# Describing Functions For Effective Stiffness and Effective Damping of Hystersis Structures

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## ABSTRACT

*For a hysteresis structure with energy dissipation devices, the force-displacement relation is nonlinear such that it is very difficult to evaluate the actual damping and stiffness coefficients, even if the force-displacement characteristic is simply perfect elasto-plastic. With the describing function method, we can linearize the nonlinear behavior of the energy dissipation devices and then obtain the equivalent damping and stiffness coefficients; In turn, the effective period and equivalent damping ratio can be attained. It is stressed that with this approach, the imaginary part of the describing function is just the energy dissipation term, which corresponds to the conventional hystersis damping derived by the energy method. Simulation results confirm the effectiveness of this proposed method.*

## KEYWORD

*describing function; hysteresis structures; energy dissipation devices; equivalent period ratio; equivalent damping ratio*

## 1 Introduction

Lots of building and bridge structures have been constructed, around the world, by using certain energy dissipation devices, the dampers and base isolators, to resist the external vibration forces. These external forces are generally induced by several factors such as the earthquake, wind, rotating machines and etc.

The base isolation techniques are used since the early 1970s. This trend has been accelerated over the past two decades. The purpose of the base isolation is to increase the natural period of the overall structure so that the acceleration response of the structure is decreased during earthquake. A further decrease in response can be achieved with the addition of damping. This shift in period, together with additional damping, can remarkably reduce the effect of the earthquake, so that less-damaging loads and deformations are imposed on the structure and its content [1-2].

Many researchers [1-4] have already proposed the determination of effective period and equivalent viscous-damping factor for base isolated systems. The American Association of State Highway Transportation Officials (AASHTO) proposes specifications to the design of base isolation systems using the effective period and equivalent damping ratio based on a secant modulus concept [2,4]. However, according to the AASHTO formula, it can be observed that the damping ratio will becomes a constant as the ductility ratio becomes large and even for the bilinear system the damping ratio will decrease as the ductility ratio increase. This is contradictory to the practical observations. Therefore, some empirical formulas have been proposed by a lot of researcher [2-3].

For the nonlinear system, the nonlinear system can be linearized by Taylor series expansion. However, the linearized model for the Taylor series expansion is valid only locally, which means it can be used in the situation of small amplitude. On the other hand, the describing

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function method has been widely used to linearize the dynamical behaviors for the nonlinear systems [5-8]. The advantages of the describing function method are that it can be even applied in the situations of large amplitude, if the linear transfer possesses low-pass filter property. This means that the describing function method is still valid during the strong earthquakes.

The problems of structural dynamics are often dealt with in the time domain. However, some dynamic characteristics can be more easily interpreted by the frequency domain analysis, e.g. the transfer function [9-10]. In this paper, we explore the dynamical properties of energy dissipation devices with perfect elasto-plastic force-displacement characteristics. In general, a steel-beam damper or rubber-bearing isolator has such property [1,11]. With the describing function method, the structure can be linearized and then according to the equivalence of the transfer function in the frequency domain, the equivalent damping coefficients and effective stiffness can be obtained. It can be noted that with this approaches, the imaginary part of the describing function is just the dissipation energy terms, which is corresponding to the conventional hystersis damping derived by the energy method. Also, it can be proved the linear transfer function possess the low-pass filter property. Moreover, the damping ratio derived by the describing function method will increase as the ductility ratio increase. Simulation results will show the effectiveness of this proposed method. The harmonic external force is considered here in the simulation.

## 2 A Structure with Hysteretic Energy Dissipation Devices

In this section, first, we will formulate the governing equation of a civil structure, which contains an energy dissipation devices as can be seen in Fig.1, such as the base isolators and dampers, to resist the external force induced by the earthquake, wind and mechanical vibration. The force-displacement characteristic for the energy dissipation devices is elasto-plastic. Assuming the whole structure is dominated by the fundamental mode, the overall governing equation of this system can be rewritten as a spring-mass-damper system, shown in Fig.1

$$m\ddot{u} + c\dot{u} + [k_0 u + \phi(u)] = f(t) \tag{1}$$

where $f(t)$ is the external force which may be induced by the earthquake, wind and rotating machine, $c$ is the coefficient for the linear viscous damping coefficient, $\phi(u)$ is the stiffness restoring force exerted by the energy dissipation devices, $k_0$ is the stiffness coefficients without considering these energy dissipation devices and $u$ is the displacement. Note that in general, the dynamic behavior of a steel-beam, steel cantilever damper or rubber-bearing isolator [] is a perfect elasto-plastic model [1], which can be shown in Fig.2.

The problem, here we posed, is that try to find equivalent damping and stiffness coefficients $c_{eq}$ and $k_{eq}$ close to the nonlinear function $\phi(u)$ shown in Fig.2, which can be described by

$$\phi(u) \approx c_{eq}\dot{u} + k_{eq}u \tag{2}$$

Then, the nonlinear system of Eq.(1) can be approximately replaced by a linear system written as

$$m\ddot{u} + (c_{eq} + c)\dot{u} + (k_{eq} + k_0)u = f(t) \tag{3}$$

where $k_0$ is the stiffness coefficient without considering these energy dissipation devices.

Consequently, in Eq.(3), the equivalent natural frequency $\omega_{eq}$ becomes

$$\omega_{eq} = \sqrt{\frac{(k_{eq} + k_0)}{m}} \tag{4}$$

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Conventionally, by considering $k_0 = 0$ and with the energy equivalence [1, 12], the equivalent damping and stiffness coefficients can be expressed by

$$c_{eq} = \frac{w_d}{\pi\omega A^2}, k_{eq} = k \tag{5}$$

where $w_d$ is the dissipation energy , $A$ is the amplitude for the displacement of $u$ and $k$ is the elastic modulus for the energy dissipation devices shown in Fig.2.

As the exciting frequency is identical to the natural frequency, then, the hysteresis damping ratio $\xi_h$ and viscous damping ratio $\xi_o$ can be expressed by

$$\xi_h = \frac{w_d}{4\pi w_s}, \xi_o = \frac{c}{2m\omega} \tag{6}$$

respectively, where $w_s = \frac{1}{2} k_{eq} A^2$ is the elastic energy for the spring constant and A is the amplitude of $u$; i.e. $u = A\sin(\omega t)$.

From Eq.(4), the effective period for the system $T_{eq}$ can be written as

$$T_{eq} = \frac{2\pi}{\omega_{eq}} = 2\pi\sqrt{\frac{m}{(k_{eq} + k_0)}} \tag{7}$$

Taking the Laplace transform of (1) for the equivalent linear system, the transfer function (frequency response function) can be expressed as

$$\frac{U(j\omega)}{F(j\omega)} = \frac{1}{(k_{eq} + k_0) - m\omega^2 + j(c_{eq} + c)\omega} \tag{8}$$

where $U(j\omega) = X(s)\big|_{s=j\omega}$ , $F(j\omega) = F(s)\big|_{s=j\omega}$ and $U(s), F(s)$ are the Laplace transform for $u(t), f(t)$ respectively.

For a perfect elasto-plastic dissipation device, the force-displacement relation can be shown in Fig.3. According to concept of secant modulus, AASHTO [4] present the equivalent stiffness for the base isolator

$$k_{eq} = \frac{kd}{A} = \frac{k}{\mu} \tag{9}$$

where $k$ is the elastic stiffness, $\mu = A/d$ is the ductility ratio and $d$ is the elastic limit.

Then, the equivalent damping ratio becomes

$$\xi_{eq} = \frac{2}{\pi}(1 - \frac{1}{\mu}) + \xi_0 \tag{10}$$

where $\xi_o$ is the viscous damping ratio defined in Eq.(6). In general $\xi_o$ is chosen as 5%. With Eq.(9) and by considering $k_0 = 0$ for the base isolator, the effective period ratio can be represented as

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$$\frac{T_{eq}}{T} = \sqrt{\mu} \tag{11}$$

In Eq.(10) of AASHTO formula, it can be observed that the damping ratio appeared in Eq.(10) becomes a constant as the ductility ratio as $\mu \rightarrow \infty$ .

and even for the bilinear system the damping ratio will decrease as $\mu \rightarrow \infty$. This is contradictory to the practical situations. Therefore, some empirical formulas have been proposed by some researchers [2-3] such as:

Iwan's Model [2-3]:

$$\frac{T_{eq}}{T} = 1 + 0.121(\mu - 1)^{0.939} \tag{12}$$

$$\xi_{eq} = 0.0587(\mu - 1)^{0.371} + \xi_o \tag{13}$$

Modified Iwan's Model [2][3]

$$\frac{T_{eq}}{T} = 1 + ln[1 + 0.13(\mu - 1)^{1.137}] \tag{14}$$

$$\xi_{eq} = 13.52(\frac{T_{eq}}{T} - 1)^{0.3952} + \xi_o \tag{15}$$

Refined AASHTO formula [3]

$$\frac{T_{eq}}{T} = [\frac{\mu}{1 + 0.15(\mu - 1)}]^{1/2}\left(1 - 0.737\frac{\mu - 1}{\mu^2}\right) \tag{16}$$

$$\xi_{eq} = \frac{1.7(1 - 1/\mu)}{\pi[1 + 0.15(\mu - 1)]}\frac{\mu^{0.58}}{4.5} + \xi_o \tag{17}$$

It can be observed that the damping ratio in Eq.(13) and Eq.(15), the damping ratio will increase as $\mu \rightarrow \infty$.

# 3 The Describing Function Method

In this section, we will focus on the derivation of the equivalent damping of a hysteresis devices by the describing function method. The derivation is dealt by the equivalence of the frequency response function (transfer function) in the frequency domain.

## 3.1 The Preview of Describing Function

The describing function method has been widely used to linearize the dynamical behavior for many nonlinear control systems [5-8]. Considering the steady state response, let's assume the displacement $u$ being a sinusoidal can be written as

$$u(t) = A sin(\omega t) \tag{16}$$

where $A$ is the amplitude and $\omega$ is the frequency.

The force output for the nonlinear dissipation devices c(t), which can be written as

$$c(t) = \phi(A sin\omega t)$$

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By using the Fourier series expansion, this periodic function c(t) can be expanded as

$$c(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

where the Fourier coefficients $a_n$ 's and $b_n$ 's are generally function of A and $\omega$, determined by

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} c(t)d(\omega t),$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} c(t)\cos(n\omega t)d(\omega t),$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} c(t)\sin(n\omega t)d(\omega t),$$

If the nonlinear function is odd, one has $a_0 = 0$. Furthermore, if the transfer function in the Lur'e problem has the low-pass proprieties, referred to as the *filtering hypothesis*. i.e.,

$$|G(j\omega)| >> |G(jn\omega)| \quad \text{for } n = 2,3,4.... \tag{19}$$

The Bode plot of the low-pass filter property can be shown in Fig.3.

Then, the output response of the system can be approximated by the fundamental component as

$$c(t) \approx a_1 \cos(\omega t) + b_1 \sin(\omega t) = M \sin(\omega t + \phi) \tag{20}$$

where

$$M(A,\omega) = \sqrt{(a_1^2 + b_1^2)} \quad \text{and} \quad \phi(A,\omega) = \tan^{-1}(a_1/b_1) \tag{21}$$

since the higher order harmonic terms are neglected.

The describing function of the nonlinear device is defined as the complex ratio of the fundamental component of the nonlinear device by the input sinusoid, i.e.

$$N(A) = \frac{Me^{j(\omega t+\phi_1)}}{Ae^{j\omega t}} = \frac{M}{A}e^{j\phi} = \frac{1}{A}(b_1 + a_1) = \frac{j}{A\pi}\int_{-\pi}^{\pi}\phi(A\sin\theta)(\sin\theta + j\cos\theta)d(\theta) \tag{22}$$

where $\theta = \omega t$ .

#### 3.1.1 Describing function of a hardening Spring

The characteristics of a hardening spring are given by

$$\phi(u) = u + u^3/2$$

with *u* being the displacement or the deformation and $\phi(u)$ being the force output. Given a displacement as $u(t) = A\sin(\omega t)$. The force output $\phi(t) = A\sin(\omega t) + A^3\sin^3(\omega t)/2$ can be expanded as a Fourier series, with the fundamental being $\phi_1 = (A + \frac{3}{8}A^3)\sin(\omega t)$ and the describing function of this nonlinear component is

$$N(A,\omega) = N(A) = 1 + \frac{3}{8}A^2 \tag{23}$$

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where $A$ is the amplitude for the sinusoidal displacement.

Remark 1: The describing function method is only valid for the case that in the feedback loop of the Lur'e problem, as shown in Fig.4, where the linear transfer function possesses low-pass filter property. As shown in Fig.4, if the linear transfer function *G(s)* possesses low-pass filter property (as seen in Fig.2.) then the high-order harmonic terms in the Fourier series can be neglected.

Note that the describing function in Eq.(23) becomes the nonlinear spring coefficients for this hardening spring since the describing function is the ratio between the force (output) and displacement (input). Moreover, even the magnitude of the sinusoidal displacement amplitude $A$ is very large, the describing function is still valid.

## 3.2 The Elasto-Plastic Base Isolator

For the perfect elastic plastic material, the force-displacement characteristic can be shown in Fig.3. As the displacement exceeds the yielding displacement $d$, the force displacement relation will reveal the hysteresis behavior.

Similarly, we can derive the equivalent damping for the elastic plastic damper by the describing function method. The describing function for a perfect elasto-plastic device (isolator or damper) is [7-8]

$$N(A)=k\{\frac{1}{2}-\frac{2}{\pi}[\sin^{-1}(1-{}^{2d}\!/_{A})+(1-{}^{2d}\!/_{A})\sqrt{(1-{}^{2d}\!/_{A})^{4d}\!/_{A}}]\}+j\frac{4kd(d-A)}{\pi A^2} \tag{24}$$

where $k$ is the elastic modulus for the hystersis damper and $d$ is the elastic limit for the nonlinear dissipation devices.

Eq.(24) can be rewritten as

$$N(A)=\{\tfrac{1}{2}-\tfrac{2}{\pi}[\sin^{-1}(1-{}^{2}\!/_{\mu})+(1-{}^{2}\!/_{\mu})\sqrt{\tfrac{4}{\mu}(2-\tfrac{1}{\mu})}]\}k+j\frac{w_d}{\pi A^2} \tag{25}$$

where $\mu={}^{A}\!/_{d}$ is the ductility ratio and $w_d=4kd(d-A)$ is the dissipation energy, i.e. the area of the hystersis loops, for the energy dissipation device.

Substituting Eq.(25) into Eq.(8) and take the Laplace transform of Eq.(8), the transfer function can be obtained as

$$\frac{U(i\omega)}{F(j\omega)}=\frac{1}{((k_{eq}+k_0-m\omega^2)+j(\frac{w_d}{\pi A^2}+c\omega)} \tag{26}$$

where $k_{eq}$ is the equivalent stiffness constant for the damper, which is defined by

$$k_{eq}=\{\tfrac{1}{2}-\tfrac{2}{\pi}[\sin^{-1}(1-{}^{2}\!/_{\mu})+(1-{}^{2}\!/_{\mu})\sqrt{\tfrac{4}{\mu}(2-\tfrac{1}{\mu})}]\}k \tag{27}$$

where the natural frequency or a resonance frequency is found to be as Eq.(7)

As the exciting frequency is identical to the natural frequency $\omega=\sqrt{\frac{(k_{eq}+k_0)}{m}}$ and by considering $k_0=0$, then, the hystersis-damping ratio can be expressed by

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$$\xi_h = \frac{w_d}{4\pi w_e} = \frac{Total\ area\ of\ \ Hystersis\ loop}{2\pi k_{eq}} \quad (28)$$

where $w_e = \frac{1}{2} k_{eq} A^2$ is the equivalent elastic energy for the spring and damper.

REMARK 2: It can be observed that the imaginary part of the describing function N(A) is $w_d / \pi A^2$ (see Eq.(25)), where $w_d$ is the dissipation energy for the devices (i.e. the area of the hystersis loop). Moreover, the imaginary part of the describing function, based on the transfer function equivalence, represents the damping coefficient, which equals the conventional hystersis-damping coefficient derived by the energy method.

Remark 3: For the perfect elasto-plastic base isolator, *d* is the elastic limit for the damper. Specifically, as $A \approx \mathbf{d}$, which means the plastic displacement is very small. Substituting the above results into Eq.(27), we have

$$k_{eq} = k \quad (29)$$

where k is the elastic modulus for the hystersis damper

Then,. The natural frequency is found to be

$$\omega = \sqrt{\frac{(k + k_0)}{m}} \quad (30)$$

Remark 4: As the plastic displacement is very large when comparing to the elastic displacement; i.e. $A >> \mathbf{d}$ then, the equivalent stiffness for the damper $k_{eq}$ in Eq.(27) becomes

$$k_{eq} \approx 0 \quad (31)$$

According to Eq.(27) and Eq.(25), It can be found that $0 \le k_{eq} \le k$, which means that for large displacement, the spring constant will vanish gradually such that the damping ratio $\xi$ will be also change gradually. Then, the natural frequency becomes

$$\omega \approx \sqrt{\frac{k_0}{m}} \quad (32)$$

From Eq.(28), we can observe that the natural frequency will drift as the amplitude of displacement becomes larger.

## 4 Effective Damping Ratio And Period Ratio

From the perturbation analysis, several literatures [1, 13] already have proved that the dominant frequency and damping ratio of the overall structure is very close to the frequency and damping ratio for the base isolator. Therefore, the effective damping ratio and period are very important for the design of earthquake resistance of the base isolation. In this section, we will explore the effective damping ratio and effective period with the describing function method. For the base isolation, let's consider $k_0 = 0$.

With the describing function method, from Eq.(26) and Eq.(27), we have the effective stiffness as

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$$k_{eq} = \{ \tfrac{1}{2} - \tfrac{2}{\pi} [\sin^{-1}(1 - 2/\mu) + (1 - 2/\mu)\sqrt{\tfrac{4}{\mu}(2 - \tfrac{1}{\mu})}\,]\}k \tag{33}$$

Then, effective period ratio as

$$\frac{T_{eq}}{T} = \frac{1}{\tfrac{1}{2} - \tfrac{2}{\pi} [\sin^{-1}(1 - 2/\mu) + (1 - 2/\mu)\sqrt{\tfrac{4}{\mu}(2 - \tfrac{1}{\mu})}\,]} \tag{34}$$

Consequently, the effective damping ratio becomes

$$\xi_{eq} = \frac{4\frac{1}{\mu}(1-\frac{1}{\mu})}{\{\pi - 4[\sin^{-1}(1 - \frac{2}{\mu}) + (1 - \frac{2}{\mu})\sqrt{\frac{4}{\mu}(2 - \frac{1}{\mu})}\,]\}} + \xi_0 \tag{35}$$

Comparing effective stiffness of Eq.(26) and Eq.(30), the effective stiffness versus ductility ratio can be shown in Fig.6. It can be observed that for $\mu < 3$, the effective stiffness of the describing function is quite close to the AASHTO formula. The effective period ratio and damping ratio are shown in Fig.7 and Fig.8 respectively.

As shown in Fig.8, the damping ratio for the AASHTO formula will become a constant as $\mu$ becomes large. This is contradictory to the practical situations [2-3].

The damping ratio of Eq.(33) derived by the describing function method is proportional to $\mu$, which can be shown in Fig.8.

## 5 Low-pass Filter Property

In the previous section, we have shown that through the transfer function equivalence, the describing function method can be applied to obtain the equivalent damping ratio and effective period ratio for an elasto-plastic isolator or damper. However, the describing function is valid only for the situation that in the feedback loop, the linear transfer function should satisfy the so-called low-pass filter property; therefore, the higher-order harmonic components can be neglected. If not, the describing function is not valid and accurate.

In this section, we will illustrate the transfer function of Eq.(1) can be transformed into a Lur'e problem and the transfer function possesses low-pass filter property.

In Eq.(1), if we take Laplace Transform of the left and right side, we have

$$\frac{U(s)}{F(s)} = \frac{1}{(cs + ms^2) + \Phi(s)} \tag{34}$$

where $\Phi(s) \equiv L\{\phi(u)\}$, L is denoted as the Laplace Transform.

Rewriting Eq.(34), we have

$$\frac{U(s)}{F(s)} = \frac{\frac{1}{(ms^2 + cs)}}{1 + \frac{1}{(ms^2 + cs)}\Phi(s)} \tag{35}$$

$$= \frac{G(s)}{1 + G(s)\Phi(s)}$$

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In Eq.(35), it can be found that the overall system is transformed to a feedback Lur'e problem as shown in Fig.4; moreover, the linear transfer function $G(s) = 1/(ms^2 + cs)$ possesses low-pass filter property, the Bode plot being shown in Fig.5, and the nonlinear block is $\Phi(s)$, which means the higher-order harmonic terms can be neglected and the describing function method is valid for the base isolation system described by the Eq.(1).

## 6 Simulation Example

In this section, we will illustrate that the effectiveness of our proposed method. We will compare the results with the formula proposed by AASHTO and some formulas of Eqs.(12-17). It can be shown that through the describing function method, the equivalent stiffness and damping ratio can be precisely estimated.

Example 1: Consider a spring mass system of Eq.(1) are shown in Fig.1, where the mass M=1 ton. The displacement vs. force behaviour for the damper is shown in Fig.2, where the slope is the elastic modulus for the damper $k = 2t/cm$. The external force is considered as the following (i) $F(t) = 4\sin(\omega t)\ \ ton$, (ii) $F(t) = 2\sin(\omega t)\ \ ton$ where $\omega$ is the excitation frequency.

The simulation model for the hysteretic damper or isolator is employed by the model presented by Wen et al (1989). The stiffness restoring force $F_s(t)$ is expressed as

$$F_s(t) = \alpha K u + (1-\alpha)kdv \tag{36}$$

where $u$ =displacement of the spring mass system, $k$ =the elastic stiffness; $\alpha$ =ratio of post-yielding to pre-yielding stiffness; *d*= yield deformation; $v$ =a non-dimensional variable introduced to describe the hysteretic component of the deformation, with $|v| \le 1$ , where

$$D\dot{v} = \delta \dot{u} - \beta[\dot{u}]|v|^{n-1} v - \gamma \dot{u}|v|^n \tag{37}$$

In Eq.(37), parameters $A, \beta, \delta$ govern the scale and general shape of the hystersis loop.

The simulation results of the frequency response function can be shown in Fig.10 and Fig.11 respectively. In the simulation of Fig.10 and Fig.11, the parameters are selected as $\xi_0 = 0, k_0 = 0, d = 0.5, k = 2\ ton/cm, \alpha = 0, \delta = 1, \beta = \gamma = 0.5, n = 10$. The viscous damping ratio $\xi_0 = 0$ means the viscous damping coefficient $c = 0$ in Eq.(1).

As shown in Fig.10 and Fig.11, the Bode plot of the actual nonlinear damper is shown as green line and the bode plot predicted by the describing function and AASHTO, Iwan's Model, Modified Iwan's Model and Refined AASHTO are shown as blue, red, pink dot, yellow and jade green lines respectively. It can be observed that with the bode plots obtained by the describing function method is very close to the actual nonlinear system.

Secondly, let's consider time response at the frequency $\omega = 0.33$ rad/sec and the external force are (i) $2\sin(\omega t), t$ (ii) $4\sin(\omega t)\ t$ respectively. The simulation results are shwon in Fig.12 and Fig.13 respectively. As shown in Fig.12,13, the time response for the actual nonlinear systen, AASHTO, the describing function method, Iwan's model, Modified Iwan's model, refined AASHTO formula are shown as green, red, blue, pink, yellow and jade green lines respectively. The percentage error of the amplitude for the time response can be shown in the following Table.

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**Table 1: The percentage errors of the amplitude for the time response**

| External Force \ Percentage Error (Formula) | AASHTO | Describing function | Iwan's Model | Modified Iwan's model | Refined AASHTO |
|---|---|---|---|---|---|
| (i) $2\,sin(\omega t)$, ton | 37.7% | 3.9% | 137.87% | 98.4% | 56.4% |
| (ii) $4\,sin(\omega t)$ ton | 24.44% | 0.88% | 41.95% | 98% | 53.2% |

It can be observed that the simulation results of the describing function are very clsoe to the actual nonlinear system. The percentage errors of the AASHTO are ten to thirty times to that for the describing function method. Also, from Fig.12 and Fig.13, the phase errors of the describing function method is smaller than other methods.

## 7 Conclusion

In this paper, we have addressed the problem of an elasto-plastic behavior in a hystersis structure. It is known that for the base isolation, the fundamental frequency of the whole structure is dominated by the natural frequency of the base isolator. Consequently, it is very important to evaluate the effective (equivalent) stiffness and damping ratio for the isolator. However, for the energy dissipation devices, the force-displacement relation, in general, is nonlinear and possesses hysteresis loop. It is difficult to evaluate the equivalent damping and stiffness. With the describing function method, we can linearize the nonlinear behavior of the hystersis structure and then the equivalent damping and stiffness coefficients can be obtained. From these damping and stiffness coefficients, the effective period ratio and equivalent damping ratio are also obtained. It can be found that the imaginary part of the describing function represents the dissipation-energy in the hystersis loop so the damping coefficient obtained from the describing functions is identical to the conventional energy method. When considering the harmonic external force, simulation results have shown the advantages of this proposed method. In the simulation example, the results of our approach is more accurate than that for AASHTO specifications and etc; moreover, it can be observed that the describing function method has the property that as the ductility ratio $\mu$ increase the damping ratio will increase also, which can be shown in Fig.9. This coincides the practical observations indicated by the literatures [2-3].

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