.

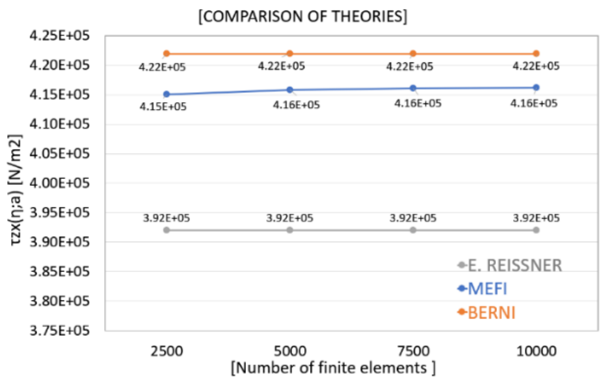

Fig. 16 Verification of the maximum shear stress τ_{zx} applying the proposed approximate equation given by Eq. (28), the equation given by Eq. (2) obtained by [Reissner,1946], and the Finite Element Method.

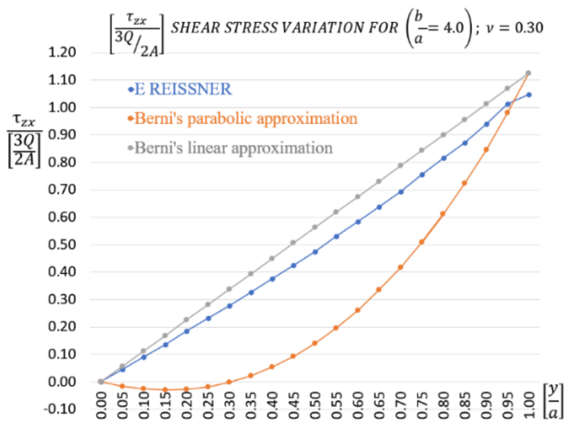

Fig. 17 Comparison of the variation of the stress law τ_{zx} proposed by [Reissner, 1946], along the depth "a" and the proposed approximations.

Table 2. Relative error rate of the linear and parabolic approximations, with theory proposed by [Reissner, 1946].

Theory	Arithmetic Mean/(3Q/2A)	Δ (%)
E. Reissner	0.4983	0.00
Berni's Parabolic Approx.	0.4513	-9.43
Berni's Linear Approx.	0.5625	+12.88

5. Discussion

- Comparing both curves, in Fig. 15, we see that the exact theory of elasticity yields higher values than the approximate theory at b/a above 7.00. In any case, its application can be considered acceptable up to b/a=10.0, where the error made is -10%. Therefore, we could establish a validity interval for the theory given by Eq. (37):

$$0.25 \le \frac{b}{a} \le 10.0 \qquad (37)$$

- According to Fig. 10 and Eq. (27), there is an interval given by Eq. (38) where the shear stresses result with a negative sign and negligible value. Within this interval, the peak value of the shear stress τ_{zx}, is given by Eq. (39).

$$0 < \frac{y}{a} \le 0.30 \qquad (38)$$

$$\tau_{zx\,max} = \tau_{zx\,(x=\frac{b}{2};\,y=0.15a)} \qquad (39)$$

- The "y" coordinate where the peak shear stress occurs is defined by Eq. (40):

$$\frac{y}{a} = 0.15 \qquad (40)$$

- Regardless of the width of the beam and within the indicated validity interval, the coordinate at which the shear stresses τ_{zx}, for the parabolic distribution, satisfies the condition of Eq. (21) always occurs at the coordinate defined by Eq. (22), as can be seen in Fig.10, for the parabolic distribution.
- From Fig. 17, it is observed that according to Eq. (2) a straight line is obtained as a linear variation law for the shear stress τ_{zx}, while with the approximate theory, according to Eq. (27), a parabolic variation is obtained, so they differ notably except in the extreme values. In addition, it was decided to incorporate a second option, where the value of the maximum peak shear stress τ_{zx} and a linear approximation are used. The expression given by Eq. (36) turned out to be much simpler than even Eq. (27), not to mention that it also fits more closely point to point with theory given by [Reissner,1946]. Table 2 shows that when considering the arithmetic mean, a good approximation has been determined with the parabolic approximation, considering only the values of positive sign and disregarding those of negative sign. Nevertheless, by using the linear approximation, the result improves considerably. Moreover, the point-to-point variation of the stresses τ_{zx}, given by Eq. (36) proved to be a better approximation than the parabolic distribution given by Eq. (27).
- From Fig. 15, from b/a=4.25, the shear stress is equal to the Collignon-Zhuravsky shear stress. Continuing with the same analysis up to b/a=10, we see that it is now equal to about 2.81 times. This revealed the great relevance of the shear stress τ_{zx} in beams of considerable width, relative to their depth. In composite beams, particularly those using metal plates as structural reinforcement, combined with timber planks, it is possible to obtain structural elements of considerable width, (when the homogenization of the composite cross-section is performed) subjected to this shear stress phenomenon τ_{zx}.

6. Conclusions

- In the energy equation given by Eq. (8), the energy contribution of the shear stress τ_{zx} was neglected because its magnitude was considered negligible compared to the value of the energy contributed by T_σ and T_τ.
- Note that the influence of Poisson's coefficient has not been considered in the approximated equation given by Eq. (15), to simplify the analysis.
- In Eq. (2), the maximum value of the peak stress occurs at the coordinates $(x = \eta; y = a)$. Unlike the proposed theory, where the maximum value occurs at (x = b/2 ; y = a) due to the parabolic law of symmetrical distribution.
- The direct application of Eq. (36) allows solving the cases of composite beams with vertical planks, arranged side by side and joined together by means of transversally arranged connectors (see Fig. 5), within the range of validity established by Eq. (37).
- Additionally, it is possible to apply Eq. (36), to solve the case of composite beams with vertical planks, arranged side by side and joined by means of adhesives or a combination of adhesives and mechanical connectors.
- The solution proposed by Eq. (36), can be perfectly extended to solve the cases of homogeneous beams of considerable width, within the range of validity established by Eq. (37).

- Currently, there are only two widespread methods to estimate the cross-section, quantity and spacing between connectors, as detailed in [DeStefano, et al., 1997], obviously without considering the finite element method. One empirical and one rational, however, interestingly, both methods dispense with the use of the shear stress τ_{zx} during the analysis of the horizontal connectors, to estimate their cross-section, quantity and spacing.
- After an exhaustive review of the specialized literature, it has been concluded that this is possibly attributable to the complexity involved in the use of expression given by Eq. (2), the application of which is a greater complexity than the set of any of the methods mentioned above. It is also interesting to note the lack of diffusion from which Eq. (2) suffers. Possibly attributable to the lack of practicality at the time of application. In addition, the search for a quick and practical solution has possibly contributed to the repeated and systematic application of empirical methods, which ignore the nature of the shear stresses τ_{zx} that develop in composite beams of considerable width. Although they have yielded satisfactory results over the years, it is also fair to mention that according to [Reissner,1946], the results are conservative.
- It is also important to highlight the simplicity and reasonable approximation of the expression given by Eq. (36), for the calculation of the shear stress τ_{zx}, for a depth "y", within the range of validity established by Eq. (37).

Nomenclature

(τ_{zy})	shear stress in the y-direction in the xy-plane;
(τ_{yz})	shear stress in the z-direction in the xz-plane;
(τ_{zx})	shear stress in the x-direction in the xy-plane;
(σ_{zz})	normal stress in the z-direction to the xy-plane;
(Δ)	maximum displacement of the section at the y=a coordinate, after rotation;
(Δ_σ)	displacement in the z-direction, at the same coordinate as the normal stress in the z-direction;
(Δ_τ)	displacement in the z-direction, at the same coordinate as the shear stress in the xz-plane;
(dT_σ)	work differential due to normal stresses in the z-direction;
(dT_τ)	work differential due to shear stresses on the xz-plane in the z direction;
(dT_M)	work differential due to the bending moment which rotates the cross-section by a certain angle;
(ψ_1)	coefficient of proportionality of the width of the cross section being studied;
(ψ_2)	coefficient of proportionality for the calculation of the rotational work of the cross-section;
(dM)	bending moment differential;
$(d\varphi)$	differential angle of rotation;
(S^n)	static moment of the cross-section;
(P)	external load applied to the beam;
(Q)	shear stress in cross section;
(Q_z)	shear stress in cross-section in the z-coordinate
(a)	half the depth of the cross-section;
(b)	half the width of the cross-section;
(A)	cross-sectional area;
(I)	moment of inertia of the cross section;
(dN)	differential normal force on the yx-plane;
(dT_{yz})	shear force differential in the z-direction, acting on the xz-plane;
(dT_{zx})	shear force differential in the x-direction, acting on the xy-plane;
(τ_o)	maximum peak shear stress in the x-direction, on the xy-plane.
$(\tau_{Collignon\ Zhuravski})$	uniform shear stress in the y-direction in the xy-plane, at the y=0 ordinate, according to [Beer et al.,2006];
(η)	value of the x-coordinate, according to [Reissner,1946] with respect to the vertical y-y axis, where the maximum horizontal shear stress value, for a y-coordinate, occurs;
$\tau_{zx}(x=\eta; y=a)$	shear stress in the x-direction in the xy-plane, at coordinates (x= η; y=a), according to [Reissner,1946].

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