eJSE International

Electronic Journal of Structural Engineering, 5 (2005)

Effects of Damping Ratio of Restoring force Device on Response of a Structure Resting on Sliding Supports with Restoring Force Device

A. Krishnamoorthy
Professor, Department of Civil Engineering
Manipal Institute of Technology, Manipal – 576 104 , Karnataka, India

ABSTRACT

Effects of damping ratio of the restoring force device on the response of a space frame structure resting on sliding type of bearing with restoring force device is studied. The NS component of the El – Centro earthquake and harmonic ground acceleration is considered for earthquake excitation. The structure is modeled considering six-degrees of freedom (three translations and three rotations) at each node. The sliding support is modeled as a fictitious spring with two horizontal degrees of freedom. The response quantities considered for the study are the top floor acceleration, base shear, bending moment and base displacement. It is concluded from the study that the displacement of the structure reduces as the damping of the restoring force device increases. Also, the peak values of acceleration, bending moment and base shear decreases as the damping of the restoring force device increases.

KEY WORDS

Base isolation, restoring force device, damping of restoring force device, El – Centro earthquake, sinusoidal ground acceleration.

1 Introduction

Base isolation is an aseismic design approach in which the structure is protected from the damaging effects of severe earthquake forces by a mechanism, which reduces the transmission of horizontal acceleration into the structure. Isolation devices are essentially classified into two types - rubber bearings and sliding bearings. Although rubber bearings have been used extensively in base isolation systems, sliding bearings have recently found increasing applications. The most attractive features of the sliding bearings are their effectiveness for a wide range of frequency inputs. Sliding bearings use rollers or sliders between the foundation and base of the structure. The shear force transmitted to the structure across the isolation interface is limited by keeping the coefficient of friction to a small value. This results in large sliding and residual displacements, which may be difficult to incorporate in structural design. The practical effectiveness of sliding bearings can be enhanced by adding suitable restoring mechanism to reduce the displacements to manageable levels. Several systems have been suggested in the past by Chalhoub and Kelly [2], Bhasker and Jangid [1] and Zayas et al. [8] to accommodate restoring mechanism in a structure isolated by sliding systems. They are in the form of high–tension springs, laminated rubber bearings or by using friction pendulum systems which provide restoring mechanism by gravity. The sliding systems perform very well under a variety of severe earthquake forces and are quite effective in reducing the large levels of the super structure acceleration without inducing large base displacements. The base displacement of the structure can be reduced further by increasing the stiffness of restoring force device. However, this results in increase in the acceleration and the force transmitted to the structure. In the present paper, the effects of damping ratio of the restoring force device on response of a structure resting on sliding type of bearing with a restoring force device is studied. Because of the non - sliding and sliding phases exist alternatively, the dynamic behavior of a sliding structure is highly non linear. Yang *et al.* [7] studied the response of the multi degree of freedom structures on sliding supports using a fictitious spring to the foundation floor. The spring was assumed to be bilinear with a very large stiffness in the non - sliding phase and zero stiffness in the sliding phase. Jangid and Londhe [3] and Jangid [4] analysed the structure resting on sliding type of bearing assuming different equations for non – sliding phase and sliding phases. Vafai *et al.* [6] analysed the multi degree of freedom structure on sliding supports by replacing a fictitious spring in the model of Yang *et al.* [7] by a link with a rigid – perfectly plastic material. In the present analysis, the space frame structure is divided into number of elements consisting of number of columns and beams and at each node six degrees of freedom (three translations and three rotations) are considered. The sliding support is modeled using a fictitious spring beneath each column. The stiffness of spring is considered as a large value in non sliding phase and is taken as zero during sliding phase.

2 Analytical modeling

Figure 1 shows the space frame structure resting on sliding bearings. The structure is divided into number of elements consisting of beams and columns connected at nodes. Each element is modeled using two noded frame element with six degrees of freedom at each node i.e., three translations along X, Y and Z axes and three rotations about these axes. For each element, the stiffness matrix, [k], consistent mass matrix [m] and transformation matrix [T] are obtained and the mass matrix and the stiffness matrix from local direction are transformed to global direction as proposed by Paz [5]. The mass matrix and stiffness matrix of each element are assembled by direct stiffness method to get the overall mass matrix [M] and stiffness matrix [K] for the entire structure. The overall dynamic equation of equilibrium for the structure can be expressed in matrix notation as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\} \qquad (1)$$

Where, [M], [C] and [K] are the overall mass, damping, and stiffness matrices. The damping of the superstructure is assumed as Rayleigh type and the damping matrix [C] is determined using the equation [C] = α [M] + β [K] where α and β are the Rayleigh constants. These constants can be determined easily if the damping ratio for each mode is known. $\{\ddot{u}\}$, $\{\dot{u}\}$, $\{u\}$ are the relative acceleration, velocity and displacement vectors at nodes and {F (t)} is the nodal load vector. $\{u\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}, \ldots u_n, v_n, w_n, \theta_{xn}, \theta_{yn}, \theta_{zn}\}$ where n is the number of nodes. The nodal load vector is calculated using the equation

$$\{F(t)\} = -[M]\{I\}\ddot{u}_g(t) \qquad (2)$$

Where [M] is the overall mass matrix, {I} is the influence vector, $\ddot{u}_g(t)$ is the ground acceleration. The sliding support is modeled using a fictitious spring of stiffness k_b, with two horizontal degrees of freedom and these springs are attached to the base of the bottom column. The restoring force device is modeled as a spring with stiffness, k_r. These springs are attached to the base of each column as shown in figure 1. The Value of the stiffness of the bearing, k_b, and stiffness of restoring force device, k_r, are added to the stiffness matrix [K] of the structure at corresponding degree of freedom to obtain the stiffness matrix of the structure and sliding bearing with restoring force device. The damping of the restoring force device, c_r, is also added to the damping matrix [C] of the structure to obtain the damping matrix of the structure and restoring force device. The value of c_r can be obtained using the equation

$$c_r = 2\zeta_r \sqrt{k_r (m_b + m_s)} \tag{3}$$

where, m_s and m_b are the mass of the structure and mass of base respectively. k_r and ξ_r are the stiffness and damping ratio of the restoring force device.

When the structure is resting on sliding type of bearing with a coefficient of friction equal to, μ, when the mobilized frictional force, F_x, at base will be resisted by the frictional resistance, F_s, which acts against the direction of mobilized frictional force. When the mobilized frictional force F_x, at base is less than the frictional resistance, F_s, (i.e. $|F_x| < F_s$) the structure will not have relative movement at base and this phase of structure is known as non – sliding phase. However, when the mobilized frictional force, F_x is equal to or more than the frictional

Table 1. Material and geometric properties of the structure

T_s (sec)	Mass ($kN\text{-}sec^2/m^2$)		Size of Column (m)		Size of Beam (m)		H(m)	E kN/m^2
	M1	M2	B	D	B	D		
0.25	2	1	0.65	0.65	0.3	0.6	4.0	$2.2x10^7$
0.50	3	2	0.6	0.6	0.3	0.6	3.0	$2.2x10^7$
1.0	5	4.5	0.5	0.6	0. 3	0.6	3.0	$2.2x10^7$

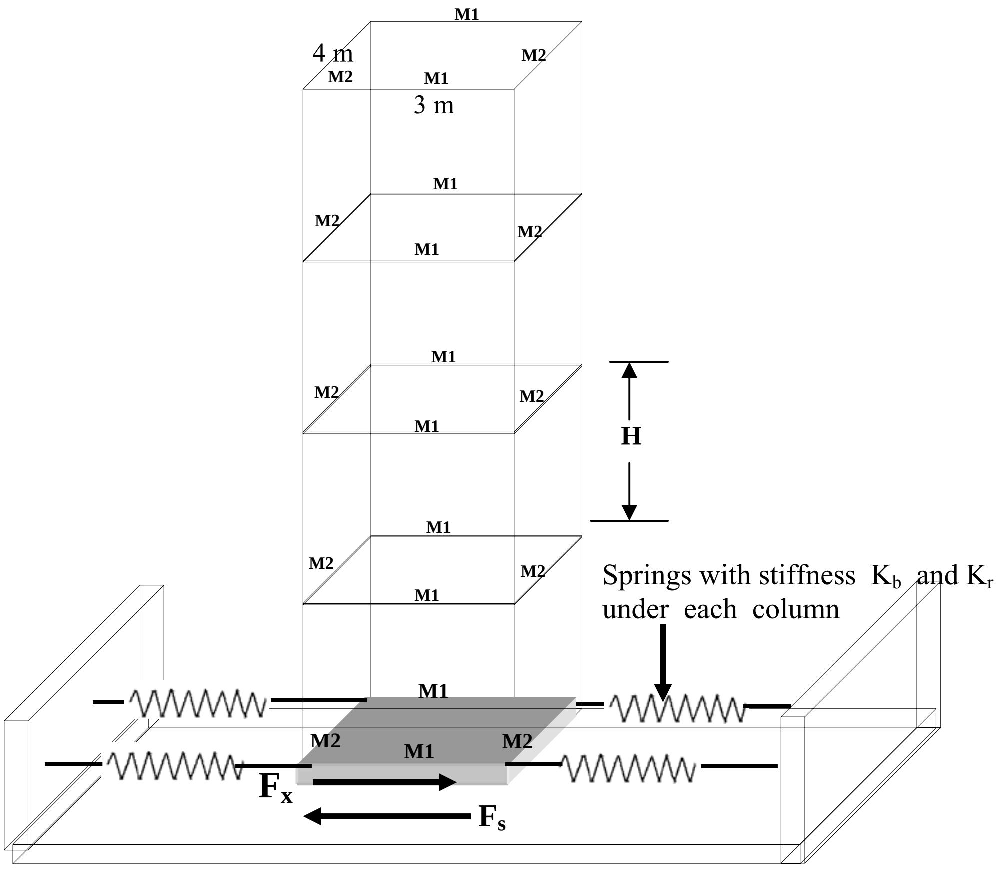

Figure 1. Modal of four story structure considered for the study

resistance, F_s (ie. $|F_x| \geq F_s$) the structure starts sliding at base and this phase of the structure is known as sliding phase. When the structure is in sliding phase and whenever reverses its direction of motion (when the velocity at base is equal to zero) then the structure may again stop its movement at base and may enter the non – sliding phase or may slide in opposite direction. In the present analysis, sliding bearing is modeled as a fictitious spring with stiffness, k_b, connected to the base of each column. The conditions for sliding and non – sliding phase are duly checked at the end of each time step. When the structure is in non – sliding phase, the stiffness of the spring, k_b, is assigned as a very high value to prevent the movement of the structure at the base whereas when the structure is in sliding phase, the value of stiffness of spring, k_b, is made equal to zero to allow the movement of the structure at the base. Thus the stiffness of the spring, k_b, may be equal to zero or very high value depending on the phase of the structure.

Also, during the non - sliding phase the relative acceleration, $\ddot{u}_b$, and relative velocity, $\dot{u}_b$, of the base is equal to zero and the relative displacement at base, u, is constant during this phase. The stiffness of the spring at base of each column are considered as very large ($k_b = 1\times10^{15}$ kN/m) during non - sliding phase. The dynamic equation of motion for the non - sliding phase is as given in equation 1. However, [K], the stiffness matrix includes the stiffness of the structure, stiffness of the spring, k_b, (k_b, being a very large value) and stiffness of restoring force device k_r.

During sliding phase, the stiffness of the spring at base of each column is considered as zero ($k_b = 0$) and the mobilized frictional force, F_x, under each column is equal to F_s and remains constant. Hence, the dynamic equations of motion for the structure during this phase is

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\}-\{F_{xmax}\} \tag{4}$$

where, [K] the stiffness matrix includes the stiffness of the structure, stiffness of spring, k_b (k_b, being equal to zero) and stiffness of restoring force device, k_r. $\{F_{xmax}\}$ is the vector with zeros at all locations except those corresponding to the horizontal degree of freedom at base of the structure. At these degrees of freedom, the vector $\{F_{xmax}\}$ will have values equal to F_s. The frictional force mobilized in the sliding system is non – linear function of the system response and hence the response of the isolated structural system is obtained in the incremental form using Newmark's method. Owing to its unconditional stability, the constant average acceleration scheme (with $\beta = 1/4$ and $\gamma = 1/2$) as adopted by Vafai *et al*. [6] is used.

2.1 Determination of mobilized frictional force and member forces

Forces in each member of the structure are obtained using the equation [k]{q}. Where [k] is the member stiffness matrix and {q} is the nodal displacement vector. The horizontal force F_{bc} at bottom node of the column in contact with the sliding bearing is the base shear under each column. Similarly the damping force, F_d, at each node can also be obtained by multiplying the damping matrix [C] of the structure and restoring force device with the nodal velocity vector $\{\dot{u}\}$. The mobilized frictional force F_x under each column when the system is in non – sliding phase is determined using the equation

$$F_x = F_{bc} + F_{bs} + F_d - F \tag{5}$$

where F_d is the damping force at base of the structure and F is applied force at base of column due to ground acceleration (ie. $F = -M_F\ddot{u}_g$, where, M_F is the base mass and $\ddot{u}_g$ is the ground acceleration).F_{bs} is the horizontal force in restoring force device. It is to be noted that the relative acceleration and velocity at base is equal to zero when the system is in non - sliding phase.

2.2 Determination of limiting frictional force

The frictional resistance, F_s is obtained using the equation $F_s = \mu W$ where μ is the coefficient of friction of the sliding material and W is the load on each column in contact with the bearing.

3 Results and discussions

The effects of damping ratio of restoring force device on response of a space frame structure subjected to harmonic ground acceleration and El Centro earthquake ground acceleration is studied. The damping ratio of the restoring force device considered for the study are 5%, 10%, 15%, 20%, 25%, 30%, 40%, and 50%. The structure with time period equal to 0.25 sec, 0.5 sec and 1.0 sec with stiffness of restoring force device equal to 100 kN/m, 300 kN/m and 600 kN/m are considered to study the effect of damping ratio of the restoring force device on response of the structure.

3.1 Effects of damping ratio of restoring force device on a structure resting on sliding bearing and subjected to harmonic ground acceleration

The variation of response with time for a structure fixed at base and for a structure isolated at base with damping of restoring force device equal to 5 % and 50 % subjected to harmonic ground acceleration of intensity $2\sin(\omega t)$ m/sec^2 is shown in figure 2. The variation of response with time for a structure isolated at base without restoring force device is also shown in figure 2. The excitation frequency, ω, is equal to 12.56 rad/sec for the structure fixed at base where as it is equal to 3.75 rad/sec for the structure isolated at base. At these values of excitation frequencies, ω, the response of the corresponding structures are maximum. The natural period of the structure, T_s, is equal to 0.5 sec and the stiffness of restoring force device is equal to 600 kN/m. The other material and geometric properties corresponding to $T_s = 0.5$ sec is tabulated as shown in table 1. The coefficient of friction of sliding material, μ, is taken as 0.05. It can be observed from the figure 2 that the acceleration, bending moment and base shear decreases considerably due to isolation. Also, the acceleration, bending moment and base shear reduces slightly as the damping of the restoring force device is increased from 5 % to 50 %. From the plot of displacement versus time relationship it can be observed that the top displacement of the structure isolated at base with damping of restoring force device equal to 5% is considerably larger than the top displacement of the structure fixed at the base. However, when the damping of restoring force device is increased to 50%, the top displacement of the structure isolated at the base reduces considerably and becomes almost equal to the top displacement of the structure fixed at base. From the plot of displacement versus time relationship it can also be observed that the structure isolated at base without restoring force device vibrates in shifted position and the structure shifts to new position after the end of earthquake where as the structure isolated at base with restoring force device vibrates in original position and will come back to original position after the end of earthquake. Thus, the restoring force device decreases the displacement and the displacement can be decreased further by increasing the damping ratio of the restoring force device. The restoring force device also restores the structure to its original position.

The variation of response of the structure isolated at base with excitation frequency, ω, when damping ratio of restoring force device equal to 5% and 50% is shown in figure 3. The variation of response with excitation frequency, ω, for the structure fixed at base and for the structure isolated at base without restoring force device is also shown in the same figure. It can be observed from figure 3 that the acceleration, bending moment and base shear of the structure fixed at base varies with excitation frequency, ω, and shows a peak values when the frequency of excitation is equal to the natural frequency of the structure ($\omega/\omega_n = 1$) where as for the structure isolated at base without restoring force device the bending moment, acceleration and

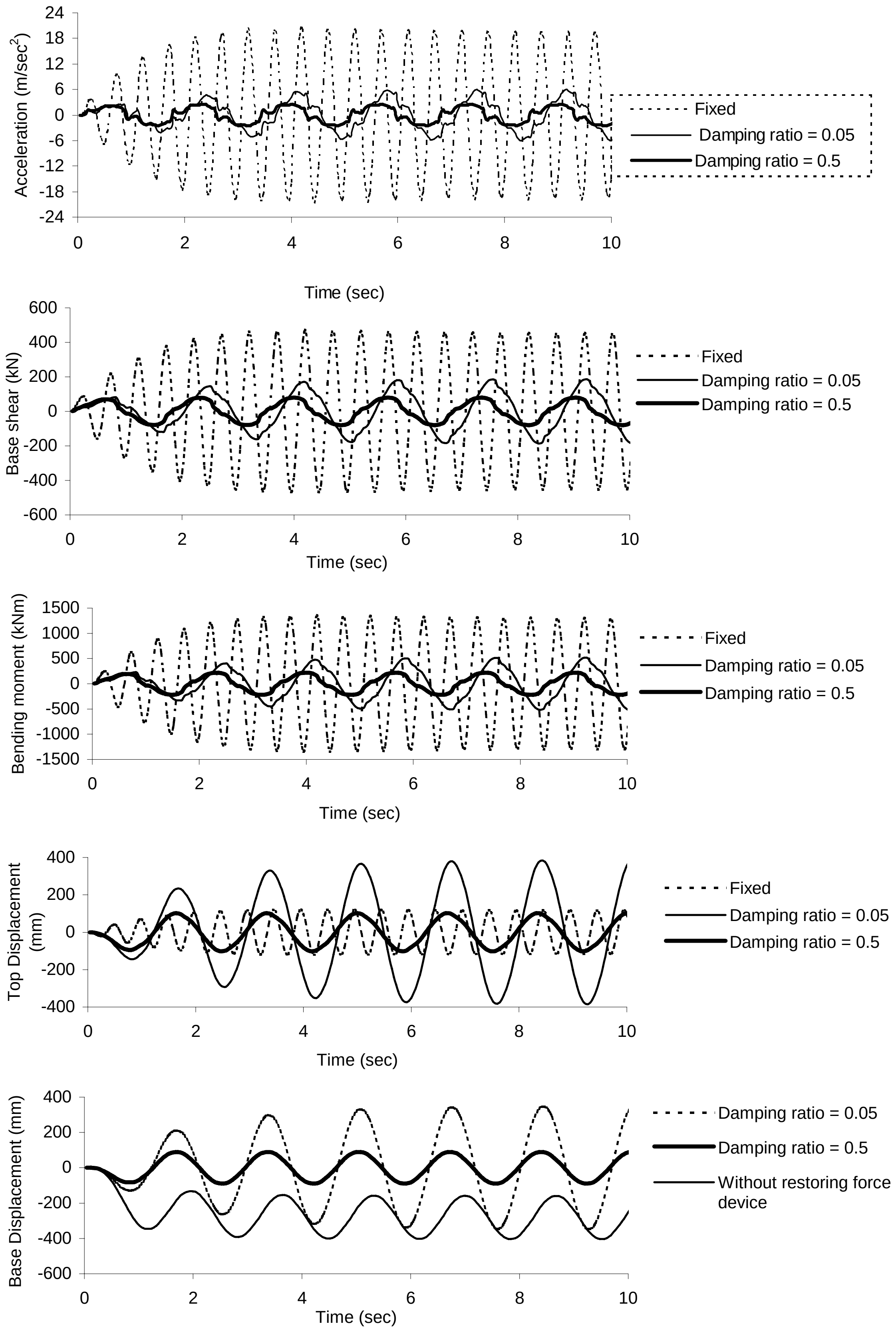

Figure 2. Variation of response with time for a structure subjected to harmonic ground acceleration

base shear will not change much with change in excitation frequency. For the structure isolated at base with restoring force device, the acceleration, bending moment and base shear varies with excitation frequency and shows a peak values when the frequency of excitation is equal to the frequency of the restoring force device. However, the peak responses of the structure isolated at base is considerably less than the peak responses of the structure fixed at base. It can also be seen from the figure that as the damping of restoring force device increases, the peak acceleration, bending moment and base shear decreases and when damping ratio of the restoring force device is equal to 50%, the acceleration, bending moment and base shear will not show peak values like a structure isolated at base without restoring force device and the variation of bending moment, acceleration and base shear becomes almost independent of the excitation frequency. At this damping ratio, the acceleration, bending moment and base shear of the isolated structure with restoring force device is almost equal to the acceleration, bending moment and base shear of the structure isolated at base without restoring force device. However, it can be observed from the figure that the acceleration of the isolated structure with damping ratio of the restoring force device equal to 50% is more than the acceleration of the isolated structure with damping ratio of restoring force device equal to 5% when excitation frequency exceeds about 12.5 rad/sec. Similarly the bending moment and base shear of the isolated structure with 50% damping of restoring force device is more than the bending moment and base shear of the isolated structure with 5% damping of restoring force device when excitation frequency exceeds about 6 rad/sec. Thus, the increase in damping ratio reduces only the peak values of acceleration, bending moment and base shear and will not reduce these values at all excitation frequencies. The maximum top displacement of the isolated structure occurs when excitation frequency is equal to the frequency of restoring force device and is considerably more than the top displacement of the structure fixed at base when damping of restoring force device is equal to 5%. However, the maximum top displacement decreases as damping of restoring force device increases and becomes less than the top displacement of the fixed base structure when damping of restoring force device is equal to 50%. It can also be seen from the figure that the maximum displacement of the structure isolated at base without restoring force device is considerably larger than the maximum displacement of the structure with restoring force device. Thus, the increase in damping ratio not only reduces the peak displacement of isolated structure but it also reduces the peak acceleration, base shear and bending moment of the isolated structure. It may also be noted that the peak values of acceleration occurs at two values of frequencies ie i) when frequency of excitation is equal to the frequency of the restoring force device and ii) when excitation frequency is about 17 rad/sec. The first peak decreases considerably with increase in damping ratio of restoring force device whereas the second peak increases slightly with increase in damping of restoring force device. The first peak also increases considerably with increase in stiffness of restoring force device whereas the second peak may decrease slightly with increase in stiffness of restoring force device as observed from figure 3.

The structure is subjected to excitation frequency varied from 2 rad/sec to 30 rad/sec to obtain the maximum response of the structure. The variation of maximum responses with damping ratio for a structure with time period, T_s, equal to 0.25 sec, 0.5 sec and 1.0 sec when stiffness of restoring force device is equal to 100 kN/m, 300 kN/m and 600 kN/m is shown in figure 4. The mass on beam and sizes of column corresponding to T_s equal to 0.25 sec, 0.5 sec and 1.0 sec are tabulated in table 1. As observed from the figure 4, the maximum acceleration, maximum base shear, maximum bending moment and maximum displacement decreases with increase in damping ratio of the restoring force device. Also, the decrease in base shear, bending moment and displacement is considerably more when time period of the structure, T_s, is equal to 1.0 sec than when time period, T_s, of the structure is equal to 0.5 sec or 0.25 sec.

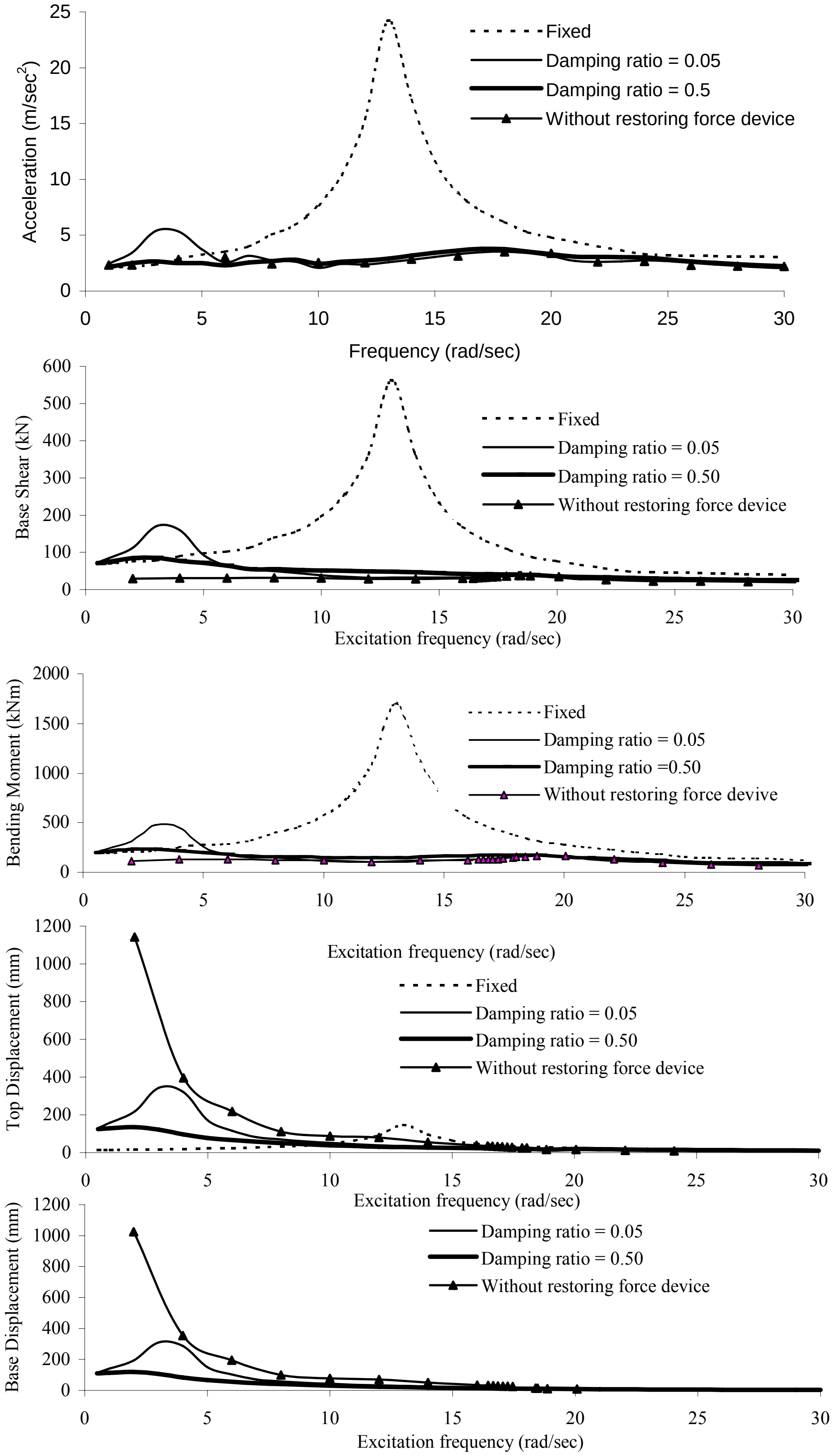

Figure 3. Variation of response with excitation frequency for a structure subjected to harmonic ground acceleration

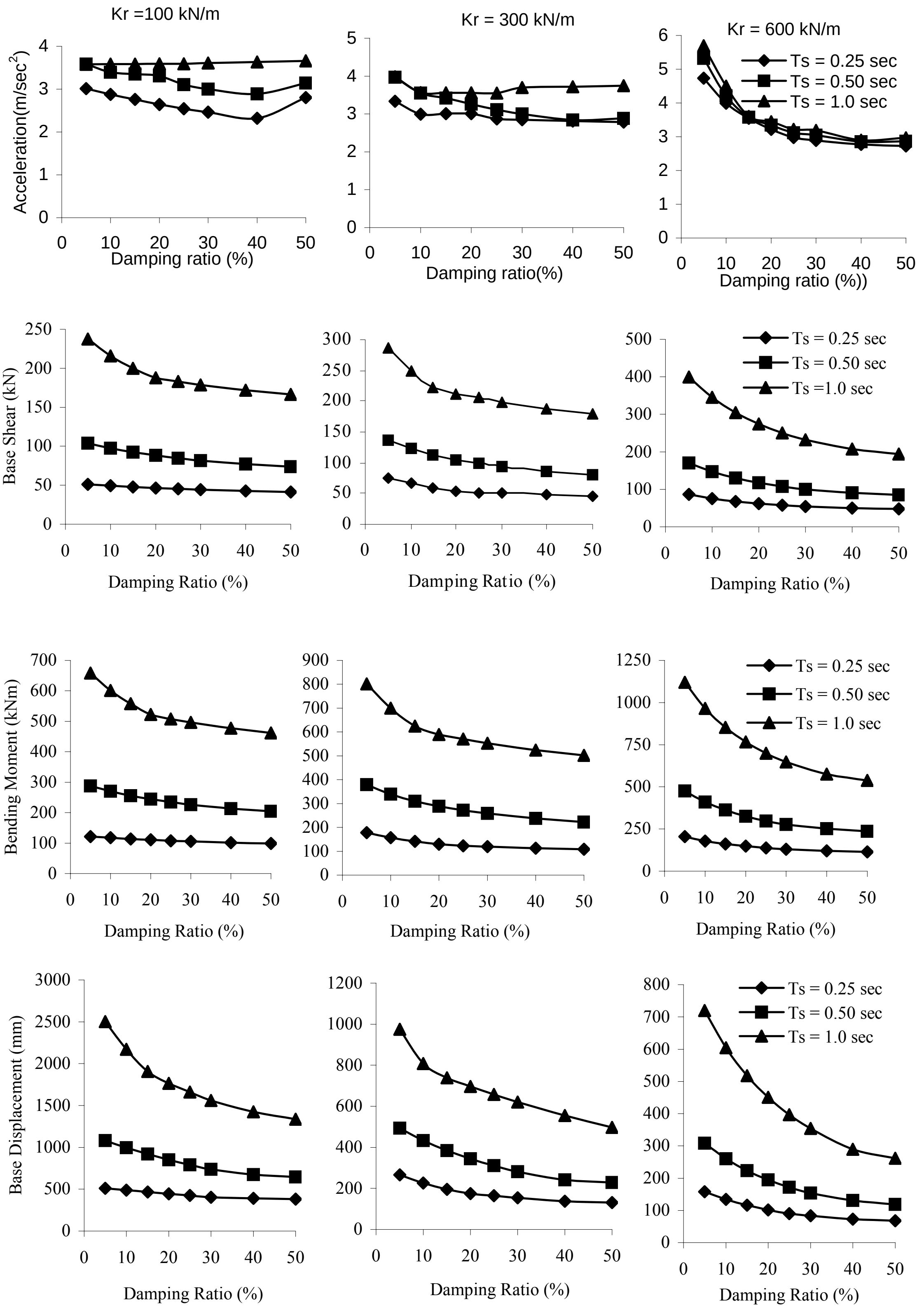

Figure 4. Variation of response with damping ratio for a structure subjected to harmonic ground acceleration

3.2 Effects of damping ratio of restoring force device on a structure resting on sliding bearing and subjected to El Centro earthquake ground acceleration

The effects of damping ratio of restoring force device are also studied when the structure shown in figure 1 is subjected to an El Centro earthquake. The accelerogram of El Centro earthquake is shown in figure 5. The variation of response with time for a structure fixed at base and for a structure isolated at base with damping ratio of restoring force device equal to 5% and 50% subjected to El Centro earthquake ground acceleration is shown in figure 6. The variation of response of the structure with time for a structure isolated at base without restoring force device is also shown in the same figure. As in the case of structure subjected to harmonic ground motion, the acceleration, bending moment and base shear decrease considerably due to isolation. However, there is not much variation in acceleration, bending moment and base shear of the structure isolated at base when damping of restoring force device is increased from 5% to 50%. The top displacement of the structure isolated at base is less than the top displacement of the structure fixed at base. The displacement of the structure at top and base reduces when the damping ratio of restoring force device is increased from 5% to 50%. It can also be seen from the figure that the base displacement of the structure without restoring force device is larger than the base displacement of the structure with restoring force device. Also, the residual displacement (displacement after the end of earthquake) reduces considerably and becomes almost zero (the structure comes to original position) due to the addition of restoring force device. The residual displacement also reduces further when damping ratio of restoring force device is increased from 5% to 50%.

The variation of maximum responses with damping ratio of restoring force device for a structure with time period equal to 0.25 sec, 0.5 sec and 1.0 sec when stiffness of restoring force device is equal to 100 kN/m, 300 kN/m and 600 kN/m is shown in figure 7. It can be observed from figure 7 that the maximum acceleration, maximum base shear and maximum bending moment will not change much with increase in damping ratio whereas the displacement decreases with increase in damping ratio.

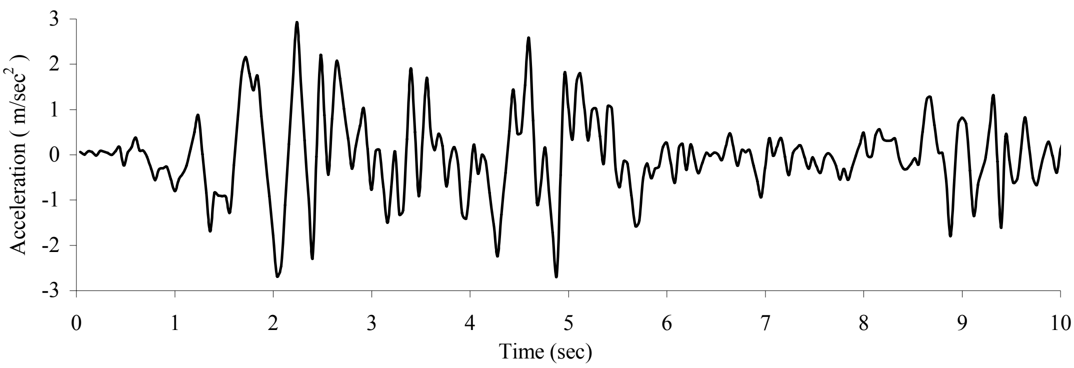

Figure 5. Accelerogram of El Centro earthquake

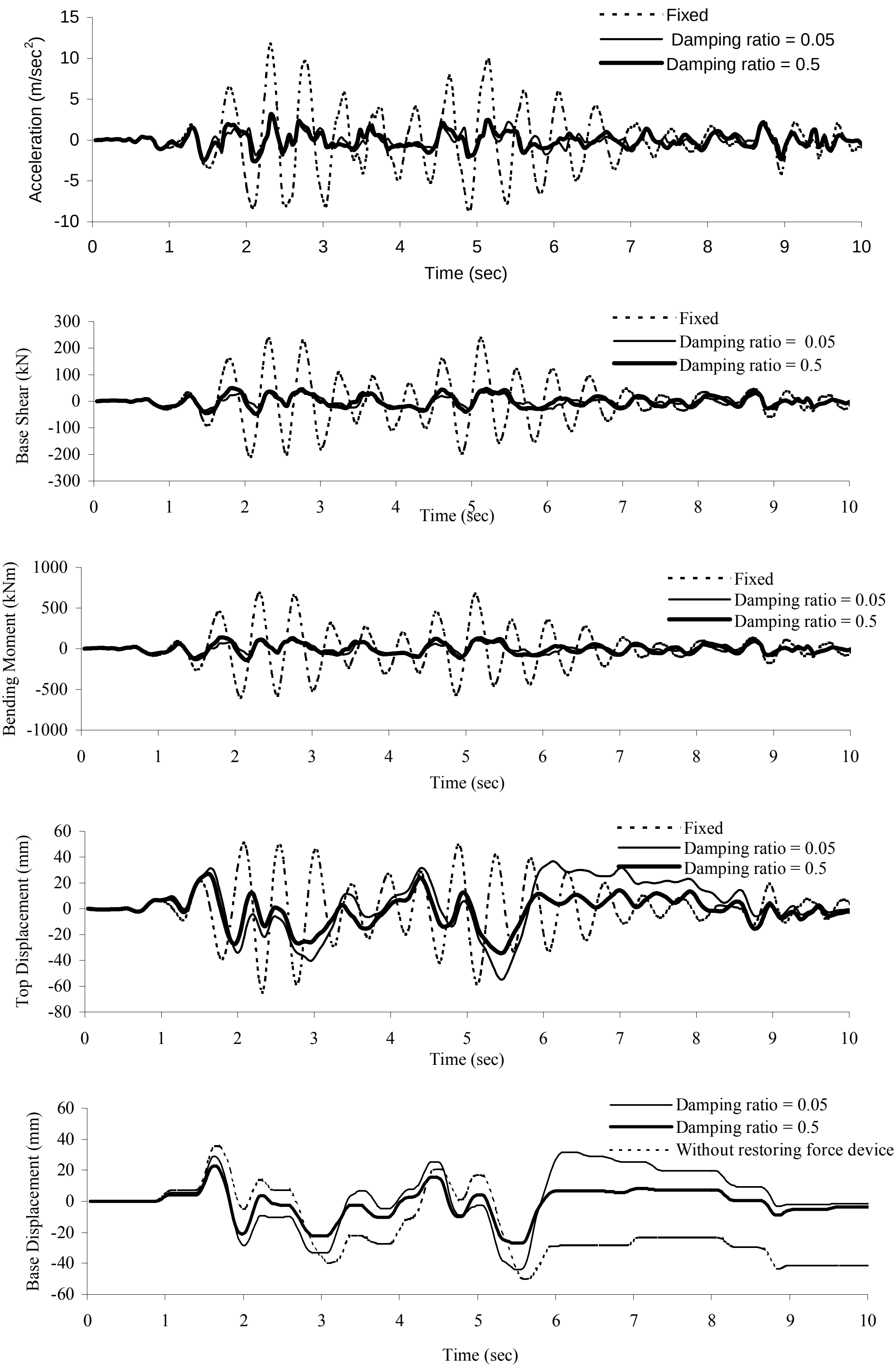

Figure 6. Variation of response with time for a structure subjected to El Centro ground acceleration

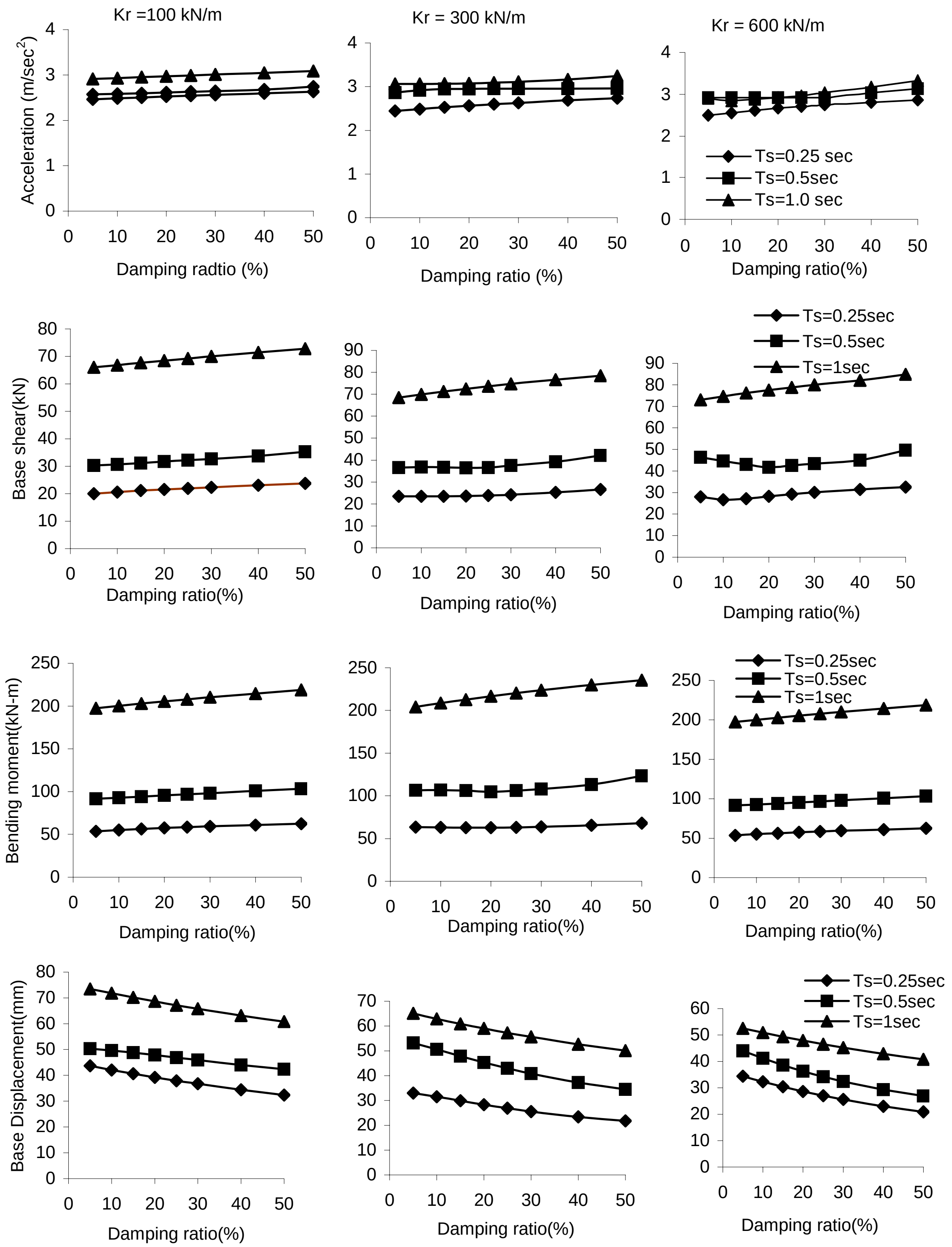

Figure 7. Variation of response with damping ratio for a structure subjected to El Centro ground acceleration

4 Summary and conclusions

The effects of damping ratio of the restoring force device on response of a space frame structure isolated at base and subjected to earthquake are studied. The peak values of acceleration, bending moment and base shear of the isolated structure reduces as damping ratio of restoring force device increases. Also, the acceleration, bending moment and base shear of the isolated structure with restoring force device will not change much with change in excitation frequency, as in the case of structure resting on sliding bearing without restoring force device, when damping ratio is increased to a particular value. The base displacement and residual displacement of the structure isolated at base also reduces as the damping of restoring force device increases. However, the effect of damping ratio of restoring force device is only when the excitation frequency is nearer to the frequency of restoring force device. At other frequency of excitation, the damping of restoring force device has not much effect on the response of the isolated structure. For the same reason, the response of the isolated structure subjected to El Centro earthquake will not vary much with variation in damping ratio of the restoring force device.

Thus, the major merits of the base isolation with restoring force device with damping as compared with the base isolation with restoring force device without damping may be summarized as follows

- i) The restoring force device with higher damping reduces the maximum acceleration, maximum bending moment and maximum base shear
- ii) It also reduces the maximum sliding displacement and residual displacement of the structure
- iii) Also, the response of the base isolated structure with restoring force device with damping is almost independent of the excitation frequency of the ground acceleration.

REFERENCES

1. Bhasker, P., Jangid R.S., "Experimental study of base – isolated structures", *Journal of Earthquake Technology* - ISET, Vol. 38, No. 1, 2001, pp. 1-15.
2.
3. Chalhoub, M.S., and Kelly J.M., "Sliders and tension controlled reinforced bearings combined for earthquake isolation system", *Earthquake Engineering and Structural Dynamics,* Vol. 19, 1990, pp. 333 – 358.
4.
5. Jangid R.S., and Londhe, Y.B., "Effectiveness of elliptical rolling rods for base isolation," *Journal of Structural Engineering* – ASCE, Vol. 124, 1998, pp. 469 – 472.
6.
7. Jangid, R.S., "Stochastic seismic response of structures isolated by rolling rods", *Engineering structures,* Vol. 22, 2000, pp. 937 – 946.
8.
9. Paz, M., "Structural Dynamics - Theory and Computation", 1991, Van Nostrand Reinhold, New York

10. Vafai, A., Hamidi M., and Ahmadi "Numerical modeling of MDOF structures with sliding supports using rigid – plastic link", *Earthquake Engineering and Structural Dynamics,* Vol. 30, 2001, pp. 27 – 42.

11. Yang, Y.B., Lee T.Y., and Tsai I.C., "Response of multi - degree – of – freedom structures with sliding supports", *Earthquake Engineering and Structural Dynamics,* Vol. 19, 1990, pp. 739 - 752.

12. Zayas, V.A., Low S.S., and Mahin, S.A., "A simple pendulum technique for achieving seismic isolation", *Earthquake Spectra,* Vol. 6, 1990, pp. 317 – 333.