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Equilibrium based evaluation of stress distribution under steel column base plates. I: Governing Equations

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ABSTRACT

The present paper, being the first part of two relevant studies, deals with the evaluation of the stress distribution under steel base-plate connections, acted upon statically by axial forces and bending moments. The whole approach is formulated on the basis of the proper equilibrium equations that are valid for both elastic and plastic plate behaviour adopting a nonlinear stress distribution function under the base plate of catenary type, depending on only one parameter. Taking into account the compatibility and geometric conditions valid for all the connection components, the aforementioned equations are fully assessed, considering the base-plate connection as semi-rigid. Numerical results, parametric studies and comparison with existing relevant investigations will be demonstrated in the companion paper, being the product of advances symbolic computations of the theoretical findings of the present work.

KEYWORDS

Steel column base plate connections, stress distribution, equilibrium equations, symbolic computations

1 Introduction

Among the various types of connections appearing in modern structural steelwork, column base plate ones exhibit the most unpredictable semi-rigid behavior. This is due to the fact that, in addition to the usual parameters of non-linearity (initial imperfections, residual stresses, non-uniform distribution of material properties, metal plastification), their basic components – base plate, anchor bolts and concrete foundation – are made from different material types, transmitting acting forces through unilateral multi-body contact. Thus, an exact evaluation of the response of such connections, accounting for all the aforementioned factors, can only be achieved through the corresponding analysis of sophisticated two or three-dimensional finite element numerical simulations, as described in earlier as well as in recent investigations [1,2,3].

On the other hand, steel base-plate connections have been and are still used extensively in engineering practice, owing to their relatively simple manufacture and assembly, while their design is mainly based on simplified methods and models, incorporating important assumptions, without necessarily reflecting the actual structural response. In the working method [4] for instance, the bearing stress distribution in concrete and bolts is assumed to be elastic, whilst the ultimate strength method adopts the assumptions that the anchor bolts are yielding and that the consequent nonlinear concrete stress distribution at failure may be replaced by an equivalent rectangular stress block. Furthermore, the complex model presented by Wald [5] is based on the analytical prediction of the elastic-plastic stress distribution under the base plate and the corresponding modeling of base plate and anchor bolts. Through this simple and practical model, after additional simplifications, three basic collapse patterns of a base-plate joint are proposed, depending on the magnitude of the applied axial load, and the values to be considered in stiffness calculation for each individual pattern are computed. In this excellent monograph, offering a rich bibliography on theoretical as well as experimental studies dealing with the static and cyclic response of column bases, one can also find a comprehensive sensitivity study, indicating that the parameters having the most significant influence on the connection behavior are the base plate thickness, the dimensions (length and diameter) of the bolts and the quality of the concrete. Similar results have been also reported in earlier and more recent relevant investigations [6,7,8,9]. An extension - improvement of the aforementioned complex model has been recently developed [10], by proposing an efficient mechanical model on the basis of the component method, described in Annex J of Eurocode 3 [11], which nevertheless requires a rather long and iterative calculation procedure in order to accurately simulate the non-linear response of column bases from the very first loading steps up to failure.

Therefore, it would be quite favorable and desirable to develop an analytical procedure, which encompasses in a comprehensive manner the non-linear stress distribution under the base plate, covers both elastic and plastic connection behavior and simultaneously provides a simple way of determining the geometric as well as force characteristics of the deformed connection throughout the loading process. An approach towards this goal was initially presented [12], in which a new methodology was developed, leading to an analytical model describing through a mathematical expression the non-linear stress distribution under the base plate in a simple manner. Classifying column bases a rigid, semi-rigid and flexible, a fourth order polynomial function was proposed as an approximation of the stress distribution, which required the determination of four (4) parameters. Thus, the assumption of specific geometrical relations concerning the position of the maximum compressive stress under the base plates was adopted, which however does not always agree with the actual response and sometimes may lead to physically unacceptable deformation patterns for the base plates.

The foregoing paper, as well as its companion one (under preparation) are also based on equilibrium equations in both elastic and plastic regimes (as far as the base plate is concerned) and tackle the whole problem uniformly, by considering base plate connections, acted upon statically by axial loads and moments, as semi-rigid, which is in fact the case in the multitude of steel joints, even for extreme configurations. Moreover, the stress distribution function employed herein is of a hyperbolic type depending only on one additional parameter, so that all the compatibility conditions related to the deformations of both base plate and concrete foundation are fulfilled. In the present (1^{st}) part of the study, the mathematical formulation of the problem is outlined and the governing equations are assessed, while introductory concepts for their solution technique are also presented. Thereafter, in the 2^{nd} part – companion paper due, a full parametric study will be demonstrated, determining the most crucial factors affecting the behavior of the base plate and hence providing a more accurate insight on the whole problem. A comparative discussion considering existing results will also be carried out, validating the proposed approach.

2 Mathematical formulation

2.1 Generalized displacements and concrete deformations

We consider at first a disk of thickness δ, cut off from a concrete compressible layer of height H, which lies on a rigid foundation, as depicted in Figure 1a. Assuming that the displacement u

along axis x is negligible, according to the notation and sign convention of Figure 1b, the deformations of the disk υ, w can be written in the form introduced by Vlasov and Leontiev [13] as follows:

$$\left.\begin{aligned} \upsilon(y,z) &= \sum_{i=1}^{m} U_i(y)\phi_i(z) \quad , \\ w(y,z) &= \sum_{k=1}^{n} W_k(y)\psi_k(z) \end{aligned}\right\} \qquad (1)$$

In the above expressions the unknown functions $U_i(y)$, $W_j(y)$ can be considered as generalized displacements. Thus, if a vertical concentrated load P acts on the elastic layer (i.e. the concrete foundation) at the origin of coordinates, it is valid that

$$\left.\upsilon(y,z) = 0 \quad , \quad w(y,z) = \sum_{k=1}^{n} W_k(y)\psi_k(z)\right\} \qquad (2)$$

since usually the horizontal displacements are quite small and thus may be neglected.

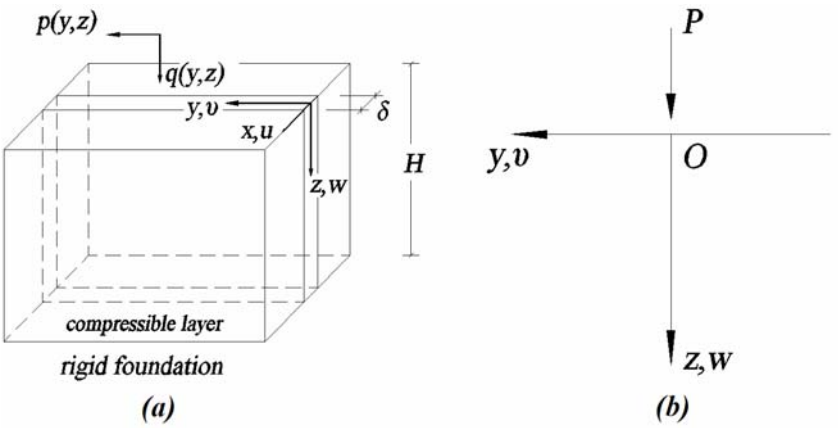

Figure 1. Geometry and sign convention for the compressible layer of a concrete foundation.

As a function of the distribution along the depth, we choose ψ_k (for $k=1$) in the form of

$$\psi_1(z) = \frac{\sinh \gamma(H-z)}{\sinh \gamma H} \qquad (3)$$

This particular function, not only satisfies the boundary conditions at z=0 and z=H, but offers a smooth distribution transition along the depth of the foundation, for values of H within practical applications. Coefficient γ is a constant of the foundation material indicating the rate of deformation decrease along the depth; a good choice for concrete is γ=2.

The virtual transverse displacements for W_k=1 are equal to

$$\overline{w}_k = \overline{w}_1 = \psi_1(z) \qquad (4)$$

while the work produced by all external and internal forces can be written as

$$\int \frac{\partial \tau_{zy}}{\partial y}\psi_1 dF - \int \sigma_z \psi_1' dF + \int p(y,z)\psi_1 dz = 0 \qquad (5)$$

Taking into account relations (2) one may write

$$\left.\begin{array}{l}\sigma_z = \dfrac{E}{1-\nu^2}\left(\varepsilon_{zz} + \nu\varepsilon_{yy}\right) = \\ \dfrac{E}{1-\nu^2}\left(\sum_{k=1}^{n} W_k\psi'_k + \nu\sum_{i=1}^{m} U'_i\phi_i\right) = \\ \dfrac{E}{1-\nu^2}W_1\psi'_1\end{array}\right\} \tag{6a}$$

$$\tau_{yz} = \tau_{zy} = \frac{E}{2(1+\nu)}\varepsilon_{yz} = \frac{E}{2(1+\nu)}\left(\frac{\partial\upsilon}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{E}{2(1+\nu)}W'_1\psi_1 \tag{6b}$$

leading, after some elaboration, to a differential equation of the form

$$2tW''(y) - kW(y) = 0 \tag{7}$$

where

$$\left.\begin{array}{l} t = \dfrac{E_0 r_{11}}{4(1+\nu_0)} \quad , \quad k = \dfrac{E_0 s_{11}}{1-\nu_0^2} \quad , \quad E_0 = \dfrac{E_c}{1-\nu_c^2} \quad , \quad \nu_0 = \dfrac{\nu_c}{1-\nu_c} \\ r_{11} = \int_0^H \psi_1^2(z)dF = \dfrac{\frac{1}{2}\sinh 2\gamma H - \gamma H}{2\gamma\sinh^2\gamma H}\delta \\ s_{11} = \int_0^H \psi_1'^2(z)dF = \dfrac{\gamma}{2}\dfrac{\left(\frac{1}{2}\sinh 2\gamma H + \gamma H\right)}{\sinh^2\gamma H}\delta \end{array}\right\} \tag{8}$$

with E_c and ν_c being the modulus of elasticity and Poisson's ratio for the concrete respectively.

Equation (7) possesses the following general integral

$$W(y) = c_1 e^{-\alpha y} + c_2 e^{\alpha y} \quad , \quad \alpha = \sqrt{\frac{k}{2t}} \tag{9}$$

Since $for\ y \to \infty : W(y) = 0 \Rightarrow c_2 = 0$ it is concluded that

$$W(y) = c_1 e^{-\alpha y} \tag{10}$$

The second integration constant c_1 is evaluated from the condition at y=0. At this point we define as the generalized shearing force $S_1(y)$ the work done by all forces acting at section y=constant over the virtual displacements $\overline{w}_1(y,z) = 1\cdot\psi_1(z)$ when $W_1(y) = 1$. This force has a discontinuity at those sections, where concentrated loads act on the elastic foundation surface, as depicted in Figure 2.

Taking into account the symmetry of the problem we find

$$S_1(0) = -\frac{P}{2}\psi_1(0) \tag{11}$$

while evidently

$$S_1 = \int \tau_{yz}\psi_1 dF = \frac{E_0\delta}{2(1+\nu_0)}W'_1\int\psi_1^2 dy = 2tW'_1 \tag{12}$$

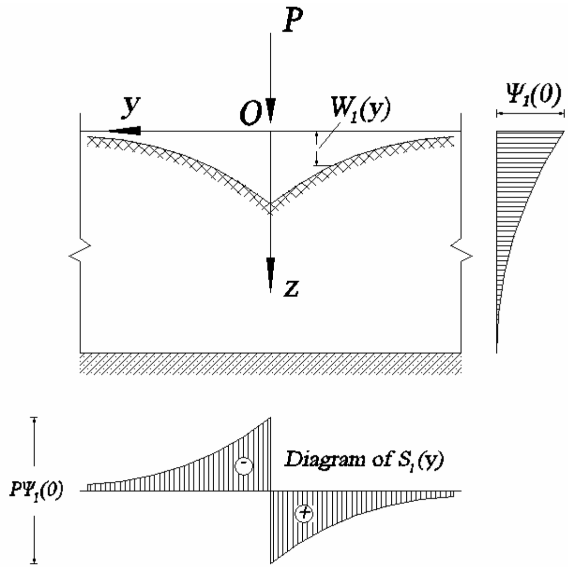

Figure 2. Elastic foundation surface exhibiting a discontinuity of the generalized shearing force due to the presence of a concentrated load.

In the sequel, combining relations (3), (10), (11) and (12) we get $c_1 = \frac{P}{4\alpha t}$ and the expression for the displacement of any point of the elastic concrete foundation due to a concentrated load P yields:

$$w(y,z) = \frac{P}{4\alpha t} e^{-\alpha y} \frac{\sinh \gamma (H-z)}{\sinh \gamma H} \quad , \quad \alpha = \sqrt{\frac{k}{2t}} \tag{13}$$

with k and t given in (8).

Furthermore, in the case of a distributed vertical loading of the form $z = \sigma(y)$ acting on the surface of the elastic foundation, as shown in Figure 3, the displacements at an arbitrary point K, with coordinates (y,0), are written according to the above as follows:

$$w(y,0) = c_1 \begin{bmatrix} \int_{d_1}^{y} \sigma(\xi) e^{-\alpha(y-\xi)} d\xi + \int_{y}^{d_2} \sigma(\xi) e^{\alpha(y-\xi)} d\xi \\ where \ c_1 = \frac{1}{4\alpha t} \psi_1(0) = \frac{1}{4\alpha t} \end{bmatrix} \tag{14}$$

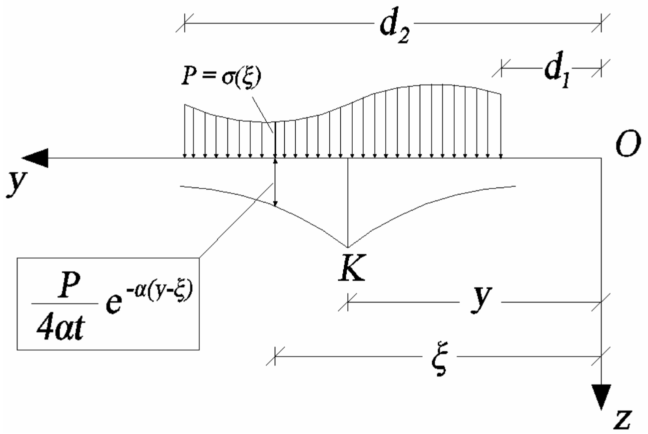

Figure 3. Deformations due to a distributed load.

2.2 Elongation of anchor bolts

For the three types of anchorage considered herein and given schematically in Figure 4, the respective bolt elongation is given by the following expressions:

$$\left.\begin{aligned} &\delta_s = k_c Z \quad for\ type\ (a)\,,\ \delta_s = \frac{k_c}{2} Z \quad for\ type\ (b)\,, \\ &\delta_s = \frac{k_c}{2} Z + \sigma_{cd} F_L k_c \quad for\ type\ (c) \\ &where\ \ k_c = \frac{L}{\eta A_s E_s} \end{aligned}\right\} \tag{15}$$

In the above expressions n is the total number of anchor bolts on the tension side, A_s the area of a single anchor bolt, E_s the modulus of elasticity of the bolt material (steel), L the length of the embedded part of each bolt, Z the total tensile force, Z_L the portion of this force carried by the angle (case c), σ_{cd} the allowable concrete stress and F_L the compressive area of the angle.

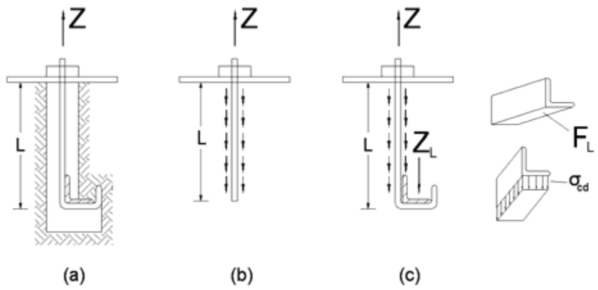

Figure 4. The three (3) types of bolt anchorage considered.

2.3 Stress distribution function

For a typical steel column base plate connection, the stress distribution diagram (corresponding to an axial load P and a strong axis bending moment M) and the deformed configuration of the plate have the form presented in Figure 5.

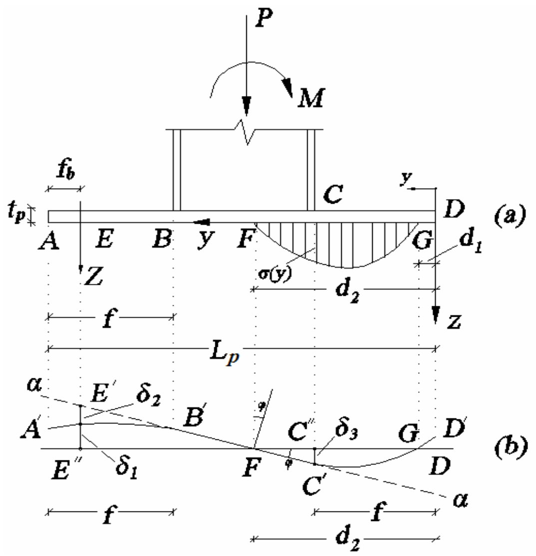

Figure 5. Stress distribution and deformation geometrical properties under a steel base plate within the elastic region.

Without serious loss of generality, we can assume that:

- The part *BC* of the base plate (directly under the column) remains rectilinear throughout deformation.
- Within the elastic region, the maximum compressive stress $max\sigma_c$ remains less than or equal to the yield stress for the concrete, i.e. $max\sigma_c \leq \sigma_f$. This usually appears in praxis for low or medium axial force loading and is commonly adopted by the majority of models proposed (see [5]).
- The neutral axis of the bending section passes through point *F* at $y=d_2$, which is usually the case under static loading.

Within the limitations and restrictions due the aforementioned assumptions, the choice of a function $\sigma(y)$, representing in the most possible accuracy the stress distribution under the base plate, must be related to the meeting of the following requirements:

- Maintenance of the strongly nonlinear character of the true (actual) distribution, for all levels of the external loading.
- Ensuring of continuity and smoothness.
- Fulfillment of all geometrical and compatibility conditions related to the problem.
- Minimal number of coefficients to be determined.

Hence, we adopt the following function for the stress distribution, similar to the expression of the well known catenary curve [14]:

$$\sigma(y) = \frac{d_2 - d_1}{4}\left\{ cosh(2+c) - cosh\left[c + 2 - 4 * \frac{y - d_1}{d_2 - d_1} \right] \right\} - (y - d_1) tan(r) \quad (16a)$$

$$\text{where } c = sinh^{-1}\left(\frac{2}{sinh 2} g(r)\right) = sinh^{-1}(0.55144 \, tan(r)) \quad (16b)$$

This particular smooth and continuous function satisfies all the above requirements, since except its obvious non-linearity and dependence only on one parameter r, it also furnishes the subsequent attributes:

$$\sigma(d_1) = \sigma(d_2) = 0 \ , \ \sigma(y) > 0 \ \text{ for } d_1 < y < d_2 \} \quad (17a,b)$$

3 Elastic behavior of base plate

In this case according to the geometrical properties and conventions perceivable in Figure 5, one may formulate the equilibrium and compatibility equations related to the specific problem. Thus, the first two equations are directly extracted from the equilibrium of forces and moments (with respect to point D) and are given by the relations that follow,

$$P + Z = B_p \int_{d_1}^{d_2} \sigma(y) y dy \quad (18a)$$

assuming that the stress distribution shown in Fig. 5 remains the same over the full width B_p of the base plate. Substituting the expression of function $\sigma(y)$ from eq.(16a), we get

$$P + Z = B_p \frac{(d \;\; - d_1)^2}{8}\left[2 cosh(2+c) - sinh(2+c) + sinh c - 4 tan r \right] \quad (18b)$$

In the same manner, one may write that

$$Z(L_p - f_b) + \frac{PL_p}{2} - M = B_p \int_{d_1}^{d_2} \sigma(y) y dy \tag{19a}$$

or

$$\left.\begin{aligned} & Z(L_p - f_b) + \frac{PL_p}{2} - M = \\ & = B_p \frac{(d_2 - d_1)^2}{8} \begin{bmatrix} (d_2 + d_1)cosh(2+c) - d_2 sinh(2+c) + d_1 sinh c \\ -2cosh(2+c) + 2coshc \end{bmatrix} - \\ & \quad - \frac{2d_2^3 - 3d_1 d_2^2 + d_1^3}{6} tan r \end{aligned}\right\} \tag{19b}$$

As far as the deformations are concerned, from the geometry of the deformed base plate configuration of Figure 5b one may easily find that:

$$\frac{\delta_3}{\delta_1 + \delta_2} = \frac{d_2 - f}{L_p - d_2 - f_b} \tag{20}$$

where

$$\delta_1 = \delta_s \tag{21a}$$

is the elongation of the anchor bolts given in relations (15),

$$\delta_2 = \frac{Z(f - f_b)^3}{3E_s J_p} \tag{21b}$$

is the deformation of point *E* (of cantilever *AB*) due to the concentrated load *Z*, in which $J_p = B_p t_p^3/12$ is the moment of inertia of the base plate, and

$$\delta_3 = w(f,0) = c_1 \left[\int_{d_1}^{f} \sigma(\xi) e^{-\alpha(f-\xi)} d\xi + \int_{f}^{d_2} \sigma(\xi) e^{\alpha(f-\xi)} d\xi \right] \tag{21c}$$

is the deformation of the concrete at point *C*, which after symbolic computations yields

$$\left.\begin{aligned} \delta_3 = & C_1 \frac{(d_2 - d_1)cosh(2+C)}{4\alpha} \left[2 - e^{-\alpha(f-d_1)} - e^{\alpha(f-d_2)} \right] \\ & + C_1 \frac{\alpha(d_2 - d_1)^3}{4\alpha^2(d_2 - d_2)^2 - 16} \left[e^{\alpha(f-d_2)} cosh(2+c) - e^{-\alpha(f-d_1)} cosh c \right] \\ & + C_1 \frac{(d_2 - d_1)^2}{2\alpha^2(d_2 - d_2)^2 - 8} \left[e^{\alpha(f-d_2)} sinh(2+c) - e^{-\alpha(f-d_1)} sinh c \right] \\ & - C_1 \frac{tan r}{\alpha} \left[2(f - d_1) + \tfrac{1}{\alpha} e^{-\alpha(f-d_1)} - \left(d_2 - d_1 + \tfrac{1}{\alpha}\right) e^{\alpha(f-d_2)} \right] \end{aligned}\right\} \tag{21d}$$

The last equation will be derived from the relation between the work produced by the external and the internal forces. Thus, it is evident that

$$\boldsymbol{E_e} = \sum_{i=1}^{4} \boldsymbol{E_i} \tag{22}$$

where $\boldsymbol{E_e}$ is the work due to the external forces, given by

$$\boldsymbol{E_e} = -M\phi = -M\frac{\delta_3}{d_2 - f} \tag{23}$$

while the contribution of the various internal forces in their corresponding work is:

3.1 Work produced by the anchor bolts: E_1

$$\boldsymbol{E_1} = \frac{LZ^2}{E_S A_S} \tag{24a}$$

3.2 Work produced by the bending of cantilever AB: E_2

$\boldsymbol{E_2} = \frac{1}{2}\int_0^{f-f_b} EJ_p z''^2 dy$ and since $-EJ_p z''(y) = M(y) = Zy$ we finally get

$$\boldsymbol{E_2} = \frac{Z(f - f_b)^2}{6EJ_p} \tag{24b}$$

3.3 Work produced by the bending of cantilever CD: E_3

Taking into account the first of the assumptions, it is clear that

$$\boldsymbol{E_3} = \frac{1}{2}\int_{d_1}^{f} EJ_p z''^2 dy = \frac{1}{2}\int_d^f \frac{M(y)^2}{EJ_p} dy \tag{24c}$$

where

$$M(y) = -B_p\int_{d_1}^{y} \sigma(\xi)(y-\xi)d\xi = -B_p(A + B + \Gamma) \tag{24d}$$

with $\sigma(y)$ defined previously and

$$\left.\begin{aligned}
A &= -\frac{(d_2 - d_1)(1 + d_1)}{4}\sinh c - \frac{(d_2 - d_1)^2}{4}\cosh c + \left(\frac{d_1^3}{6} + \frac{d_1^2}{2}\right)\tan r \\
B &= \frac{(d_2 - d_1)(y^2 - d_1 y)}{8}\cosh(2 + c) + \left(\frac{y^3}{3} + (1 - d_1)\frac{y^2}{2} - d_1 y\right)\tan r \\
\Gamma &= \frac{(d_2 - d_1)^2}{4}\cosh\left(c + 2\frac{y - d_1}{d_2 - d_1}\right) - y\frac{(d_2 - d_1)}{4}\sinh\left(c + 2\frac{y - d_1}{d_2 - d_1}\right)
\end{aligned}\right\} \tag{24e}$$

3.4 Work produced by the deformation of the concrete: E_4

$$\boldsymbol{E_4} = \frac{B_p}{2}\int_{d_1}^{d_2} \sigma(y)w(y,0)dy \tag{24f}$$

in which the value of w(y,0) is directly derived from relation (21d) by setting $f \to y$.

Solving the system of equations (18b), (19b), (20) and (22) with respect to d_1, d_2, r and Z one can describe the elastic behavior of the base plate in a more accurate and comprehensive manner.

4 Plastic behavior of the base plate

In this case $max\,\sigma_c = \sigma_f$ and the corresponding stress distribution pattern is depicted in Figure 6. The plastified length of the base plate strip is defined by its origin point P_1' at $y=p_1$ and its end point P_2' at $y=p_2$. We assume that the form of the curve $\sigma(y)$ (parts of which are the curves GP_1 and P_2F) are described by eq.(16) with quantities d_1, d_2 and r to be sought and generally different from those evaluated for the elastic behavior of the plate. Following a procedure similar to the one outlined above, the number of equations to be derived are in this case increased to six (6), since the corresponding unknown quantities are: d_1, d_2, r, p_1, p_2, Z.

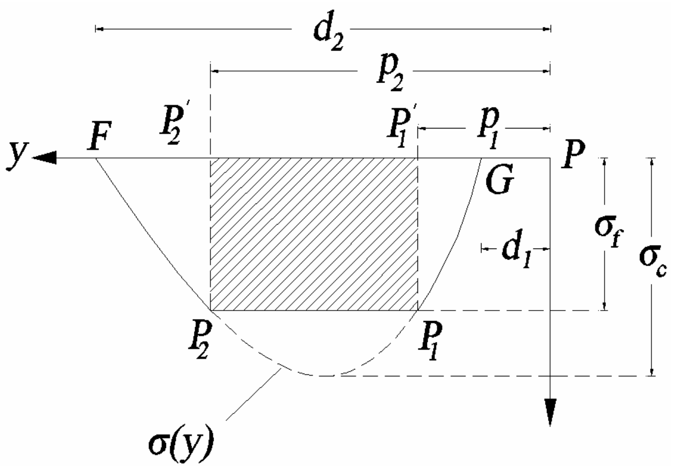

Figure 6. As in Figure 5, but within the plastic region.

Equations (20) and (21a-b) are still valid for the deformations, while δ_3 is now equal to

$$\delta_3 = C_1\left[\begin{array}{l}\int_{d_1}^{p_1}\sigma(\xi)e^{-\alpha(f-\xi)}d\xi + \dfrac{\sigma_f}{\alpha}\left(2 - e^{-\alpha(f-p_1)} - e^{-\alpha(f-p_2)}\right)\\ + \int_{p_2}^{d_2}\sigma(\xi)e^{\alpha(f-\xi)}d\xi\end{array}\right] \tag{25}$$

and additionally it is obvious that

$$\sigma(p_1) = \sigma_f = \sigma(p_2) \tag{26a,b}$$

Furthermore, the equilibrium equations for the forces and moments (again with respect to point D, as shown in Figure 6) are for this case given by:

$$P + Z = B_p\left[\int_{d_1}^{p_1}\sigma(y)dy + (p_2 - p_1)\sigma_f + \int_{p_2}^{d_2}\sigma(y)dy\right] \tag{27}$$

$$Z(L_p - f_p) + \frac{PL_p}{2} - M = B_p\left[\int_{d_1}^{p_1} y\sigma(y)dy + \frac{p_2^2 - p_1^2}{2}\int_{p_2}^{d_2} y\sigma(y)dy\right] \tag{28}$$

The last equation is again derived from the work produced, as prescribed by expression (22), in which relations (23) and (24a-b) are still valid, while the work produced by the bending of cantilever *CD* and the concrete deformations are respectively equal to:

$$\boldsymbol{E_3} = \frac{1}{2}\int_{d_1}^{f}\frac{M(y)^2}{EJ_p}dy \tag{29a}$$

with

$$\left.\begin{array}{l} M(y) = -B_p\left(\int_{d_1}^{p_1} \sigma(\xi)(y-\xi)d\xi + \dfrac{\sigma_f}{2}(f-p_1)^2\right) \\ \quad for\ p_1 < f < p_2 \end{array}\right\} \tag{29b}$$

and

$$\boldsymbol{E_4} = \frac{B_p}{2}\int_{d_1}^{d_2} \sigma(y)w(y,0)dy \tag{30a}$$

in which

$$w(y,0) = c_1\int_{d_1}^{y} \sigma(\xi)e^{-\alpha(y-\xi)}d\xi + c_1\int_{y}^{p_1} \sigma(\xi)e^{\alpha(y-\xi)}d\xi \text{ , for } d_1 \leq y \leq p_1 \tag{31a}$$

$$w(y,0) = c_1\begin{bmatrix} \int_{d_1}^{p_1} \sigma(\xi)e^{-\alpha(y-\xi)}d\xi \\ +\dfrac{\sigma_f}{2}\left(2 - e^{-\alpha(y-p_1)} - e^{\alpha(y-p_2)}\right) \\ +\int_{p_2}^{d_2} \sigma(\xi)e^{\alpha(y-\xi)}d\xi \end{bmatrix} \text{ for } p_1 \leq y \leq p_2 \tag{31b}$$

$$w(y,0) = c_1\int_{p_2}^{y} \sigma(\xi)e^{-\alpha(y-\xi)}d\xi + c_1\int_{y}^{d_2} \sigma(\xi)e^{\alpha(y-\xi)}d\xi \text{ , for } p_2 \leq y \leq d_2 \tag{31c}$$

The system of equations (20) [along with (21a,b) and (25)], (26a,b), (27), (28) and (30) [in conjunction with eqs.(23), (24a,b), (29a,b) and (31a,b,c)] is the one that applies for the plastic behavior of the base plate.

5 Solution of the governing equations via symbolic computations

Evidently, the most difficult task inherent for the numerical solution of the governing equations is the evaluation of the integral components involved. More specifically, the strongly nonlinear character of these equations in conjunction with the existence of complicated integral functions, requires delicate symbolic manipulations within the context of modern commercial mathematical software, combined with the adequate computer environment. In what follows in the companion paper to appear in the near future, the well known *Mathematica* package will be utilized, running on the computer facilities, located at the Structural Engineering Computer Laboratory of the Civil Engineering Department of the University of Thessaly.

6 Conclusions

The present study offers an equilibrium based theoretical approach concerning the evaluation of the stress distribution under steel column base plates, acted upon statically by axial force and bending moment. A system of strongly nonlinear equations for the stress distribution function in both elastic and plastic regimes is fully assessed, which may be treated by powerful modern commercial mathematical software to produce numerical results. The latter – to appear on a companion paper under preparation – will be mainly used for studying the accuracy of existing relevant simplified analyses and secondarily, through a parametric study, for producing alternative tabulated data with ready to use applications in everyday engineering practice for structural design purposes.

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