FE Analysis of Complex Discontinuous and Jointed Structural Systems (Part 2: Application of the Method – Development of a 3D Model for the Analysis of Unreinforced Masonry Walls)

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ABSTRACT

In this part, a non-linear 3D finite element model for the analysis of unreinforced masonry walls subjected to static and seismic loads is presented, in order to demonstrate the applicability and potential of the method proposed in Part 1. The work reported here could also stand "on-its-own", due to the detailed investigation made on this particular subject. The model developed considers masonry as a two-phase material, treating bricks and mortar joints separately, thus allowing for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints. The influence of the mortar joints is taken into account by using 'interface' elements to simulate the time-dependent sliding and separation along the interfaces. Analytical and experimental solutions available in the literature have been employed to verify the results obtained from the present finite element model, showing that it is capable of a high degree of accuracy.

KEYWORDS

Discontinuous structures, Interface element, Masonry Wall, Non-linear behaviour.

1 INTRODUCTION

In Civil Engineering, inherently three-dimensional structures with joints or discontinuities subjected to static, dynamic, or seismic loads, present an interesting but complex problem, especially when their material behaviour is non-linear. Masonry structures provide a familiar example of this. Masonry walls, designed as vertical load-bearing elements primarily to resist in-plane compressive forces, are frequently subjected to considerable lateral loads, arising from one or more of the following: eccentricity of the applied in-plane forces, earth pressure, wind loads, and seismic or blast forces.

Use of three-dimensional finite element analysis requires a considerably greater effort in comparison with processes based on two-dimensional idealisations, which provide convenient and economical solutions. However, many engineering problems demand, by virtue of their configuration, loading or behaviour, a three-dimensional solution for a realistic prediction of their behaviour. Furthermore, a three-dimensional solution can be, and is sometimes used to verify the accuracy of the corresponding idealised two-dimensional solution.

A review of the available literature that follows reveals that only two-dimensional analyses with plane stress formulation have so far been adopted for the finite element analysis of masonry walls. Further, in the absence of a suitable model to represent its behaviour, in the past masonry was assumed to be an isotropic elastic continuum; consequently, the influence of the mortar joints acting as planes of weakness, could not be addressed. Indeed, it is only recently that analytical procedures, which account for the non-linear behaviour of masonry under static loads, have been developed.

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An extension of these procedures to static and dynamic/seismic response analysis of masonry walls in three-dimensions is reported here. A non-linear three-dimensional finite element model for masonry structures under static loads and seismic base inputs is presented, to demonstrate the applicability of the three-dimensional interface element (developed in Part I of the study) to the analysis of discontinuous systems. The model adopted treats bricks and mortar joints separately and allows for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints.

The influence of mortar joints has been taken into account by using interface elements to simulate the time-dependent sliding and separation along interfaces. The constitutive model of the interface elements reflects the limiting value of the shear stress/normal stress ratio at the interfaces. The proposed method of analysis is applicable to both static and dynamic problems in three-dimensions and is suitable for analysis of response, including the effects of material non-linearity and discontinuous contact surface. It is particularly suited to cases in which extensive stress redistribution occurs due to non-linear material behaviour and local failure.

The inelastic non-linear behaviour of masonry, subjected to an in-plane bending load, has been studied first. A loading of this nature produces high gradients of stress, with accompanying stress redistribution from local joint failure. The proposed finite element model has then been used to predict the non-linear response of an unreinforced masonry wall panel to an in-plane earthquake excitation. Analytical and experimental solutions available in the literature have been employed to verify the results obtained from the present finite element model.

2 LITERATURE REVIEW

2.1 Model for masonry

Analytical and experimental studies of the behaviour of masonry walls to in-plane static loads have been the focus of activity of a number of investigators for many years. Early studies concentrated mainly on experimental investigations of necessity, and these have been hampered by the large number of variables that must be considered in the overall behaviour of the masonry. The great number of the influencing factors, such as dimension and anisotropy of the bricks, joint width and arrangement of bed and head joints, material properties of both brick and mortar, and quality of workmanship, make the simulation of plain brick masonry extremely difficult.

Several investigators have dealt with the structural behaviour of masonry in recent years using the finite element method. Most analyses have considered masonry to be an assemblage of bricks and mortar with average properties, and isotropic elastic behaviour has been assumed to simplify the problem [1, 2] ignoring the influence of mortar joints acting as planes of weakness. Assumptions like that were useful in predicting deformations at low stress levels, but not at higher stress levels where extensive stress redistribution caused by non-linear material behaviour and local failure would occur. Material models, based on average properties and with the influence of mortar joints ignored but including the possibility of local failure, have also been developed by some investigators [3, 4].

More recently, Dhanasekar et al. [5] proposed a non-linear finite element model for solid masonry based on average properties derived from biaxial tests on brick masonry panels. The model is capable of reproducing the effects of material non-linearity and progressive local failure, but the masonry is modelled as a continuum with average properties, with each element in the finite element subdivision including several bricks and joints. The model, therefore, has limitations when local effects are important and cannot be used, for example, to predict the behaviour of masonry subjected to concentrated loads where local stresses and stress gradients are high.

Other investigators have produced more refined models, including elastic analyses with bricks and mortar joints being modelled separately [6, 7, 8]. A method that accounts for the non-linear behaviour of masonry, considering masonry as a two-phase material, was first developed and applied to solid

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masonry by Page [9] and to grouted and hollow concrete masonry by Hegemier et al. [10]. The method was also used by Ali & Page [11, 12] to study the non-linear behaviour of masonry subjected to concentrated loads.

A theoretical parametric study of the behaviour of storey height solid masonry walls, subjected to concentrated loads, has been performed by Ali and Page [13] using multi-level substructuring and mesh-refinement techniques for the efficient modelling of the walls. In the parametric study of the wall behaviour, the influence of the loaded area ratio, the load location, and the wall geometry have been considered and design relationships have been proposed.

The majority of the proposed constitutive models for masonry, therefore, can be classified in two categories: (i) the *one-phase* material models, treating masonry as an ideal homogeneous material with constitutive equations that differ from those of the components; and (ii) the *two-phase* material models where the components are considered separately to account for the interaction between them.

The constitutive models of the first category are relatively simple to use and require less input data, and the failure criterion has normally a simple form. On the other hand, their constitutive equations are relatively complicated and are suitable at best for the study of the global behaviour of masonry. The ‘two-phase’ material models are relatively costly to use due to the great number of the degrees of freedom, require more input data, and their failure criterion has a complicated form due to the brick-mortar interaction. The constitutive equations of the components have normally a simple form, on the other hand, and they are suitable for the study of the local behaviour of masonry.

Conclusions regarding the relative merits of various models and algorithms should be based on direct comparison of the methods, when applied to identical problems. Depending on the kind of problem therefore, the degree of accuracy required versus the simplicity desired, one could use the appropriate model and method to analyse the problem in hand.

2.2 Material behaviour

(a) Static loading

The development of improved models of material behaviour was made possible by the increased sophistication of numerical methods of stress analysis. A complete model requires the elastic properties of masonry, a yield criterion, inelastic stress-strain relations, and a failure criterion. Most of the studies to date have concentrated on finding a failure criterion rather than on studying the deformation characteristics of the material, and they have been restricted to tests under monotonic loading conditions. Masonry elements are subjected to biaxial states of stress produced by in-plane loading, and only recently there have been attempts to study the material properties of brick masonry subject to biaxial stress states.

Information on the stress-strain relations for brick masonry is limited and restricted to data on the elastic properties. The average elastic properties of brick masonry have been reported by Dhanasekar et al. [14] after a series of tests on masonry panels. Extensive tests were conducted on grouted concrete masonry by Hegemier et al. [15], who determined the elastic properties and the failure surface under biaxial stress state.

The failure of masonry under uniaxial compression, combined shear and compression, and tension, has been studied extensively in the past by many investigators [16, 17, 18, 19]. These failures all represent particular points on the general failure surface. The development of a general failure criterion for masonry is difficult, because of the difficulties in developing a representative biaxial test and the large number of tests involved. Yokel and Fattal [20] discussed the problem with reference to the failure of shear walls, and, in their study, various failure hypotheses were compared with the results of tests on single-wythe clay brick walls.

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Page [21] proposed a failure surface for brick masonry subjected to biaxial tension-tension, using a non-linear finite element model accounting for joint failure. He also reported experimentally derived failure surfaces for half-scale brick masonry subjected to compression-compression and tension-compression stress states [22, 23, 24]. A complete failure surface was developed later by Dhanasekar et al. [25] as an extension of these experimental results, and an approximate method of establishing a conservative failure surface from a reduced number of uniaxial and biaxial tests was proposed.

Samarasinghe and Hendry [26] also obtained a failure surface from tests on one-sixth scale of brick masonry subjected to biaxial tension-compression. The shape of both these failure surfaces was found to be critically dependant on the bed-joint orientation and the relationship between the shear and tensile bond strengths of the mortar joints. The influence of the orientation of the applied stresses to the joints has also been investigated for grouted and ungrouted concrete masonry by Hamid et al. [27] and failure criteria were proposed as a generalised form for masonry taking into consideration its anisotropic nature as a composite material [28]. The tensile strength of concrete masonry has also been investigated by the same authors [29], and tension failure criteria for unreinforced concrete block masonry have been developed [30] which account for strength variation due to the anisotropic nature of masonry as a composite material.

Asteris and Tzamtzis [31] have developed a methodology for the non-linear 'macroscopic' analysis of unreinforced masonry walls under biaxial stress state using the finite element method. The methodology focuses on the definition / specification of a general anisotropic (orthotropic) failure surface of masonry under biaxial stress, using a cubic tensor polynomial, as well as on the numerical solution of this non-linear problem. The characteristics of the polynomial used, ensure the closed shape of the failure surface which is expressed in a unique mathematical form for all possible combinations of plane stress, making it easier to include it into existing software for the analysis of masonry structures [32, 33]. The work is now under review and it will be reported in due course.

Ganz and Thurlimann [34] suggested a failure surface for brick masonry constructed from highly perforated bricks in terms of the direct stresses parallel and perpendicular to the bed joint, and shear stress on the bed joint. The failure surface was defined by four separate functions of these stresses, related to four distinct failure modes.

Non-linear stress-strain relations for brick masonry have been derived by Dhanasekar et al. [35] from the results of a large number of biaxial tests on square panels with various angles of the bed joint to the principal stress axes. It has been found that although the initial elastic behaviour is close, on average, to isotropic, the non-linear behaviour is strongly influenced by joint deformations and is best expressed in terms of stresses and strains referred to axes normal and parallel to the bed joint orientation.

Comparatively little research has been reported on the behaviour of brick masonry under cyclic compressive loading. The effects of repeated compressive loading are particularly applicable to brick masonry structures having large live load to dead load ratio. Recently, an investigation of the behaviour of brick masonry under cyclic, uniaxial and biaxial compressive loading has been presented by Naraine and Sinha [36, 37, 38]. They established that the uniaxial and biaxial stress-strain history possess a locus of common points where the reloading part of any cycle crosses the unloading part of the previous cycle which can be used to define the permissible stress levels for brick masonry under cyclic loading. A generalised approach is also proposed by the same authors [39] to determine the envelope, common-point, and stability-point curves on the absolute stress-strain coordinate system, using a general stress-strain equation for each principal stress ratio.

(b) Dynamic loading

The behaviour of masonry walls under earthquake forces is of primary importance for construction in earthquake-prone areas. Devastating damage to masonry structures in the last thirty years due to

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earthquakes has caused engineers to consider masonry as a structural material, to recognise its weaknesses and to consider ways to overcome these so that such damage can be significantly reduced in the future. Experimental and analytical research on the seismic behaviour of masonry structures during the last fifteen years has revealed that their earthquake performance can be significantly improved by properly reinforcing against tensile stresses which cause brittle failure because of the low tensile strength of masonry.

The state of knowledge concerning shear strength and shear load-displacement behaviour of masonry is far less advanced than that concerning behaviour in compression, even though shear is the dominant mode of failure observed in many masonry buildings subjected to lateral loading due to earthquakes, wind or other causes [40]. Most of the research conducted to date with regard to masonry shear behaviour has been limited to determining the peak shear strength and the factors that affect it. Additional information on the shear stiffness for both initial and repeated loading states, peak and residual strength values, repeated shear reversals, and dynamic effects are required in order to construct analytical models to simulate response under realistic seismic loading conditions. At present there appears to be very little knowledge about the dynamic mechanical properties of masonry.

The horizontal bed joint shear failure mode and the shear load-displacement behaviour of unreinforced brick masonry during static and cyclic loading have been examined by Atkinson et al. [41]. Results of tests by various researchers [42, 43] on reinforced masonry cantilever walls, subjected to in-plane lateral loads, indicate the existence of two basic failure modes: shear failure, which is characterised by diagonal cracking of the masonry along lines of principal tensile stresses in the wall plane; and flexural failure, which is characterised by either yielding of tension steel, followed by crushing of the masonry, or by crushing of the masonry alone at the compression toe.

The behaviour of masonry piers under in-plane cyclic loadings has been reported by many investigators in relation to the seismic design of buildings. These experimental studies are aimed at modifying and improving the existing codes for masonry structures. Mayes et al. [44, 45] were concerned with the strength, failure modes, and cyclic shear behaviour of masonry piers, and Hidalgo et al. [46, 47] have studied the cyclic behaviour of masonry piers subjected to load reversals. They tested various fixed-end masonry piers, some of which developed high bearing stresses leading to a shear mode of failure, and observed that the structural behaviour of the specimens failing in shear was much more complicated than those failing in flexure.

Gulkan et al. [48, 49] tested single-story masonry houses on a shaking table and investigated their behaviour by subjecting them to earthquake excitations. Manos et al. [50] studied the effect of combined in-plane and out-of-plane action on the earthquake performance of the same models used by Gulcan et al., as well as to vertical input motion.

Priestley [51] made an attempt to explain the behaviour of unreinforced masonry walls under seismic loading, with particular emphasis being given to face-load response. The response of unreinforced masonry walls to out-of-plane (face-load) seismic excitation is one of the most complex and little understood areas in seismic analysis. Extensive dynamic analyses and shake-table testing of face-loaded walls, performed by Kariotis et al. [52, 53], indicated that the walls could sustain levels of excitation acceleration far greater than that predicted by elastic or ultimate strength calculations. They found that a correlation could be found between the strength of face-loaded walls and the spectral velocity of the input acceleration. This indicates that energy considerations are important. An analytical model was developed by Button et al. [54] to predict the out-of-plane seismic behaviour of reinforced masonry walls. The model was validated by comparing its predicted response with those obtained from full-scale dynamic tests on clay-brick walls performed by Blondet et al. [55], and tests on concrete-block walls performed by Adham et al. [56].

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The first attempt in developing a mathematical model to predict the response of masonry walls to dynamic excitation was made by Sucuoglu et al. [57, 58, 59]. Two mathematical models were presented by the authors for predicting the linear in-plane dynamic behaviour of masonry walls. The first, called ‘mixture model’, recognises the two-material composition of masonry and predicts its response accurately for a wide range of frequencies. The theoretical framework of the model was established by Mengi and McNiven [60]. The second, called the ‘effective modulus’ model, is simpler and accurate for a smaller frequency range. The study was presented in three stages: experimental observations, selection of the mathematical model, and the determination of model parameters through optimisation analysis.

Later, a model for predicting the non-linear in-plane behaviour of clay brick masonry walls, when subjected to dynamic excitations, was proposed by the same authors [61] by extending the previously developed linear model to cover behaviour in the non-linear range. In the study of the mechanical properties of masonry, it has been found that the values established when masonry is responding to dynamic forces are radically different from those when the masonry is responding to static forces. These values remain constant until cracking begins, after which the shear modulus decreases and the damping coefficient increases. Results and optimisation show that these changes are almost linear. The same study was published again by the authors in two parts [62, 63] in 1989. Part 1 was devoted to a description of the appropriate experiments and to the development of the mathematical model, including use of experimental data. In Part 2 the model was completed by establishing the ‘parameter functions’ appearing in it.

3 FINITE ELEMENT MODEL

In the present study, a special three-dimensional interface element (developed in Part 1 of the study) has been used to simulate the mortar joints present in a masonry wall assemblage. The proposed interface element is three-dimensional and it does not appear to have been used in the non-linear analysis of three-dimensional masonry structures subjected to static or dynamic forces. A work of this nature is to be reported by the authors in due course [64].

In order to test the element’s performance, it would be logical to use it to analyse a truly three-dimensional problem for which analytical or experimental results are known or available. Unfortunately, no such work, analytical or experimental, appears to have been done to date with which present results of a three-dimensional finite element analysis could be compared. Reliable results of tests on two-dimensional masonry walls are, on the other hand, available, under static and dynamic loading conditions. For this reason, two-dimensional masonry walls analysed by other investigators have been chosen as the test problem, in both static and dynamic analyses.

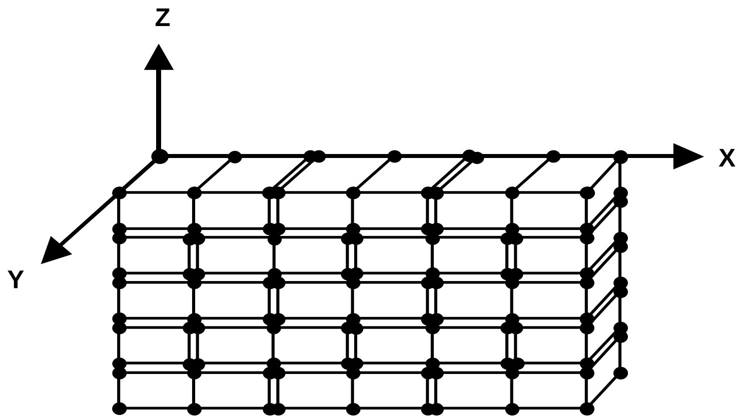

Figure 3. Typical finite element subdivision

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A three-dimensional formulation has been used instead, to demonstrate the level of accuracy the proposed interface element is capable of. A typical discertization of a masonry wall is shown in Figure (3). It was discertized using 8-noded isoparametric 'brick' elements to represent the bricks, while the mortar joints (shown exaggerated), where represented by three-dimensional interface elements (Fig. 1).

4 STATIC ANALYSIS

4.1 The problem and method of analysis

This study deals with the analysis of unreinforced masonry walls subjected to in-plane static loading, a problem commonly encountered in the design of masonry structures, for which experimental and analytical results are available. Bending tests on a masonry deep beam have been performed by Page [9] to verify his two-dimensional finite element model for the analysis of masonry walls subjected to in-plane loading. Since the present study deals with the extension and development of a similar model in three-dimensions, the experimental results obtained by Page [9] were used here to verify the results obtained from the present three-dimensional model. Figure (4) shows the test arrangement of the masonry wall used for the experiments, and adopted in the present study.

Incremental analysis is able to give full illustration of the loading processes and has obvious advantages in describing complicated processes such as loading-unloading-reloading and so on, and for this reason it has been employed in the present study. The element types described have been incorporated into an incremental finite element computer program especially developed for the analysis. The load was applied incrementally in terms of equivalent displacements at the relevant nodes, in order to avoid numerical instability.

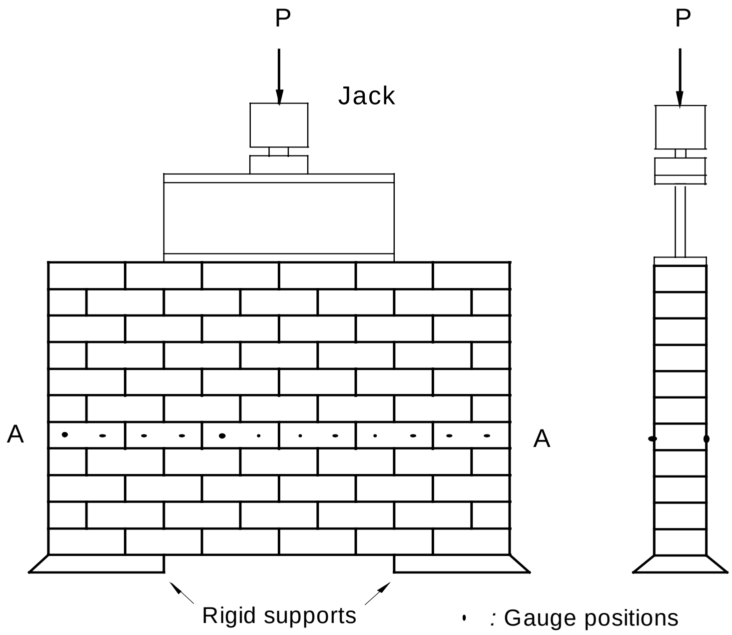

Figure 4. General arrangement of masonry deep beam test [9]

At a given load level, iteration was continued until stresses generated by the loads satisfied the yield or failure criteria within prescribed tolerances. Convergence was taken to have been achieved when the displacement increment vector, from one iteration to the next, was less than the chosen tolerance. A further increment of load was applied once convergence was achieved, and the process repeated as required. Non-linear material characteristics, for both bricks and joints, were modelled using the *initial stress* method.

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Inelastic behaviour of the bricks was also introduced in the finite element analysis. Non-linear finite element analysis of three-dimensional masonry prisms is a complex and time-consuming exercise, which relies on a detailed knowledge of the stress-strain relationships of both the blocks and the mortar under a multi-axial state of stress. Since such information is difficult and often not possible to provide, elastic-perfectly plastic behaviour has been assumed in the present analysis for simplicity. The failure criterion used for the bricks is the Mohr-Coulomb criterion, which is best suited for brittle materials with properties similar to concrete. This criterion is also used because of its inherent simplicity and convenience.

4.2 Interface behaviour

For the efficient non-linear analysis of masonry, it is necessary to consider relative slip, debonding and cycles of closing and opening of the interfaces. To account for this behaviour, a special interface element has been proposed and presented in Part I of the study. The stiffness properties of such an interface element can be derived from tests that can simulate the transfer of shear stresses.

For wall thickness t_w and mortar joint thickness t_m, the normal and shear stiffnesses required to define the material property matrix in Eq. (13), can be represented by the following expressions [9]:

$$k_{sx} = k_{sy} = \frac{G\,t_w}{t_m}; \quad k_{nz} = \frac{E\,t_w}{t_m} \tag{13}$$

in which E and G are the instantaneous tangent elastic and shear moduli at the particular value of normal and shear stress being considered. The non-linear behaviour of the joints can therefore be treated by assigning the joint properties corresponding to the level of stress obtained from the last load step in a step-by-step loading analysis procedure.

The failure criteria of a joint depend mainly on the relative magnitudes of the normal and shear stresses present in the joint. The relationship between the normal stress in a joint and its ultimate shear strength can be obtained from tests on masonry prisms with the load inclined to the bed joints. A failure criterion of this type, obtained by Page [9], has been adopted in the present analysis (Figure 5).

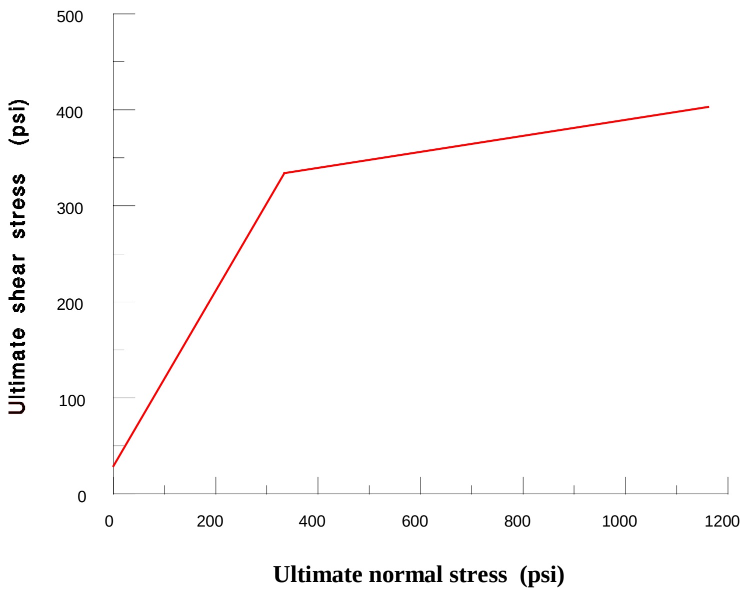

Figure 5. Joint failure envelope (1 psi = 6.89 kN/m^2)

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When used in the analytical model, this criterion allows progressive joint failure to occur. If the criterion for the failure of a joint element is violated, the element properties are modified and the problem is resolved. If failure occurs due to a normal tensile stress in the joint, the joint element stiffness is set equal to zero in both the normal and shear directions. If failure occurs under a combination of compression and shear stresses, the stiffness of the joint in the normal direction is assumed to remain unchanged, and a reduced value for the shear stiffness is allocated, depending mainly on the compression stress in that joint. As the normal stress diminishes, the residual shear stiffness is assumed to reduce to zero under a condition of pure shear. Introducing the above constitutive laws in the finite element program, the inelastic behaviour of the mortar joints can be represented.

4.3 Results - verification of the model

Vertical stress distributions at level A-A (see Figure 5), obtained from experiments performed by Page [9], are compared to the results obtained using the three-dimensional finite element model developed in section 5.3. Stress distributions at two levels of applied load, P, are shown (Figure 6). Results obtained from a conventional finite element analysis, with the masonry modelled as a continuum with average properties and isotropic elastic behaviour, are also included. The computed progression of cracks, as predicted by the finite element model, is also shown in Figure (6), with P increasing from 40 KN to 60 KN.

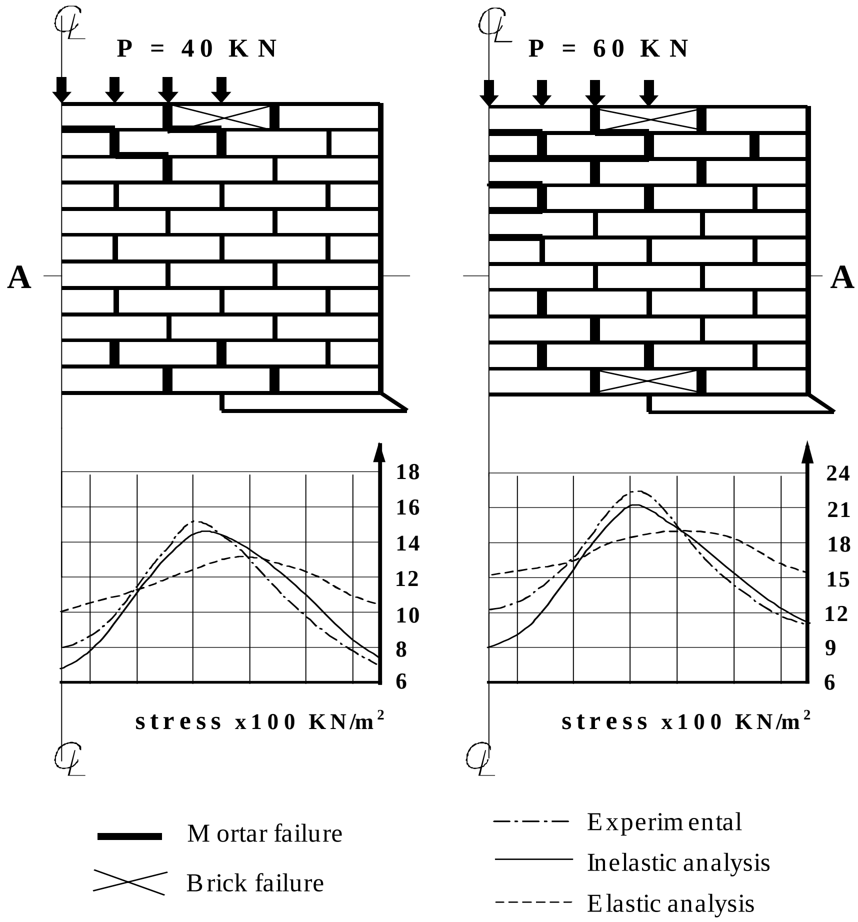

Figure 6. Comparison of present analytical results with the experimental results of Page [9]

Clearly, there is a good agreement between the results of the present inelastic analysis and the experimental results of Page [9], particularly in view of the inherent variability of actual material properties, joint strengths and the various simplifications that have been made in the proposed

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analytical model (stress-strain relationship, failure criterion, etc.). It is also clear that the elastic solution deviates grossly from both experimental and the inelastic results, and this confirms that material behaviour is significantly non-linear, especially that of the mortar joints.

5 SEISMIC ANALYSIS

5.1 The problem and method of analysis

The calculation of the dynamic response of a discontinuous system subjected to time-varying loads such as blast, impact or earthquake loading is generally a complicated process, especially if the non-linear behaviour of the system is simulated.

Given that by far the majority of earthquake fatalities world-wide are caused by the failure of masonry structures, and that masonry is probably the least understood of all construction materials, the need for developing an accurate model of the seismic behaviour of masonry structures cannot be over-stated. Unfortunately, information on the material properties of masonry under dynamic/seismic loading is both complex and scarce. At present there appears to be very little knowledge about the dynamic properties of masonry, especially in the non-linear regime. The problem is further compounded by the unavailability of results, experimental or analytical, with which present results could be compared. Indeed, to the best of author's knowledge, the only relevant work available with which present results could be compared, is the shaking table test on a brick wall, shown schematically in Figure (7), due to Mengi and McNiven [61, 62, 63].

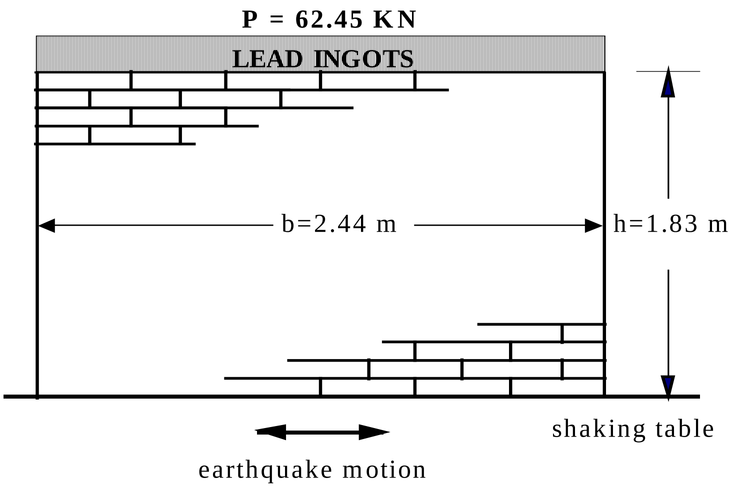

Figure 7. Schematic diagram of the seismic test setup [61]

The finite element model described above has been used to predict the in-plane response of a masonry wall to a given earthquake excitation. The bricks in Figure (7) were modelled with 8-noded 'brick' elements, and the joints between adjacent bricks with the interface element of Figure (1). The exciting base input to the system was taken to be the modified El-Centro earthquake record shown in Figure (8), used by Mengi and McNiven [61] for experiment 11 in their study. The modification involved the reduction of the time-scale by a factor of $\sqrt{3}$ so that it was possible to study the wall behaviour for a wider range of frequencies.

Since material behaviour under dynamic loading is very complex and experimental information is scare, in attempting to perform an analysis of a dynamically-loaded structure we must look for an idealised material model where some compromises can be made. The simplest stress-strain law has been implemented in the present finite element analysis and involves elastic-perfectly plastic material behaviour.

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Isotropic conditions have been assumed with the following material properties: assuming that the condition G/E=0.5, obtained by Mengi and McNiven [61] for the dynamic case holds in general, the value of the Poisson ratio has been taken as zero, and the value of the Young's modulus (considered to be the same for both bricks and mortar) has been taken as E=48780 psi.

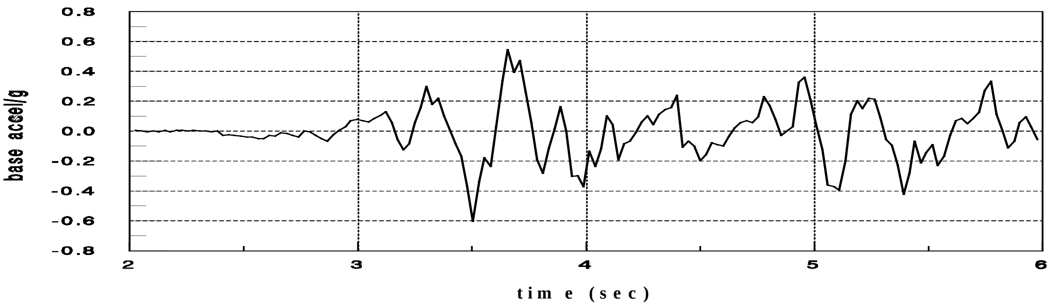

Figure (8) Seismic record of the modified El Centro earthquake (taken from Reference 61).

Both the Wilson-theta and Newmark average acceleration methods (i.e. α=1/4 and δ=1/2) have been employed in the present study for computing system response. The optimised size of the time-step, Δt, was taken as 0.025 sec, and the value of theta used in the Wilson-theta method as 1.4 in order to ensure the unconditional stability of solution. The size of errors, which may be introduced by any numerical integration scheme, depends on the characteristics of the dynamic loading and the time step. The general nature of the computational errors may be expressed in terms of an artificial change of period and a reduction of amplitude. These period-elongation and amplitude-decay, caused by the numerical integration errors, can be important and should be taken into account when comparing the results of the analysis.

Very limited information is available on damping in linear solid mechanics problems, and there is even less data available on damping in non-linear situations, especially for materials subjected to seismic loads. However, since the effect of damping is small compared to those of inertia and stiffness, the damping matrix [C] may be represented by simplified expressions. In this analysis it has been assumed, therefore, that the damping matrix is proportional to the mass and stiffness matrix. This is known as Rayleigh damping and we have

$$[C] = a[M] + b[K] \tag{14}$$

in which **a** and **b** are proportionality constants selected to control the damping ratios of the lowest and highest modes expected to contribute significantly to the response. For the problem under consideration, the value of **a** was taken as zero, and, following Reference [61], the value of **b** was taken as 0.24. Rayleigh modelling of damping is rather poor since constants **a** and **b** are fixed for all modes of vibration and the effect on the response history of the structure has to be acknowledged.

The stress level in the structure before the application of the seismic base excitation was assumed to be due to extra loading on top of the wall (Figure 7). The influence of this mass on the seismic behaviour of the wall was considered by taking into account the mass of the lead ingots attached to the top face of the wall. Prior to the seismic analysis, the initial stresses were calculated using a static finite element program. The stress state for every Gauss integration point was recorded and added to the input data for seismic analysis.

5.2 Response results

The experimental response of the brick masonry wall of Figure (7), subjected to the El Centro earthquake motion (Figure 8), was obtained from an experimental program on a shaking table performed by Mengi and McNiven [61]. It has been used here as the basis for comparison of the results. This is given in Figure (9), and is compared with the system response as predicted by the present finite element method using both Wilson-theta and Newmark's implicit algorithms.

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Inspite the various simplifications and assumptions that have been made in the present finite element analysis (material properties, stress-strain behaviour, failure criterion, Rayleigh damping, etc.), the computed response using both the algorithms are in a very good agreement with that found from the experiments. Unfortunately, not all the material properties necessary to define the present finite element model could be found, and therefore, some of the properties had to be assumed. This is probably the reason for the discrepancy between test and present results. In addition, experimental conditions could not be reproduced accurately, since the test specimen was composed of two parallel walls connected to a steel base frame, which in turn, was bolted to the shaking table. The finite element model used in the present analysis simulates only one of the walls with half of the imposed weight placed on it.

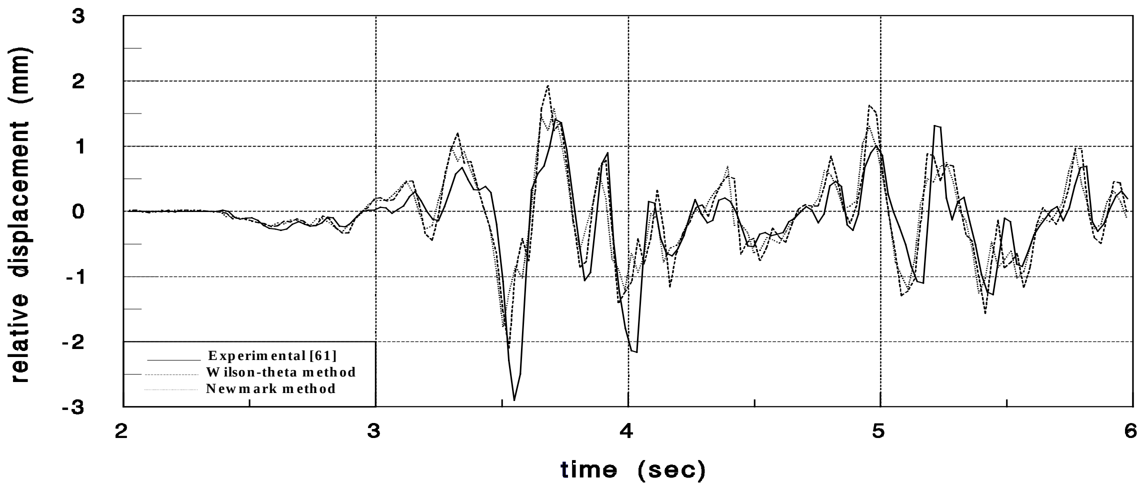

Figure 9. Comparison of the computed seismic responses of the wall of Fig. (7) to the excitation of Fig. (8)

6 SUMMARY AND CONCLUSIONS

A method for the non-linear finite element analysis of masonry, subjected to static and earthquake loading conditions, has been developed in this part of the study. The problem is offered as a case study in order to demonstrate the applicability of the three-dimensional interface element presented in Part 1, to the analysis of discontinuous systems. The model developed is three-dimensional and considers masonry as a two-phase material, treating bricks and mortar joints separately, thus allowing for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints. The influence of the mortar joints has been taken into account by using interface elements to simulate the time-dependent sliding and separation along the interfaces.

Stress distributions from an in-plane bending test on a masonry panel performed by other investigators, were reproduced to a reasonable degree of accuracy by the present finite element model. The accuracy and potential of the proposed three-dimensional model were also demonstrated by using the model to predict the non-linear response of an unreinforced masonry wall to in-plane earthquake excitations. Analytical and experimental solutions available in the literature have been employed to verify the results obtained from the present finite element model, showing that it is capable of a high degree of accuracy.

The use of the interface element in the finite element analysis of masonry structures, employed to simulate the actual joint behavioural features of the system, may eventually lead to a more rational design of these structures. The model adopted offers a more realistic alternative to an analysis that assumes masonry as continuum with average properties, and it has a potential as a research tool since

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the properties of the masonry components can be varied individually and their significance to the overall behaviour of the masonry assemblage investigated.

REFERENCES

1. Rosenhaupt, S., and Sokal, Y., 'Masonry Walls on Continuous Beams', J. Structural Division, ASCE, Vol. 91, No ST1, 1965, pp. 155-171.
2. Saw, C. B., 'Linear Elastic Finite Element Analysis of Masonry Walls on Beams', Building Science, Vol. 9, 1974, pp. 299-307.
3. Ganju, T. N., 'Nonlinear Finite Element Analysis of Clay Brick Masonry', Proc. 6th Australasian Conf. on Mech. of Structures and Materials, 1977, pp. 59-65.
4. Samarasinghe, W., Page, A. W., and Hendry, A. W., 'A Finite Element Model for the In-Plane Behaviour of Brickwork', Proc. Inst. of Civil Engineers, Vol. 73, 1982, pp. 171-178.
5. Dhanasekar, M., Page, A. W., and Kleeman, P. W., 'A Finite Element Model for the In-Plane Behaviour of Brick Masonry', Proc. 9th Australasian Conference on Mechanisms of Structures, 1984, pp. 262-267.
6. Smith, B. S., Carter, C., and Chowdhury, J. R., 'The Diagonal Tensile Strength of Brickwork', The Structural Engineer, Vol. 48, No 4, 1970, pp. 219-225.
7. Smith, B. S., and Rahman, K. M. K., 'The Variation of Stress in Vertically Loaded Brickwork Walls', Proc. Inst. of Civil Engineers, Vol. 43, 1972, pp. 689-700.
8. Ali, S., and Page, A. W., 'An Elastic Analysis of Concentrated Loads on Brickwork', Masonry Int., Vol. 6, 1985, pp. 9-21.
9. Page, A. W., 'Finite Element Model for Masonry', Journal Str. Div., Proc. ASCE, Vol. 104, No ST8, 1978, pp. 1267-1285.
10. Hegemier, G. A., et al., 'On the Behaviour of Joints in Concrete Masonry', Proc. North American Conference, Masonry Society, USA, 1978, pp. 4.1-4.21.
11. Ali, S., and Page, A. W., 'Non-linear Finite Element Analysis of Masonry subjected to Concentrated Loads', Proc. Inst. of Civil Engineers, Vol. 83, 1987, pp. 815-832.
12. Ali, S., and Page, A. W., 'Finite Element Model for Masonry Subjected to Concentrated Loads', Journal Str. Div., Proc. ASCE, Vol. 114, No ST8, 1988, pp. 1761-84.
13. Ali, S., and Page, A. W., 'Concentrated Loads on solid Masonry Walls - A Parametric Study and Design Recommendations', Proc. Inst. of Civil Engineers, Vol. 85, 1988, pp. 271-289.
14. Dhanasekar, M., Page, A. W., and Kleeman, P. W., 'The Elastic Properties of Brick Masonry', Int. Journal of Masonry Construction, Vol. 2, No 4, 1982, pp. 155-160.
15. Hegemier, G. A., Nunn, R. O., and Arya, S. K., 'Behaviour of Concrete Masonry Under Biaxial Stresses', Proc. North American Masonry Conference, 1978, pp. 1.1-1.28.
16. Khoo, C. L., and Hendry, A. W., 'Strength Tests on Brick and Mortar under Complex Stresses for the Development of a Failure Criterion for Brickwork in Compression', Proc. British Ceramic Society, Load Bearing Brickwork (4), 1973, pp. 51-66.
17. Grimm, C. T., 'Strength and Related Properties of Brick Masonry', Journal of the Structural Division, ASCE, Vol. 101, No ST1, 1975, pp. 217-231.
18. Hendry, A. W., 'A Note on the Strength of brickwork in Combined Racking Shear and Compression', Proc. British Ceramic Soc., 1978, Load Bearing Brickwork (6), pp. 47-52.
19. Smith, B. S., and Carter, C., 'Distribution of Stresses in Masonry Walls subjected to Vertical Loading', Proc. 2nd Int. Brick Masonry Conference, 1970, pp. 119-124.
20. Yokel, F. Y., and Fattal, S. G., 'Failure Hypothesis for Masonry Shear Walls', Journal of the Structural Division, ASCE, Vol. 102, No ST3, 1976, pp. 515-532.
21. Page, A. W., 'A Biaxial Failure Criterion for Brick Masonry in the Tension-Tension Range', Int. Journal Masonry Constr., 1980, 1(1), pp. 26-29.
22. Page, A. W., 'The Biaxial Compressive Strength of Brick Masonry', Proc. Inst. of Civil Engrs., Vol. 71, 1981, pp. 893-906.
23. Page, A. W., 'An Experimental Investigation of the Biaxial Strength of Brick Masonry', Proc. Sixth Int. Brick Masonry Conf., Rome, 1982, pp. 3-15.

© 2004 EJSE International. All rights reserved.

24. Page, A. W., 'The Strength of Brick Masonry under Biaxial Compression-Tension', Int. Journal of Masonry Construction, Vol. 3(1), 1983, pp. 26-31.
25. Dhanasekar, M., Page, A. W., and Kleeman, P. W., 'The Failure of Brick Masonry under Biaxial Stresses', Proc. Inst. of Civil Engineers, Vol. 79, Part 2, 1985, pp. 295-313.
26. Samarasinghe, W., and Hendry, A. W., 'The Strength of Brickwork under Biaxial Tension-Compression', Proc. 7th Int. Symp. on Load Bearing Brickwork, London, 1980.
27. Hamid, A. A., Drysdale, R. G., and Heidebrecht, A. C., 'Shear Strength of Concrete Masonry Joints', Journal of the Structural Division, Proc. ASCE, Vol. 105, No ST7, 1979, pp. 1227-1240.
28. Hamid, A. A., and Drysdale, R. G., 'Proposed Failure Criteria for Concrete Block Masonry under Biaxial Stresses', Journal of the Structural Division, Proc. ASCE, Vol. 107, No ST8, 1981, pp. 1675-1687.
29. Drysdale, R. G., Hamid, A. A., and Heidebrecht, A. C., 'Tensile Strength of Concrete Masonry', Journal of the Structural Div., Proc. ASCE, Vol. 105, No ST7, 1979, pp. 1261-1275.
30. Drysdale, R. G., and Hamid, A. A., 'Tension Failure Criteria for Plain Concrete Masonry', Journal of Structural Engineering, ASCE, Vol. 110, No 2, 1984, pp. 228-244.
31. Asteris, P. G., and Tzamtzis, A. D., ‘On the Use of a Regular Yield Surface for the Analysis of Unreinforced Masonry Walls’, Electronic Journal of Structural Engineering, Vol. 3, 2003, pp. 23-42 .
32. Asteris, P. G., “Analysis of anisotropic nonlinear masonry”, PhD thesis, Dept. of Civ. Engrg., National Technical University of Athens, Greece, 2000.
33. Syrmakezis, C. A. and Asteris, P. G., ‘Masonry failure criterion under biaxial stress state’, ASCE J. Mat. in Civ. Engrg., Vol. 1, No 13, 2001, pp. 58-64.
34. Ganz, H. R., and Thurlimann, B., 'Strength of Brick Walls under Normal Force and Shear', Proc. 8th Int. Symp. on Load Bearing Brickwork, London, 1983.
35. Dhanasekar, M., Kleeman, P. W., and Page, A. W., 'Biaxial Stress-Strain Relations for Brick Masonry', Journal of Structural Engineering, ASCE, Vol. 111, No 5, 1985, pp. 1085-1100.
36. Naraine, K., and Sinha, S. N., 'Test of Brick Masonry Models Under Cyclic Compressive Loading', Proc. 8th Int. Brick Masonry Conf., Dublin, Ireland, Vol. 1, 1988, pp. 395-405.
37. Naraine, K., and Sinha, S. N., 'Behaviour of Brick Masonry under Cyclic Compressive Loading', Journal of Struct. Engineering, ASCE, Vol. 115, No 6, 1989, pp. 1432-1445.
38. Naraine, K., and Sinha, S. N., 'Loading and Unloading Stress-Strain Curves for Brick Masonry', Journal of Structural Engineering, ASCE, Vol. 115, No 10, 1989, pp. 2631-2644.
39. Naraine, K., and Sinha, S. N., 'Stress-Strain Curves for Brick Masonry in Biaxial Compression', Journal of Structural Engineering, ASCE, Vol. 118, No 6, 1992, pp. 1451-1461.
40. Kariotis, J. C., Ewing, R. D., and Johnson, A. W., 'Strength Determination and Shear Failure Modes of Unreinforced Brick Masonry with Low Strength Mortar', Proc. 7th Int. Brick Masonry Conference, 1985, Melbourne, Australia.
41. Atkinson, R. H., et al., 'Response of Masonry Bed Joints in Direct Shear', Journal of Structural Engineering, Vol. 115, No 9, 1989, pp. 2276-2296.
42. Meli, R., 'Behaviour of Masonry Walls under Lateral Loads', Proc. Fifth World Conf. on Earthquake Engineering, Rome, 1973.
43. Priestley, M. J. N., and Bridgeman, D. O., 'Seismic Resistance of Brick Masonry Walls', Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 7, No 4, 1974, pp. 167-187.
44. Mayes, R. L., Omote, Y., and Clough, R. W., 'Cyclic Shear Tests of Masonry Piers, Vol. I - Test Results', EERC Report No. 76/8, University of California, Berkeley, California, 1976.
45. Mayes, R. L., Omote, Y., and Clough, R. W., 'Cyclic Shear Tests of Masonry Piers, Vol. II - Analysis of Test Results', EERC Report No. 76/16, University of California, Berkeley, California, 1976.

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46. Hidalgo, P. A., et al., 'Cyclic Loading Tests on Masonry Piers, Vol. I -Height to Width Ratio of 2', EERC Report No. 78/27, University of California, Berkeley, California, 1978.
47. Hidalgo, P. A., et al., 'Cyclic Loading Tests on Masonry Piers, Vol. III -Height to Width Ratio of 0.5', EERC Report No. 79/12, University of California, Berkeley, California, 1979.
48. Gulkan, P., Mayes, R. L., and Clough, R. W., 'Shaking Table Study of Single-Story Masonry Houses, Vol. I-Test Structures 1 and 2', EERC Report No. 79/23, University of California, Berkeley, California, 1979.
49. Gulkan, P., Mayes, R. L., and Clough, R. W., 'Shaking Table Study of Single-Story Masonry Houses, Vol. II-Test Structures 3 and 4', EERC Report No. 79/24, University of California, Berkeley, California, 1979.
50. Manos, G. C., Clough, R. W., and Mayes, R. L., 'Shaking Table Study of Single-Story Masonry Houses', EERC Report No. 83/11, University of California, Berkeley, California, 1983.
51. Priestley, M. J. N., 'Seismic Behaviour of Unreinforced Masonry Walls', Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 18, No 2, 1985, pp. 191-205.
52. Kariotis, J. C., Ewing, R. D., and Johnson, A. W., 'Predictions of Stability for Unreinforced Brick Masonry Walls Shaken by Earthquakes', Proc. 7th Int. Brick Masonry Conference, Melbourne, Australia, 1985, pp. 1175-1184.
53. Kariotis, J. C., Ewing, R. D., and Johnson, A. W., 'Methodology for Mitigation of Earthquake Hazards in Unreinforced Brick Masonry Buildings', Proc. 7th Int. Brick Masonry Conference, Melbourne, Australia, 1985, pp. 1339-1350.
54. Button, M. R., and Mayes, R. L., 'Out-of-Plain Seismic Response of Reinforced Masonry Walls', Journal of Structural Engineering, ASCE, Vol. 118, No 9, 1992, pp. 2495-2513.
55. Blondet, J. M., Mayes, R. L., and Kelly, T. E., 'Out-of-Plain Testing of Slender Hollow Clay Masonry Block Walls', Proc. 5th North American Masonry Conf., Univ. of Illinois, 1990.
56. Adham, S. A., et al., 'Out-of-Plain Dynamic Testing of Concrete Masonry Walls', Proc. 5th North American Masonry Conference, Univ. of Illinois, 1990.
57. Sucuoglu, H., Mengi, Y., and McNiven, H. D., 'A Mathematical Model for the Response of Masonry Walls to Dynamic Excitations', EERC Report No. 82/24, University of California, Berkeley, California, 1982.
58. Mengi, Y., Sucuoglu, H., and McNiven, H. D., 'A Linear Mathematical Model for the Seismic Inplane Behaviour of Brick Masonry Walls. Part 1: Theoretical Considerations', Earthquake Engineering and Structural Dynamics, Vol. 12, 1984, pp. 313-326.
59. Sucuoglu, H., Mengi, Y., and McNiven, H. D., 'A Linear Mathematical Model for the Seismic Inplane Behaviour of Brick Masonry Walls. Part 2: Determination of Model Parameters Through Optimization Using Experimental Data', Earthquake Engineering and Structural Dynamics, Vol. 12, 1984, pp. 327-346.
60. Mengi, Y., and McNiven, H. D., 'A Mathematical Model of Masonry for Predicting its Linear Seismic Response Characteristics', EERC Report No. 79/04, University of California, Berkeley, California, 1979.
61. Mengi, Y., and McNiven, H. D., 'A Mathematical Model of Masonry for Predicting the Nonlinear Response of Unreinforced Masonry Walls to In-Plane Earthquake Excitations', EERC Report No. 86/07, University of California, Berkeley, California, 1986.
62. Mengi, Y., and McNiven, H. D., 'A Mathematical Model for the In-Plane Non-Linear Earthquake Behaviour of Unreinforced Masonry Walls. Part 1: Experiments and Proposed Model', Earthquake Engineering and Structural Dynamics, Vol. 18, 1989, pp. 233-247.
63. McNiven, H. D., and Mengi, Y., 'A Mathematical Model for the In-Plane Non-Linear Earthquake Behaviour of Unreinforced Masonry Walls. Part 2: Completion of the Model', Earthquake Engineering and Str. Dynamics, Vol. 18, 1989, pp. 249-261.
64. Tzamtzis, A. D, and Asteris, P. G., ‘A 3D Model for Non-linear ‘Microscopic’ FE Analysis of Masonry Structures’, Proceedings of the Sixth International Masonry Conference, London, No 9 November 2002, pp. 493-497.

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