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Decision Tools for the Engineering of Steel Structures

Michael Gedig[1]

AMEC Dynamic Structures, Ltd., Port Coquitlam, BC, Canada
E-mail: mike.gedig@amec.com

Siegfried Stiemer

University of British Columbia Department of Civil Engineering, BC, Canada

ABSTRACT: This paper gives an overview of currently available tools for decision making in the field of steel structures engineering. The theory of decision making in business, economics, and politics is well established. There is a wide array of methods and software that is used to implement the theory. In particular, the realm of decision making under conditions of uncertainty is of particular importance to the practicing engineer. This paper describes methods and software that may be used to inform and guide the decision making process in engineering practice. Examples are provided to illustrate the application of these tools to steel structures engineering.

KEYWORDS: steel structures; engineering; decision analysis; decision theory; decision making

1 INTRODUCTION

Most engineers have been exposed to the idea at some point in their education that theories exist for making rational decisions in business and economics. This exposure invariably occurs during a course in undergraduate engineering economics, which is now likely associated with vague memories of urns filled with coloured balls and spinning roulette wheels. Games of chance and contrived laboratory experiments are often used to explain statistical decision theory, the science of making decisions under uncertainty. For most structural engineers, decision making theories remain largely forgotten and of little practical importance. This paper describes currently available techniques and software that may be used to effectively work with decision theory in order to improve the quality of decisions made in engineering practice. Specific examples are given to show how these tools can be used in the field of structural steel engineering.

Decision making under uncertainty pervades all engineering phases, from conceptual design development through fabrication and installation to operation and maintenance. Although many design and analysis procedures are well defined and deterministic, the reality in engineering practice is that most key engineering decisions must be made before such procedures have been completed. For example, decisions on whether to bid on a design-build job and how to select a bid design concept are generally made with very limited available time. Economic realities usually do not allow a number of competing design options to be fully developed before making a decision on which option to pursue for the final design. It is generally well understood that the earlier decisions are made in the engineering design process the greater the economic implications to a project. As a project advances through its various design stages from concept to completion, significant changes become more difficult and expensive to make. Tools that may be used to help make better decisions, particularly at the uncertain early stages of a project, have the potential to sig-

[1] Corresponding author: mike.gedig@amec.com; 1515 Kingsway Ave., Port Coquitlam, B.C., Canada, V3C 1S2

nificantly impact the technical and financial outcome of a project.

It should be stated that the application of decision making theories to practical situations in business and economics has been the subject of much debate. Decision science is a relatively modern science, and has evolved considerably over the past century. Both game theory, developed in the 1940s by von Neumann and Morgenstern, and statistical decision theory, developed in the 1960s by Raiffa, are examples of the normative, or prescriptive branch of decision theory. Normative theories such as these are often formal mathematical treatments describing the idealized behaviour of rational decision makers with access to perfect information. In contrast, the descriptive branch of decision theory has its roots in human behavioural research and attempts to explain the behaviour of people in realistic decision scenarios. Extensive laboratory testing was conducted in the 1970s to develop and test hypotheses for descriptive decision theory. Some of this research has been criticized on the basis that the often complicated and contrived scenarios presented to students bear little resemblance to real world situations, where the decision maker has considerable insight and stake in the decision process. Some also argue that there is no reason to assume that human decision makers are essentially rational, that they are more strongly influenced by emotion, instinct, imitation, habit, suggestion, and other illogical forms of thinking. The predominating view is that prescriptive decision theories have greater value as method of structuring and analyzing a decision than as a way of finding an irrefutable prescription for action. Significant differences between a prescribed solution and the views of the decision makers often give important clues about how the decision model is structured, and force decision makers to better understand their biases and inconsistencies.

2 PROBABILITY

Probabilities are classified as objective or subjective depending on how they were derived. In the classical probability theory, the probability of an event occurring is defined as the number of outcomes which lead to the event divided by the total number of possible outcomes. Thus the probability of drawing a jack out of a standard deck of 52 cards is 4/52 = 0.0769 according to the classical approach. In the relative frequency approach the probability of an event occurring is estimated by repeating an experiment a large number of times or by gathering data and determining the frequency with which an event has occurred in the past. For example, if a weld inspector tests 100 identical components and finds 4 require weld repair then this information suggests that the probability is 4/100 or 0.04 that a component has a faulty weld. Of course, this estimate is only valid if all welding conditions remain unchanged. Both the classical approach and the relative frequency approach describe objective probabilities.

Clearly, objective probabilities are of limited application in structural engineering decision problems. In structural engineering, most decision problems concern unique events or one-off decisions. Generally there is not enough statistical data on past problems, or else conditions for those problems were not similar enough to the present situation to be able to determine a probability using the relative frequency approach. For these reasons, subjective probabilities are normally used in structural engineering decision problems. A subjective probability is an expression of an individual's degree of belief that a particular event will occur. Subjective probabilities vary from individual to individual, even when they have access to the same information.

Some people are skeptical about subjective probabilities. Research has shown that subjective assessment of probabilities may be of poor quality and strongly influenced by the method in which they are obtained (Goodwin & Wright, 1991). Subjective probabilities are used extensively in decision analysis for several reasons. In many cases, they represent the best information available to the decision maker. Experts in a given field have accumulated a body of knowledge and experience so that their judgments about subjective probabilities are usually based on sound facts and principles. Representing a judgment numerically in the form of a probability rather than verbally, for example, allows a much less vague assessment to be made. Furthermore, the resulting statement can be precisely communicated to others, and allows a decision maker's views to be challenged and explored. When using subjective probabilities it is essential to perform a sensitivity study to understand how the outcome of a decision model changes in response to the chosen probability value. Often, sensitivity analysis indicates that major changes can be made to probabilities in decision models before affecting the recommended course of action. Finally, a systematic method is available in Bayes' Theorem to test and refine the hypothesis suggested by a subjective probability as more information becomes available.

Although in most practical problems the probabilities will be subjective, these probabilities must still conform to the underlying axioms of probability theory, including the Kolmogorov axioms (Kolmogorov, 1956), restated here as follows:

1 The probability of an event occurring must be non-negative;
2 The probability of an event which is certain to occur is 1;
3 The probabilities of two or more mutually exclusive independent events ($p(A \cap B)=0$) can be added, i.e. $p(A \cup B) = p(A) + p(B)$;
4 The probability that 2 or more independent events will occur together in succession is the product of all the individual probabilities i.e. $p(A \cap B) = p(A) \cdot p(B)$ (joint probability);
5 The conditional probability of event A, given event B, is defined by $p(A|B) = p(A \cap B)/p(B)$ on condition $p(B) \neq 0$; if A and B are independent, $p(A|B) = p(A)$.

The first 2 axioms imply that the probability of an event occurring must be at least zero and no greater than 1.

3 GRAPHICAL TOOLS

Decision analysis models are often graphically represented using decision trees and influence diagrams. Although similar, decision trees and influence diagrams show different types of information and are used for different purposes. Decision trees (Magee, 1964) provide a detailed structure in which different possible scenarios or decision paths are shown as sequential branches linked from left to right in the order they would occur. Influence diagrams (Howard & Matheson, 1981) define the general structure of the model, showing the decision variables and the dependencies between those variables. Figure 1 shows a decision tree and influence diagram used to model a simplified problem of how a company might choose to bid on a new project.

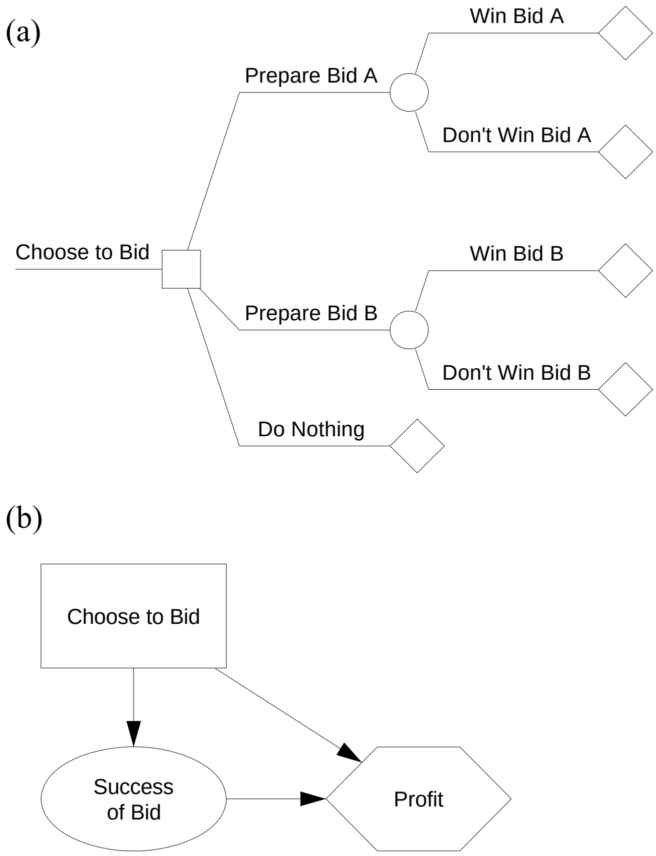

Figure 1: Graphical representation of a bid decision (a) decision tree, and (b) influence diagram

Decision trees provide a detailed picture of the decision problem, clearly showing the different possible scenarios and the sequence in which decisions and chance events occur. Decision trees contain three types of nodes (decision , chance, and utility), and paths directed from left to right between nodes. Nodes in a decision tree represent different types of variables. Decision nodes, usually drawn as rectangles, represent variables controlled by the decision maker. Chance nodes, usually drawn as circles or ellipses, are random variables that represent uncertain quantities in the decision model. Utility nodes, also known as terminal or value nodes, are usually drawn as diamonds. Utility nodes correspond to the leaves of the decision tree, and represent the value or utility of an outcome in the decision process. The use of decision trees contributes to comprehensive analysis and clear communications between decision makers. Decision trees clearly outline the risk and uncertainty associated with different decision paths and promote a comprehensive organization of alternative strategies.

Similar to decision trees, influence diagrams contain three types of nodes (decision, chance, and utility). Influence diagrams contain two types of arcs (influences and informational arcs). A directed arc in an influence diagram normally indicates an influence, where the node at the tail of the arc influences the state of the node at the head of the arc. Arcs from a decision node to a chance node imply that the decision will impact the probability distribution of the random variable. Arcs coming into decision nodes are informational arcs that do not denote influences but temporal precedence. Informational arcs reflect the sequence in which decisions are made and the information that will be available when a decision is made.

Although clear, intuitive and detailed, decision trees have several disadvantages in comparison to influence diagrams. First, the number of nodes in a decision tree increases exponentially with the number of decision and chance variables, and even very small decision scenarios require a large tree. Often decision trees contain many identical subtrees. Although strategies exist for collapsing subtrees and for pruning complex trees, these operations result in some loss of clarity. A further limitation of decision trees is that all variables must be treated as discrete (with a finite number of alternatives), even if they are continuous. The influence diagram is a more compact representation of a decision problem than a decision tree. Influence diagrams were originally developed as a method of compactly representing decision trees for symmetric decision scenarios (decision trees with similar branches); however, they are now seen more as a decision tool for use with Bayesian networks.

4 EXPECTED VALUE

The expected value is a standard measure for selecting a preferred alternative in decisions involving uncertainty. The expected value is a weighted average of outcomes, calculated as the sum of the products of all outcomes from independent states multiplied by their associated probabilities of occurrence. The expected value EV for an alternative i is given by

$$EV(i) = \sum_j P(j)O(i,j), \text{ and } \sum_j P(j) = 1, \qquad (1)$$

where P(j) is the independent probability of state j, and O(i,j) is the outcome of state j for alternative i. The expected value can be interpreted as an average value which will result if a process is repeated a large number of times. Despite this assertion, expected values are often used in unique situations. When it is possible to assign probabilities to future states the expected value criterion may be used to select a preferred alternative. When there is insufficient reason to believe one state is more probable than another, then each should be assigned an equal probability of occurrence, following the “equal-likelihood” criterion.

A decision tree showing expected value calculations for the bid decision is shown in Figure 2. In this example, Bid A leads to a project that produces an expected profit of \$740 000. The cost of preparing Bid A is \$10 000. For Bid A, the expected value is 0.20(\$740 000) + 0.80(\$-10 000) = \$140 000. Similarly, the expected value for Bid B is \$120 000. If the goal is to maximize expected value, the firm should choose to submit Bid A.

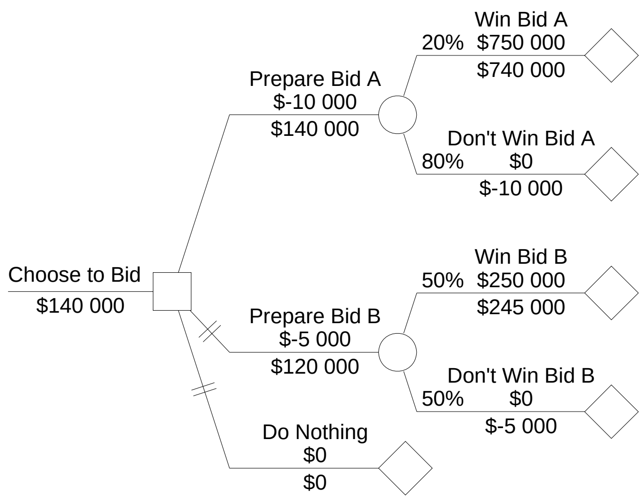

Figure 2: Decision tree with expected value

One important feature of the expected value criterion is that it fails to take into account the decision maker’s attitude to risk. This feature is illustrated by the St. Petersburg Paradox (Martin, 2004). The St. Petersburg game is played by flipping a fair coin until it comes up tails. The total number of flips determines the payoff, which is $\$2^n$. For example, if the coin is flipped three times before a tail appears on the fourth, then n=4 and the payoff is $\$2^4=\$16$. The expected payoff for n=4 is $(1/2)^4(\$16) = \$1$, as it is for every possible consequence. The expected value of the game is infinite since there is an infinite number of possible consequences n = 1,2,3, and so on. Most people would not pay even $20 to enter such a game, which indicates that there may be significant discrepancies between the expected value criterion and human behaviour. This problem, known as the St. Petersburg Paradox, was discovered by the Swiss mathematician Daniel Bernoulli in the eighteenth century. Bernoulli observed that the expected value calculations are in error because they use dollar value outcomes instead of the expected utilities of each consequence. He introduced the now widely accepted principle that money has a decreasing marginal utility, and suggested that a realistic measure of the utility of money might be given by the logarithm of the money amount. The function proposed by Bernoulli indicates an unwillingness to gamble for a very small chance at a very large prize.

5 UTILITY THEORY

In decision theory, utility is a measure of the "desirability of consequences of courses of action in a decision made under uncertain conditions" (Krippendorff, 1986). The underlying assumption in Utility Theory is that the decision maker always chooses the alternative for which the expected utility is maximized. To determine the expected utility, a utility has to be assigned to each of the possible consequences of each alternative. A utility function maps utility to the range of outcomes of a decision, depending on the decision maker's preferences and attitude toward risk. Utility Theory is therefore intrinsically related to the concepts of risk and uncertainty in decision making.

Utility Theory was formalized mathematically by the classic work of von Neumann and Morgenstern (1944), *Theory of Games and Economic Behavior*. Von Neumann and Morgenstern introduced a set of necessary and sufficient axioms, and promoted the development of methods to measure utilities on numerical scales. In their theory of expected utility, they defined an expected utility function over lotteries or gambles, in contrast to Bernoulli's utility function, which was defined over money. There are several methods for eliciting utility functions from individuals. In the method of certainty equivalence, a utility rating is assigned to a certain-to-be-received amount that is considered equivalent to a gamble at given probabilities of a certain gain or loss. In this method, an individual is assumed to have a utility function with the following key properties: 1) if outcome A is preferred to outcome B, the utility of A is greater than the utility of B and the converse is true; and 2) if an individual has a contract that carries a payoff of A with a probability of p and a payoff of B with a probability of (1-p), the utility of the contract is the expected value of the utilities of the payoffs. Using the second property, the utility of a contract, U(C), can be calculated from the utilities of the payoffs, U(A) and U(B), and their probabilities as:

$$U(C) = U(A)\cdot p + U(B)\cdot(1-p) \qquad (2)$$

For example, say that an analyst elicits a decision maker's utility function for monetary values in the range $0 to $40 000, so that U($0)=0 and U($40 000)=1. If the decision maker would pay $10 000 for a hypothetical lottery ticket which gave a 50% chance of $0 and a 50% chance of $40 000, this implies that U($10 000) = 0.5·U($0) + 0.5·U($40 000) = 0.5(0) + 0.5(1) = 0.5. This type of analysis is repeated to generate the complete utility function.

Utility functions can be used to establish risk tolerance, a mathematical quantity which describes a decision maker's attitude toward risk. A concave utility function (where the inside of the curve faces down on the utility function graph), such as the logarithmic function proposed by Bernoulli, implies that an individual is risk-averse. Such a person will accept a certain outcome, say $10, that is lower in value than the expected value of a risky gamble that may result in winning less or more, for example, a 0.5 chance of winning $30 and a 0.5 chance of receiving nothing. On the other hand a convex utility function indicates a person is risk seeking and will prefer to gamble rather than settle for a sure thing. Although a risk averse attitude is shared by many people, it cannot be used as the basis for decision making behaviour, since there are also many people who exhibit risk-seeking behaviour. For example, many people buy lottery tickets even though the cost of a ticket is more than the expected utility. Attitude toward risk varies between people and business organizations, and depends on the circumstances under which the risk occurs.

Utility Theory was proposed as a normative, rational model as opposed to a descriptive model of human behaviour. The fact that people make inconsistent judgments and are sometimes not rational should not invalidate the theory. As mentioned, there are several different methods for eliciting utility functions from individuals. Goodwin & Wright (1991) state that "utilities appear to be extremely sensitive to the elicitation method which is adopted." Risks involving losses tend to produce different utility functions than risks involving gains. Different utility functions are produced when one is asked to buy or sell the hypothetical lottery ticket used in the elicitation process. Research by Tversky and Kahneman (1981) indicated that the way in which choice is framed affects the decision maker's response, the so-called framing problem. They discovered that choices involving statements about gains tends to produce risk-averse responses, while those involving losses are often risk-seeking. Goodwin & Wright (1991) propose several methods for overcoming problems in the elicitation of utility functions. He suggests several methods should be used, and that inconsistencies should be investigated. Also, in order to avoid the framing problem, questions should be phrased in a way that values are closely related to the values in the actual decision problem. Kahneman and Tversky (1996) proposed Prospect Theory as an alternative to Utility Theory. According to Prospect Theory, people value a certain gain more than a probable gain with an equal or greater expected value. The function relating the subjective value and the corresponding losses is steeper than that for gains.

Utility Theory has important implications for engineering decision making. Utility Theory forms the basis for developing utility scales and methods that assign numbers to intangibles. Engineering decision making often involves complex tradeoffs between performance, cost, and intangible items. Values such as time, prestige, knowledge, experience and other indirect costs and benefits are examples of intangibles that are often overlooked in engineering decision making. Utility Theory provides a mathematical basis for making these complex tradeoffs between different attributes. Also, the theory may be used to force the decision maker to look at whether an additional unit of money, time, performance or other attribute produces a linear increase in utility.

Multi-attribute Utility Theory (MAUT) extends the application of Utility Theory to problems involving more than one attribute. Although several methods have been proposed for this analysis, the methods of Keeney and Raiffa (1976) are widely known and accepted. An early application of the theory was to study alternative locations for a new airport in Mexico City in the early 1970s. The study considered cost, capacity, access time, safety, social disruption and noise pollution. The U.S. military has used MAUT in the design of major weapons systems to make tradeoffs of cost, weight, durability, lethability and survivability (Chelst & Edwards, 1987). To perform the analysis of decisions involving multiple attributes, the utility function for each attribute must be derived individually. A multiattribute utility function is then defined. An important simplifying assumption in the definition of the multiattribute utility function is that of mutual utility independence. Multiattribute problems can become complex if the attributes are not mutually utility independent.

Multi-attribute Utility Theory belongs to the field of Multi-Criteria Decision Making (MCDM). Other alternative methods for rank ordering alternatives are Analytic Hierarchy Process (AHP), due to Saaty (1982) and the Simple Multi-attribute Rating Technique (SMART), developed by Edwards (1971). Both methods have the advantage of being less complicated and more transparent than the MAUT approach. AHP and SMART are widely used to facilitate public policy decisions.

6 VARIANCES

Statistical analysis provides other tools in addition to the expected value criterion for evaluating uncertain options. Measures of the variation of outcomes from the mean give an indication of the risk inherent in a decision alternative. For two options with the same expected value, the one with outcomes having less variation from the mean is considered to have less risk.

For a random variable X with expected value E[X], the variance of X, denoted by Var(X) or □2, is given by

$$Var(X) = E[x^2]-(E[x])^2 \quad (3)$$

The standard deviation is simply $\sigma = (Var(X))^{1/2}$. The variance of the sum of 2 random variables, say *X* and *Y*, is

$$Var(X+Y) = Var(X) + 2\,Cov(X,Y) + Var(Y) \quad (4)$$

where *Cov(X,Y)* is the covariance of the product of the variables *X* and *Y*. If the 2 variables are independent, then $Var(X+Y) = Var(X) + Var(Y)$. Note also that $Var(X-Y) = Var(X) + Var(Y)$. The equation for the variance of the product of two independent random variables *X* and *Y* is given by:

$$\sigma^2_{XY} = E[X]^2\sigma^2_Y + E[Y]^2\sigma^2_X + \sigma^2_X\,\sigma^2_Y. \quad (5)$$

There is no general formulation for the variance of a product of two random variables where covariance exists between the variables.

The above expressions for the sums and products of random variables may be used to create stochastic models for use in decision making. For example, say that a pin connection in a structure has a pin with diameter A and a hole with diameter B. The diametral clearance C between the pin and hole is B-A. Assuming that the as-manufactured dimensions of the pin and bore are normally distributed random variables, the risk that pin does not fit can be calculated from the probability distribution for the variable C using the expressions above. For simple stochastic models such as this, direct analytical methods using statistical analysis can be employed. As models become more complicated, such as when nonlinear functions or more variables are involved, direct analytical methods quickly become infeasible. In these situations, simulation is the preferred approach.

7 SIMULATION

Monte Carlo simulation is a numerical method used to find solutions to mathematical problems using random numbers. Often the method is used when the problem involves uncertainty, a large number of variables, or nonlinearities, or other features which make it difficult to solve analytically. Because of this, it is well suited to real-world problems, which often have complex nonlinear relationships with no closed-form solutions. The method becomes more efficient compared to other numerical methods as the dimension of the problem increases. Monte Carlo simulation is classified as a sampling method because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population. The simulation is an iterative process where sampling is repeated until a statistically significant distribution of outputs is obtained. In the pin connection problem discussed in the previous section, Monte Carlo simulation could be used to sample values from the probability distributions defining the diameter of the pin and hole. The clearance is then calculated from each pair of dimensions, leading to a distribution of clearance values.

The Monte Carlo method is a method of analyzing stochastic uncertainty propagation, which is used to determine the impact of uncertainties such as random variation, lack of knowledge, or errors, on the performance of a system. Given representative probability distributions as input, the method generates distributions that are useful in the decision making process. Similar to decision trees, Monte Carlo simulation results in an expected value that can be used in selecting the most attractive course of action. In addition to expected value, the simulation results can be converted to error bars, reliability predictions, tolerance zones, and confidence intervals. The usefulness of the results obviously depend on the input distributions, which should closely match existing data or represent the current state of knowledge.

Monte Carlo simulation requires a relatively large number of model evaluations which are not practical without the use of the computer. In fact, the history of the Monte Carlo method is tightly linked to the history of the computer. The first significant application of the Monte Carlo method was run on the first electronic computer, the ENIAC, to solve a problem in thermonuclear physics in 1947 (Metropolis, 1987). Von Neumann and Ulam developed a statistical approach to solving the problem of neutron diffusion in fissionable material. The method was named by Ulam, who implemented the solution with Metropolis (1949).

Monte Carlo methods have been used to solve problems in a wide range of areas, including finance, business, physics, mathematics, chemistry, medicine, manufacturing and engineering. Monte Carlo simulation is used extensively in business for risk and decision analysis to account for uncertainties in market trends and cash flows. The method is used in the modeling of materials and chemicals to study grain growth in metallic alloys, the behaviour of polymers, and protein structure predictions (*Impact of Monte Carlo methods on scientific research*, 2004). Sampling methods may be used to efficiently evaluate complicated and many-dimensional integrals. In engineering applications, Monte Carlo simulation is used for uncertainty analysis, optimization, and reliability-based design.

A relatively large number of samples must be completed using the Monte Carlo method, since increasing the sample size reduces the standard error of the results. In some cases the method is not practical because of the computation requirements, which may limit the complexity of the model. Research has been directed at the problem of improving the speed and efficiency of the method. Variance reduction is a technique that may be used to decrease computation time. The Latin Hypercube Sampling method is an alternative that modifies the way in which the input distribution is sampled in order to reduce the number of solutions required compared to the simple Monte Carlo method (Isukapalli, 1999). In this method, the range of probable values for each uncertain input parameter is divided into ordered segments of equal probability which are sampled in such a way as to generate samples from all the ranges of possible values, producing information about the extremes of the output probability distributions. Structural reliability analysis, a well-known engineering application of sampling methods, involves very low probabilities of failure and requires a large number of samples. More efficient sampling alternatives to Monte Carlo simulation, including first-order and second-order reliability methods (FORM and SORM), as well as response surface methods (RSM), are often used in structural reliability problems.

8 OPTIMIZATION

A vast array of analytical and numerical techniques are available to determine the optimum assignment of resources to minimize or maximize some aspect of a system. Optimization techniques may be classified in a number of different ways, based on the types of variables and equations used in the mathematical model of the system. Systems may be modeled as linear or nonlinear, continuous or discrete, deterministic or stochastic, and constrained or unconstrained. Optimization techniques are also classified as global or local, depending on whether they find the nearest local minimum relative to a given starting point, or whether they find the minimum value over all possible values of input. Table 1 lists a number of the available optimization techniques. Rao (1996) has written a thorough description of most commonly used engineering optimization techniques. A comprehensive online guide to numerical optimization techniques and software is maintained as part of NEOS, the Network Enabled Optimization System (2004).

Optimization has been used in structural engineering to reduce costs while satisfying safety and serviceability constraints. Most structural optimization work has employed local optimization techniques using relatively simple objective functions, generally to minimize material weight. Although there is a range of different techniques for doing local optimization, many practical problems in engineering have nonlinear components that can create a solution space with several locally optimum solutions. At this point there seems to be no sound systematic way of solving such problems. As Pinter (1998) indicates, "even the most advanced mathematical programming and scientific computing environments lack a universal and proven direct solver capability to tackle continuous problems which possess multiple optima."

Table 1: Optimization methods

- Linear Programming
    - Simplex Method
    - Revised Simplex Method
    - Primal-Dual Simplex Method
    - Dual Simplex Method
    - Interior Point Methods
    - Decomposition Method
    - Sensitivity Analysis
    - Parametric Programming
    - Quadratic Programming
- Nonlinear Programming
    - Analytical Methods
        - Equality Constraints
            - Lagrange Multiplier Method
        - Inequality Constraints
            - Kuhn-Tucker Conditions
    - One-Dimensional Minimization Methods
        - Elimination Methods
        - Unrestricted Search
        - Exhaustive or Simultaneous Search
        - Dichotomous Search
        - Fibonacci Method
        - Golden Section Method
    - Interpolation Methods
        - Quadratic Interpolation
        - Cubic Interpolation
        - Newton's Method
        - Quasi-Newton Method
        - Secant Method
    - Unconstrained Optimization Methods
        - Direct Methods
            - Random Search Method
            - Grid Search Method
            - Univariate Method
            - Pattern Search Methods
            - Powell's method
                - Hooke-Jeeves' Method
                - Rosenbrock's Method
            - Simplex (Polytope) Method
        - Descent Methods
            - Steepest Descent (Cauchy) Method
            - Fletcher-Reeves Method
            - Newton's Method
            - Marquardt Method
            - Quasi-Newton Methods
                - Davidon-Fletcher-Powell (DFP) Method
                - Broyden-Fletcher-Goldfarb-Shanno (BFGS) Method
    - Constrained Optimization Techniques
        - Direct Methods
            - Random Search Methods
            - Heuristic Search Methods
            - Sequential Linear Programming Method
            - Sequential Quadratic Programming Method
            - Methods of Feasible Directions
                - Zoutendijk's Method
                - Rosen's Gradient Projection Method
            - Generalized Reduced Gradient Method
        - Indirect Methods
            - Transformation of Variables
            - Sequential Unconstrained Minimization
                - Interior Penalty Function Methods
                - Exterior Penalty Function Method
                - Exterior Interior Penalty Function Method
                - Penalty Function Method for Parametric Constraints
                - Augmented Lagrange Multiplier Method
- Geometric Programming
- Dynamic Programming
- Integer Programming
    - Cutting Plane Method
    - Branch and Bound Method
    - Balas Method
    - Generalized Penalty Function Method
    - Sequential Linear Discrete Programming Method
- Stochastic Programming
- Separable Programming
- Multiobjective Optimization
    - Pareto Optimum Solution
    - Utility Function Method
    - Global Criterion Method
    - Lexicographic Method
    - Goal Programming Method
- Global Optimization
    - Exact Methods
        - Naïve Approaches
        - Enumerative Search Strategies
        - Homotopy (Parameter Continuation), Trajectory Methods, and Related Approaches
        - Successive Approximation (Relaxation) Methods
        - Branch and Bound Algorithms
        - Bayesian Search Algorithms
        - Adaptive Stochastic Search Algorithms
        - Interval Analysis Methods
    - Heuristic Methods
        - Global Extensions of Local Search Methods
        - Genetic Algorithms
        - Simulated Annealing
        - Neural-Network-Based Methods
        - Fuzzy Systems
        - Tabu Search
        - Scatter Search
        - Approximate Convex Global Underestimation
        - Continuation Methods
        - Sequential Improvement of Local Optima
- Meta-Heuristic Methods

8.1 *Sensitivity analysis*

Sensitivity analysis is used to study the impact of changes in a model's input parameters on the solution. In models involving uncertain inputs, sensitivity analysis is critical in determining the impact of assumptions about uncertain quantities on the quality of the results. Sensitivity analysis results are often presented in graphical form, which intuitively shows which inputs are the most critical. Sensitivity analysis helps to guide the decision of where resources can be most efficiently utilized in refining the analysis model and collecting more input data.

Sensitivity analysis may be performed using a number of different techniques, some more informal than others. Sensitivity testing may be done by studying the response of a model to changes in the formulation of the model, and to changes in input parameter values and combinations. Analytical methods involve the differentiation of equations governing the model, and the subsequent solution of the resulting sensitivity equations. Sampling-based methods require multiple model runs using different input parameter combinations, or sample points, and the sensitivity is estimated using the output at those points.

9 SOFTWARE

A large number of software applications are available which implement the methods discussed in this paper. A survey of software with potential application to engineering decision making is given in the Appendix. For each software package, the capabilities are presented in tabular format. The applications were not rated or evaluated hands-on, and the capabilities shown generally rely on information provided by the software manufacturer.

As a general comment, some of the software packages may not be suitable for use in the typical structural engineering office. Some may be too expensive, specialized, or have hardware requirements unsuited for the decision making scenarios that occur in engineering practice. The table is presented as an attempt to give insight into the capabilities of various commercial software products that might be useful in engineering decision making, both in research and in practice.

Spreadsheet applications deserve special attention in the context of decision making. The ubiquitous Microsoft Excel program has some capabilities that might not be apparent to the casual user. What-if analysis, expected utility calculations, Monte Carlo methods, optimization and sensitivity analysis can be done directly in Excel without additional third-party add-ins. Although there exist better programs for large-scale problems, the spreadsheet is an invaluable tool for everyday use and for gaining familiarity with the various techniques. Excel comes with a general-purpose optimizer for small-scale linear, integer, and nonlinear problems, and can do linear programming and nonlinear programming with continuous or discrete variables.

10 APPLICATIONS

There is a range of potential practical applications of decision analysis tools in structural steel engineering. Engineering by its nature involves many uncertainties, and the engineer is often expected to decide between competing requirements in uncertain conditions. A number of applications for engineering decision making under risk and uncertainty are described in this section.

10.1 *Design*

Design decisions made early in the life of a project have greater economic impact than those made later on. The early stages of a project have the greatest uncertainty. Decisions made at the conceptual stage may have profound impacts on detail design, fabrication, shipping and erection of the structure. It is becoming increasingly important for engineers to make decisions which take into account the true cost drivers of a structural project. Decision analysis tools may be used to help make more rational decisions early, and to quantify the risk associated with different design options.

10.2 *Optimization*

When steel structures are optimized for weight independent of other cost considerations, the most efficient structure is one characterized by many different member cross-sections and connection configurations. Taken to the extreme, minimum weight optimization leads to structures which are usually not the most cost-effective solution overall. Optimization for minimum cost generally favours structures with more common components: a limited set of member sizes, bay or module sizes, and connection types. Symmetry of design is often a key to cost-effective structures, even when the loading is not strictly symmetrical and weight optimization would lead to non-symmetric structures. Currently the engineer has few tools to approach cost optimization in a systematic way; intuition, experience, and heuristics are the norm in practice. Some structural design programs allow minimum weight optimization with constraints that favour common member sizes; however, there are many additional cost-drivers at the fabrication and erection stages which these programs do not consider.

As a further illustration of the caveats of minimum weight optimization, consider the design of structures using hollow structural section (HSS) members. Minimum weight member design favours relatively thin-walled sections. Thin-walled sections are more prone to local buckling and wall yielding limit states. Increasing connection resistance by using internal stiffener plates is generally a highly inefficient option. The most cost-effective approach with HSS structural design is to choose heavier wall sections to alleviate the need for external or internal stiffening plates. This situation is not limited to structural design with HSS members; a similar situation arises in beam to column moment connections using wide flange members. The most cost-effective solution often involves selecting heavier columns than dictated by the minimum weight solution in order to avoid the need for column stiffener plates and doubler plates.

As a general observation, optimization to minimize overall costs produces more conservative designs than optimization to minimize member weight. Using more conservative connection design details for example may result in greater numbers of common connection details, less detailing time and less shop fabrication time. Reduced activities durations improve project schedule performance and reduce overall costs.

10.3 *Overall costs*

Knowledge of overall costs throughout all stages of a project produces better designs. Although much of this information may be tightly bound to experience and intuition, decision analysis offers techniques for quantifying complex decision factors and allowing them to be queried and reused. It is possible to incorporate the impact of design decisions on a structural engineering project into a decision model. Such a model could be capable of modeling the impact of design choices on: design, fabrication and erection time; fabrication costs, based on the ease of fabrication and possibility of errors and related rework; the cost of fabrication jigs; inspection costs, depending on accessibility and method; estimating errors, accounting for higher risk of error with unfamiliar designs; shipping costs; erection falsework costs; and safety, and related costs of injury claims.

Overall costs are not limited to hard costs; the value of intangibles should be factored into engineering decisions as well. Utility Theory provides a basis for quantifying non-monetary values. Examples of intangible factors in structural engineering include: ease of maintenance, inspectability, employee satisfaction, and the impact of repetition (there may be an element of greater satisfaction in designing one versatile component as opposed to many similar, but different designs).

10.4 *Estimating*

Accurate cost estimation of steel structure fabrication plays a key part in successful bidding of new jobs and in setting baselines for production planning and control. Estimators forecast the costs of producing components as a function of piece size and weight, based on specific information about shop conditions, including overall layout, equipment, and manpower resources. Traditionally, estimators use average values based on historical data for known operations in a particular shop. Since the shop layout is typically fixed, it is possible to create simulation models to study the flow of work through a given shop to understand production bottlenecks and other inefficiencies. Many computer programs have been written to optimize production planning and control. Rather than use historical data, a promising new avenue for estimating steel fabrication costs is through virtual shop modeling. Virtual shop simulation models can be integrated into the engineering decision making process to provide accurate cost information

10.5 *Tolerances*

Tolerances on structural steel encompass mill tolerances, fabrication tolerances due to imprecision in the measuring, cutting, fitting and welding processes, and erection tolerances. In particular, the welding process is highly uncertain and there is no reliable method of quantitatively predicting distortion in steel weldments due to weld shrinkage. Tolerances specified incorrectly by the engineer may have considerable impact on overall structural costs. Fabrication tolerances that are too narrowly specified unnecessarily increase costs because of additional labour and possibly unnecessary additional jigs. On the other hand, tolerances specified too loosely may cause fit problems, also increasing costs due to additional time and rework. Tolerances have a similar impact in the field, having the potential to significantly increase costs and back charges. Tolerance specification has implications to safety as well, as steel members that that do not fit might be forced into position, putting additional unaccounted stresses into a structure and increasing risk of injury to the erector. In critical applications, structural tolerances can be simulated by Monte Carlo simulation or more general reliability methods. Tolerances can also be included in overall structural cost models to balance tradeoffs between the additional cost of poorly fitting members and the additional cost of maintaining tighter tolerances in the shop.

10.6 *Metrology*

Metrology, the scientific study of measurement, is a topic where uncertainty plays a key role. Measurement instruments have limited accuracy, and there are the questions of how the measured point represents a theoretical datum on the object being measured, and on the impact of temperature on the object. Of course, it is always possible to spend more to achieve a more accurate measurement, but there is value in being able to reason with uncertain information and make logical predictions. For example, in erecting a structure subject to measurement uncertainty and temperature fluctuations, how much time should be spent aligning components, and what is the best that can be achieved with the components at hand, understanding their accuracy limitations? Structural metrology problems can be approached using simulation methods. Bayes' method, which uses conditional probabilities to make statistical inferences about the state of a model, may be used to improve measurement accuracy by efficiently incorporating new data into existing sets of measurements.

10.7 *Testing*

Material and component testing is often a cost-effective and less risky alternative to further analysis studies. In addition, testing is often necessary when dealing with designs not covered by applicable codes and standards, new materials and designs, or components that were incorrectly fabricated or installed but costly to remake or rework. Test results may be summarized using classical statistical methods and incorporated into decision analysis models.

10.8 *Planning and scheduling*

Risk analysis can be incorporated into planning and scheduling in order to estimate the probabilities of meeting key project milestones. It is common practice to use only deterministic scheduling tools which do not reflect uncertainty in data inputs and do not perform sensitivity analysis.

10.9 *Forecasting*

Forecasting is an important activity for determining business plans in virtually any industry. Statistical analysis tools such as regression analysis and correlation models may be used to analyze historical data to predict future occurrences. Recent fluctuations in the price of steel highlight the need to accurately forecast and include the risks of possible future variations on current decisions.

10.10 *Material and component selection*

Decision analysis tools may be used to develop a rational basis to select materials and components for a design. The Analytical Hierarchy Process (AHP) has been used to characterize the selection of materials for bridges by highway officials in the U.S. (Smith, Bush, & Schmoldt, 1997).

CONCLUSION

Many well-established decision making tools are currently used in business, economics and politics, however these tools have yet to find widespread application in the engineering office. Decision making theories have a wide range of potential applications in structural engineering, from design through to fabrication and installation. Several potential applications of decision analysis tools to structural steel engineering have been presented here. Decision theories are particularly well suited to conditions of uncertainty, and offer significant economic benefits by improving the quality of decisions made early in the design process. Further research work in improving decision making during conceptual and preliminary engineering stages is therefore recommended.

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APPENDIX: SOFTWARE SURVEY

Software		Vendor	Functions
DECISION ANALYSIS			
1	Analytica	Lumina Decision Systems Inc.	Visual tool for creating, analyzing, and communicating decision models; influence diagrams; create and manage multidimensional tables; Monte Carlo simulation and sensitivity analysis http://www.lumina.com/software/influencediagrams.html
2	Comlab Games	Carnegie-Mellon University	Game theory based software for instructors and students to design, run, and analyze the outcomes of games played over the internet; modules for strategic form, discrete form, free form, market game http://www.cmu.edu/comlabgames/
3	Criterium Decision Plus	InfoHarvest	Decision management software using multi-criteria analysis (AHP and SMART) and uncertainty analysis http://www.infoharvest.com/ihroot/index.asp
4	Crystal Ball	Decisioneering	Decision analysis and simulation add-in for Microsoft Excel; what-if analysis, monte carlo simulation, sensitivity analysis, trend charts, linear and nonlinear global optimization, time-series forecasting (cyclic pattern analysis) using linear regression http://www.decisioneering.com
5	DecisionPro	Vanguard	Integrated application for building models that aid in decision making; decision tree analysis; Monte Carlo Simulation (custom and predefined input distributions, automatic distribution fitting to historical data, correlated inputs, unlimited number of stochastic inputs); forecasting (regression-based curve fitting, exponential smoothing; Fourier analysis); Markov simulation; linear and integer programming optimization (Simplex method); general business modeling http://www.vanguardsw.com
6	DecisionTools	Palisade	Risk analysis and simulation add-in suite for Microsoft Excel; @RISK Monte Carlo simulation, sensitivity and scenario analysis; PrecisionTree influence diagrams and decision tree add-in; TopRank what-if analysis and Tornado Charts; RiskOptimizer combined genetic optimization algorithms and Monte Carlo simulation; BestFit data fitting routines http://palisade.com
7	DPL	Syncopation Software	Sequential decision analysis and real option valuation using influence diagrams, decision trees and Bayesian networks; supports multiple objective analysis; links with Microsoft Excel; Monte Carlo simulation with continuous random variables; mix continuous chance nodes with discrete chance nodes and decisions; outputs can be exported in XML format http://www.syncopationsoftware.com/
8	ERGO	TechnologyEvaluation	MAUT decision support environment; single-user; detailed knowledge bases for decision support activities in ERP, CRM, HR, SCM, PLM, EAM, financial, and security systems http://www.arlingsoft.com
9	EVOLVER	Palisade	Optimization using genetic algorithm technology, a stochastic directed searching technique; add-in for Microsoft Excel; solves linear, nonlinear, stochastic, combinatorial, noisy, or probabilistic problems http://palisade.com
10	Expert Choice	Expert Choice	Decision support software employing Analytic Hierarchy Process (AHP), a mathematically rigorous application for prioritization and decision making; supports group decision making; graphical-based presentation of decision criteria hierarchy http://www.expertchoice.com

Software		Vendor	Functions
11	GAMBIT	Texas A & M University	Open source library of game theory software and tools for the construction and analysis of finite extensive and normal form games; mission of the Gambit Project is to provide libraries and software tools both for teaching game theory, and for doing original research in game theory and its applications http://econweb.tamu.edu/gambit/
12	GeNIe/SMILE	University of Pittsburgh	SMILE class library implementing graphical probabilistic and decision theoretic models, including Bayesian networks, influence diagrams, and structural equation models; GeNIe creates decision theoretic models intuitively using the graphical interface, supports chance nodes with General, Noisy OR/MAX and Noisy AND distribution; complete integration with Microsoft Excel http://www2.sis.pitt.edu/~genie/
13	HUGIN	Hugin Expert A/S	System for construction, maintenance and usage of knowledge bases, based on Bayesian network technology; construction of knowledge bases using Bayesian networks and influence diagrams technology; supports development of object oriented Bayesian networks; automated learning of knowledge bases from databases; wizard for generation of probability tables http://www.hugin.com
14	HIPRE	Helsinki University of Technology	Decision support software integrating the Analytic Hierarchy Process (AHP) and the Simple Multiattribute Rating Technique (SMART); run both methods independently or combine them in one model; visual and customizable graphical interface http://www.hipre.hut.fi/
15	ISMAUT Tools	University of Michigan	Software application of Imprecisely Specified Multi-Attribute Utility Theory in engineering design http://www.eecs.umich.edu/techreports/cse/1996/CSE-TR-289-96.pdf
16	JAVABayes	CMU	Set of tools for the creation and manipulation of Bayesian networks; composed of a graphical editor, a core inference engine and a set of parsers; engine produces: the marginal probability for any variable in a Bayesian network, the expectations for univariate functions (for example, the expected value of a variable), configurations with maximum a posteriori probability; produces marginal distributions and expectations using two different algorithms: variable elimination and bucket tree elimination; conducts robustness analysis on top of inferences; distributed under the GNU License http://www.cs.cmu.edu/~javabayes/Home/
17	Logical Decisions	Logical Decisions	Multiple objective decision analysis; multi-attribute utiliy theory (MAUT); multiple tradeoff and preference weighting techniques including swing weighting, direct entry, SMART, SMARTER, and AHP; preference tradeoff analysis; multi-user capabilities http://www.logicaldecisions.com/
18	Lumenaut	Lumenaut	Decision analysis and statistical analysis add-in for Microsoft Excel; decision trees; sensitivity analysis; parametric and non-parametric statistics including paired tests, ANOVA, regression, correlation, and time series http://www.lumenaut.com
19	MACBETH	Technical University of Lisbon	Interactive approach for converting qualitative judgments to quantitative values for decision analysis; employs an initial, interactive, questioning procedure that compares two elements at a time, requesting only a qualitative preference judgment; automatically verifies consistency as judgments are added; a numerical scale is generated that is entirely consistent with all the decision maker 's judgments; through a similar process weights are generated for criteria; software provides tools to facilitate complete model structuring, management of complex problems involving qualitative value scores and weights, and interactive sensitivity and robustness analyses http://www.m-macbeth.com/Msite.html

Software		Vendor	Functions
20	MSBN	Microsoft Research	Application for Bayesian belief network construction and inference tool called Microsoft Belief Networks; developed by Decision Theory & Adaptive Systems Group (DTAS), focused on investigating the use of probability and utility theory to enhance computer applications and platforms http://www.research.microsoft.com/dtas/msbn/
21	Netica	Norsys	Program for belief networks and influence diagrams; user interface for drawing networks; relationships between variables may be entered as individual probabilities, equations, or learned from data files; compiles belief (Bayesian) networks into a junction tree of cliques for fast probabilistic reasoning; utility-free sensitivity analysis; produces a confusion matrix, error rate, logarithmic and quadratic (Brier) scoring rule results, calibration table and surprise indexes for each node desired; finds optimal decisions for sequential decision problems; extensive built-in library of probabilistic functions and other mathematical functions; facilities for the easy discretization of continuous variables http://www.norsys.com
22	Predict!	RiskDecisions	Database application for recording and managing risks, opportunities and mitigation strategies; web-enabled and client-server versions on Oracle or Sybase http://www.riskdecisions.com
23	REMBRANDT	Delft University of Technology	System for Multi-Criteria Decision Analysis (MCDA) using a direct rating of preferences on a logarithmic scale; created as an amalgamation of the Multiplicative AHP and SMART methods; preference ratios can be expressed in their original magnitudes on a geometric scale or in orders of magnitude on an arithmetic scale; designed for group decision making, with power coefficients assigned to the respective members http://www.inescc.pt/~ewgmcda/Lootsma.html
24	TESS	TechnologyEvaluation	Same as ERGO but with remote file sharing for collaborative decision making (Technology Evaluation Support System) http://www.arlingsoft.com
25	TreeAge	TreeAge Software	Decision trees; influence diagrams; Markov models; multi-attribute models; sensitivity analysis (1-, 2-, 3-way, tornado diagrams); Monte Carlo simulation; Bayes' revision; threshold analysis; spreadsheet links http://www.treeage.com
26	TreePlan	Decision Support Services	Decision modeling add-ins for Microsoft Excel; decision trees; sensitivity analysis (simple plots, spider charts, tornado charts); Monte Carlo simulation http://www.treeplan.com
27	VISA	Simul8 Corp.	Multi-criteria decision analysis tool (MCDA); visual interactive sensitivity analysis http://www.simul8.com/products/visa.htm
SIMULATION			
1	ARENA	Rockwell Software	Process modeling and simulation software; 3D animation; OptQuest optimization includes sampling techniques and advanced error control, and incorporating algorithms based on tabu search, scatter search, integer programming, and neural networks http://www.arenasimulation.com
2	ProModel	ProModel Solutions	Discrete-event simulation and modeling package; manufacturing and logistics; factory and supply chain modeling; VAO (Visualize Analyze Optimize) technology; probabilistic multiple element simulation optimization; probabilistic scheduling; 2D process animation http://www.promodel.com/solutions/m&lhome.asp
3	RDD-100	Holagent	Requirements management software with discrete event simulation capability http://www.holagent.com/products/features.html

Software		Vendor	Functions
4	Simphony	University of Alberta	Decision and risk analysis software; computer simulation platform for construction industry; result of over 10 years research in simulation-based planning in industry; applied to virtual shop model for structural steel fabrication http://irc.construction.ualberta.ca/
5	Simul8	Simul8 Corp.	Discrete-event simulation and modeling; optimization; data fitting; integrated planning, scheduling and simulation; AutoMod link for 3D virtual reality simulation http://www.simul8.com/products/s8prof.htm
GENERAL OPTIMIZATION			
1	AMPL	AMPL Optimization LLC	A comprehensive algebraic modeling language for linear and nonlinear optimization problems, in discrete or continuous variables. Linear programming, network, mixed integer programming, quadratic programming, and general nonlinear programming problems http://www.ampl.com
2	CPLEX	ILOG	High-performance, robust, flexible optimizers for solving linear, mixed-integer, and quadratic programming problems in mission-critical resource allocation applications; Simplex algorithms (primal, dual, network); Barrier solver (primal-dual interior point algorithm including a predictor-corrector strategy, and a crossover algorithm to convert mid-face solutions to basic, vertex solutions); mixed integer programming (variety of branching and node selection techniques, including cuts, heuristics, and branch-and-bound algorithms) http://www.ilog.com
3	Excel	Microsoft	General-purpose optimizer for small-scale linear, integer and nonlinear programming problems; linear programming (dense Simplex method with bounds on the variables); nonlinear programming (variant of Lasdon and Waren's GRG2 Generalized Reduced Gradient code); integer linear and nonlinear programming (branch and bound method using Simplex or GRG2 for subproblems); Premium Solver includes an extended Simplex-based solver for quadratic programming; Large-Scale Solver uses sparse Simplex method using LU decomposition with dynamic Markowitz refactorization and two-sided bounds on both variables and constraints http://www.microsoft.com
4	FQSP	AEM Design	Nonlinear and minmax constrained optimization, with feasible iterates; algorithms based on the concept of feasible sequential quadratic programming http://www.aemtechnology.com/aemdesign/FSQPframe.htm
5	GAMS	GAMS Development Corp.	A high-level modeling system for mathematical programming problems; consists of a language compiler and a stable of integrated high-performance solvers; tailored for complex, large scale modeling applications; linear and nonlinear programming, nonlinear programming with discontinuous derivatives; mixed-integer programming (linear and nonlinear); mixed complementarity problems; constrained nonlinear systems; quadratically constrained problems; mixed integer quadratically constrained problems http://www.gams.com
6	MATLAB Optimization Toolbox	MathWorks	Extends the MATLAB environment to provide tools for general and large-scale optimization; linear programming, quadratic programming, nonlinear least-squares, and nonlinear equations; unconstrained nonlinear minimization; constrained nonlinear minimization, including minimax, goal attainment, and semi-infinite minimization problems; quadratic and linear programming; nonlinear least-squares and curve-fitting with bounds nonlinear system of equations solving; constrained linear least-squares; specialized large-scale algorithms for solving large sparse problems; data fitting using curve fitting, nonlinear least-squares, nonlinear zero finding, and nonlinear

Software		Vendor	Functions
			systems of equations http://www.mathworks.com/products/optimization/
7	MOSEK	MOSEK ApS	Designed to solve large-scale mathematical optimization problems; linear problems (integer constrained variables allowed); conic quadratic problems; quadratic and quadratically constrained problems (integer constrained variables allowed); general convex nonlinear problems; simplex and interior-point based algorithms (continuous problems); branch & bound & cut algorithm (mixed integer problems); interior-point optimizer is capable of exploiting multiple processors; C/C++ API, command line; JAVA API and Microsoft .NET API interfaces http://www.mosek.com
8	OptQuest	OptTek Systems Inc.	Optimization software sold primarily as a value-added module imbedded in simulation software (i.e., Crystal Ball, Frontline, Arena, Simul8 etc); uses a combination of metaheuristic procedures from methods such as Tabu search, neural networks, and scatter search find global optimal solutions to nonlinear problems including uncertainty http://www.opttek.com/products/optquest.html
9	OSL	IBM	Open-source operations research software including linear, convex quadratic, mixed integer and stochastic programming programs; BCP parallel branch-cut-price framework; CGL cut generation library; CLP native simplex solver; DFO package for solving general nonlinear optimization problems when derivatives are unavailable; IPOPT interior point algorithm for general large-scale nonlinear optimization; Multifario continuation method for computing implicitly defined manifolds; NLPAPI subroutine interface for defining and solving nonlinear programming problems; OSI open solver interface layer; OTS open framework for tabu search; SBB branch and cut code; SMI stochastic modeling interface for optimization under uncertainty http://oss.software.ibm.com/developerworks/opensource/coin/
10	PCx	Optimization Technology Center	Freely available primal-dual interior-point code for linear programming; implements Mehrotra's predictor-corrector algorithm; solution of a linear system with a large, sparse positive definite coefficient matrix performed with the sparse Cholesky package of Ng and Peyton (Oak Ridge National Laboratory), with minor modifications to handle small pivot elements http://www-fp.mcs.anl.gov/otc/Tools/PCx/
11	SAS/OR	SAS	Linear and mixed-integer programming (revised simplex algorithm with LU factorization of the basis, interior-point algorithm, branch and bound algorithm for integer variables, sparse column representation, crash routine, special ordered sets); network flow programming (primal simplex network algorithm, primal partitioning algorithm, primal-dual predictor-corrector interior-point algorithm, crash routines); quadratic programming (linear complementary, active set techniques); general nonlinear programming (trust region, Newton-Raphson with line search, Newton-Raphson with ridging, quasi-newton methods, double-dogleg method, conjugate gradient methods, Nelder-Mead Simplex method); nonlinear least-squares (Levenberg-Marquardt, hybrid quasi-Newton methods) http://support.sas.com/rnd/app/or/MP.html

Software		Vendor	Functions
12	SOL/UCSD	Stanford Business Software Inc.	A suite of packages for solving linear, quadratic, and nonlinear programs; MINOS for large-scale optimization problems (linear and non linear programs); SNOPT general-purpose software for optimization problems involving many variables and constraints; NPSOL is a set of C dlls for minimizing a smooth function subject to constraints, which may include simple bounds on the variables, linear constraints and smooth nonlinear constraints; QPOPT is a set of subroutines for solving the quadratic programming problem http://www.sbsi-sol-optimize.com/#
13	TOMLAB	TOMLAB Optimization	General purpose development environment in Matlab for research, teaching and practical solution of applied optimization problems; solves sparse and dense problems in the following areas: mixed-integer linear, quadratic and nonlinear programming; semidefinite programming with bilinear matrix constraints; semidefinite programming with LMI (linear matrix inequalities); constrained nonlinear parameter estimation, Minimax and L1 data fitting; global optimization (several minima), box-bounded, nonlinear and integer constraints; costly global nonconvex optimization; linear and nonlinear least squares; nonsmooth optimization; unconstrained optimization; approximation of empirical data to positive sums of exponential functions; mixed complementarity problems http://tomlab.biz/about/
14	XPRESS-MP	Dash Optimization	Suite of optimization software, used to solve linear, integer, quadratic and non-linear optimization problems; linear (LP), mixed integer (MIP), quadratic (QP), mixed integer quadratic (MIQP), non-linear (NLP), and mixed interger non-linear programming problems (MINLP); Simplex optimizer includes primal and dual methods, solves LP problems, and is also used within a branch-and-bound framework; Newton barrier interior point method for solving LP and QP problems; sparse matrix handling; presolve procedure to reduce problem size before it is solved; ability to solve numerically hard or unstable problems which are common in process industries; MIP/MIQP optimizer uses a sophisticated branch-and-bound algorithm http://www.dashoptimization.com/
DESIGN OPTIMIZATION			
1	COSMOS/SM	SRAC	Finite element optimization; sensitivity analysis to examine the effect of various design variables on the results of the optimization analysis (global, local and offset); support for multi-disciplinary optimization analysis involving results from static, dynamic, thermal, fatigue, nonlinear, and buckling analyses; optimization using multiple load cases; modified feasible direction method; singular value decomposition technique; automatic determination of polynomial type; linear, quadratic, and cubic approximations; restart and restore engines; convergence and sensitivity plots http://www.cosmosm.com/pages/products/modules_OPTSTAR.html
2	DesignSpace Extra	ANSYS Inc.	Topology, shape and size optimization; probabilistic design; subproblem approximation (zero order) method; first order method; random design generation; sweep generation; factorial evaluation (statistical tool for design of experiments) using a 2-level, full and fractional factorial analysis; gradient evaluation http://www.designspace.com
3	DesignXplorer VT	ANSYS Inc.	Finite element optimization using Variational Technology, sensitivity analysis, what-if analysis, six sigma design, parametric CAD integration; automatically calculate the entire design envelope within a single finite element solution; handles over 100 discrete parameters, in comparison to traditional design of experiments (DOE) approach which is limited to

	Software	Vendor	Functions
			about 10 parameters; handles geometric, discrete, element property and material variations; uses Uncertainty Variables to support probabilistic analysis and design http://www.ansys.com/ansys/designxplorer_vt.htm
4	EPOGY	Engineous	Process Integration and Design Optimization (PIDO) solution; automatically link applications and iterate the impact that thousands of variables have on the design of a product or process; runs on all operating systems and supports the most relevant CAD, CAE, FEA and CFD packages http://www.synaps-inc.com
5	GENESIS	Vanderplaats Research and Development Inc.	Topology, topography, shape and size optimization; DOT and BIGDOT optimization algorithm http://www.vma.com
6	ISIGHT	Engineous	Integrates and manages the computer software required to execute simulation-based design processes, including commercial CAD/CAE software, internally developed programs, and Excel spreadsheets http://www.engineous.com
7	LINDO	Lindo Systems Inc.	Interactive modeling environment for linear, integer, and quadratic programming problems http://www.lindo.com
8	LINGO	Lindo Systems Inc.	Optimization package including solvers for linear, nonlinear (convex & nonconvex), quadratic, quadratically constrained, and integer optimization http://www.lindo.com
9	OPTISHAPE	Quint	Topology, shape and size optimization http://www.quint.co.jp/english/
10	OptiStruct	Altair Engineering	Topology, topography, shape and size optimization http://www.altair.com
11	STRAP	ATIR Engineering Software	Select member sizes to satisfy drift/deflection criteria; optimization based on cost-factors for each property group; optimization driven by members that have most impact on sway/cost criteria http://www.atirsoft.com/
12	VisualDOC	Vanderplaats Research and Development Inc.	Gradient and non-gradient based optimization, response surface optimization, design-of-experiments (DOE) optimization http://www.vma.com
13	What'sBest	Lindo Systems Inc.	Microsoft Excel add-in for large-scale linear, nonlinear and mixed integer programming; linear solvers (primal and dual simplex; barrier or interior point method); nonlinear solvers (generalized reduction gradient algorithm, crash procedure, steepest edge/descent option, sequential linear programming); automatic selection of algorithm based on problem http://www.lindo.com