

# Engineering Properties and Prediction of Strength of High Performance Fibre Reinforced Concrete using Artificial Neural Networks

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**ABSTRACT:** This paper presents the experimental and numerical studies on high performance fiber concrete (HPFRC) with water-cementitious materials (w/cm) ratios of 0.4- 0.3, steel fiber volume fraction (Vf) varying from 0- 1.5%, polypropylene fiber volume fraction varying from 0- 1% and silica fume replacement at 10% and 15%. Experimental results showed high improvements in 28 day cylinder compressive strength and flexural strength of steel fiber reinforced concrete at fiber volume fraction of 1.5%; for polypropylene (PP) FRC improvement in compressive and flexural strengths are marginal and moderate, respectively. Statistical models developed for compressive strength ratios and flexural strength ratios of HPSFRC indicate the prediction capabilities of the models. Due to the complex mix proportions of HPSFRC and the non-linear relationship between the concrete mix proportions and properties, research on HPSFRC has been empirical and no models with reliable predictive capabilities for its behavior have been developed. Based on the large data collected for HPSFRC mixes, a trained artificial neural network (ANN) model which adopts a back propagation algorithm to predict 28-day compressive strength of HPSFRC mixes was employed. This paper describes the comparison of the experimental results obtained for various mixes. Multiple linear regression (MLR) model with  $R^2 = 0.78$  was also developed for the prediction of compressive strength of HPSFRC mixes. On validation of the data sets by NNs, the error range is within 2% of the actual values. ANN models give the significant degree of accuracy compared to MLR model, and can be easily used to estimate the strength of concrete mixes.

**Key words:** High performance fiber reinforced concrete, silica fume, PP fibers, steel fibers, mechanical properties, empirical equations, neural networks, modeling, prediction.

## 1 INTRODUCTION

High performance concrete (HPC) and ultra-high performance concrete (UHPC) are widely used in construction industry. The intrinsic brittle nature of HSC/ HPC/ UHPC represents a limitation for its use, which can overcome by addition of discrete steel fibers in the concrete matrix [9, 12, 14, 26- 29]. The addition of steel fibers in HPC/ HSC enhances the mechanical properties of concrete at normal and elevated temperatures, and significantly improves the ductility and toughness of concrete [14, 17, 22, 23, 26, 28, 29, 31]. HPC contains supplementary cementitious materials (SCM), which enhances the strength and improves the durability of the matrix and also has financial and environmental benefits [9, 12, 29, 31]. HPFRC/ HPHyFRC/ UHPFRC is becoming a new superior material and has wide range of applications such as pavements, industrial floors, hydraulic and marine structures, infrastructures, retrofitting of RC structures and slope stabilization works.

The 28-day compressive strength of concrete is a common index of concrete strength, which is considered as a prime data in the analysis and design of concrete structures. The strength of concrete is related to the mix proportions and mix preparation techniques. Because of complex mixture proportions, and lack of theoretical relationships between the mix proportions and measured properties of HPC/ HSC/ HPFRC, properties are often described using statistical models (empirical equations) [9, 20, 27, 28, 30, 31, 32].

High performance steel fiber reinforced concrete (HPSFRC) is such a highly complex material that modeling its behavior is a difficult task. Due to non-linear relationship between concrete mix proportions and properties, statistical methods have failed to accurately predict the properties of mixes. Furthermore choosing suitable regression equation involves techniques and is not an easy thing. Concrete strength is influenced by many factors and a mapping model considering many factors to the 28-day compressive strength can be created using neural networks (NNs) [19, 33]. Ni H. Gaung and Wang Ji-Zong [25] have

developed a model to predict 28-day compressive strength of concrete by using multi-layer feed forward neural networks. Yeh [35] developed an ANN model to estimate strength of HPC and found that the model was more accurate than that of regression model. Kim et al. [19] developed a back propagation neural network model to estimate compressive strength of concrete mix proportions of two companies. Ghaboussi et al. [11] modeled the behavior of concrete under a state of plane stress using monotonic biaxial compressive loading with a back propagation neural networks (NNs). In civil engineering, neural networks have been applied to the detection of structural damage and structural system identification [8, 10, 11, 21]. Moncef Nehdi et al. [24] have developed ANN model to study the performance of self-compacting concrete. Cheng Yeh [7] used BP-NN models for 28 day strength and workability and used GA for optimization of HPC.

The objective of this paper is to investigate the performance of HPSFRC with w/cm varying from 0.4- 0.25 at 10% silica fume replacement and steel fiber volume fraction varying from 0- 1.5%, and to provide a methodology by incorporating most of the fundamental aspects of NNs to predict the compressive strength of HPSFRC. The multi-layer feed forward neural network is one of the most commonly used artificial neural network models and applications are based on the back propagation paradigm [15, 16, 33]. Training and testing patterns of NNs were prepared using the data set containing mix proportions obtained from experimental results and different sources. The proposed back propagation neural network model has been validated with series of experiments and compared with the MLR model. It was shown that BP-NN model can efficiently be used as a new predictive tool by the concrete mix designers and technologists to solve the complex non-linear mapping to estimate the strength of the concrete mix proportions.

## 2 MATERIALS AND METHODS

### 2.1 Materials and Mixture proportions

Ordinary Portland cement- 53 grade with a 28-day compressive strength of 54.5 MPa and specific grav-

ity of 3.15, and condensed silica fume as SCM having specific surface area of 23000 m<sup>2</sup>/kg and specific gravity of 2.7, were used. The chemical analysis of silica fume is given in Table 1.

Table 1. Chemical Analysis of Silica fume

Silicon dioxide, SiO <sub>2</sub>	Moisture content	Loss of Ignition @ 975 °C	Carbon	Fineness (by residue on 45µ)
88.7 %	0.7 %	1.8 %	0.9 %	2 %

Fine aggregate of locally available river sand passing through 4.75mm IS sieve, conforming to grading zone-II of IS: 383-1978 was used. Coarse aggregate of crushed blue granite stones with 12.5mm maximum size was used. Sulphonated naphthalene formaldehyde condensate having specific gravity of 1.20 as HRWR admixture conforming to ASTM C494 was used.

Fibers used in this investigation are crimped steel fibers of length = 36 mm and diameter = 0.45 mm, with an aspect ratio of 80 and ultimate tensile strength,  $f_u = 910$  MPa and PP fibers of length= 20 mm with an aspect ratio of 600.

Mixtures were proportioned using guidelines and specifications given in ACI 211.4R-93 [1], and recommended guidelines of ACI 544.3R-1993 [2]. Mixture proportions used in this investigation are listed in Table 2. For each water- cementitious materials ratio (w/cm), one HPC mix and 3 steel fibrous concrete mixes having fiber volume fraction,  $V_f = 0.5, 1.0$  and  $1.5$  % by volume (39, 78 and 117.5 kg/m<sup>3</sup>) and 3 PP fibrous concrete mixes with  $V_f = 0.25, 0.5$  and  $1\%$  were prepared. Super-plasticizer with dosage range of 1.75 to 2.5% by weight of cementitious materials has been used. 16 series of high performance steel fiber reinforced concrete (HPS-FRC) mixes and 8 series of HP-PP fiber reinforced concrete (HPSFRC) mixes were used in this investigation. For each mix at least three 150 mm diameter cylinders and three 100 x 100 x 500 mm prisms were produced.

Table 2. Data of mix proportions of HPFRC for Strength Analysis (Kg/ m<sup>3</sup>)

Mix	w/c m	cm, kg	FA, kg	CA, kg	SF, kg	W, kg	SP (%)	Steel fiber V <sub>f</sub> (%)
M1-10-0	0.4	438	645	1088	43.8	175	1.75	0
M1-10-0.5	0.4	438	641	1079	43.8	175	1.75	0.5
M1-10-1	0.4	438	636	1071	43.8	175	1.75	1.0
M1-10-1.5	0.4	438	632	1062	43.8	175	1.75	1.5
M2-10-0	0.35	486	639	1088	48.6	170	2	0
M2-10-0.5	0.35	486	635	1079	48.6	170	2	0.5
M2-10-1	0.35	486	630	1071	48.6	170	2	1.0
M2-10-1.5	0.35	486	626	1062	48.6	170	2	1.5
M3-10-0	0.3	550	601	1088	55	165	2.5	0
M3-10-0.5	0.3	550	597	1079	55	165	2.5	0.5
M3-10-1	0.3	550	592	1071	55	165	2.5	1.0
M3-10-1.5	0.3	550	588	1062	55	165	2.5	1.5
M4-10-0	0.25	640	542	1088	64	160	3	0
M4-10-0.5	0.25	640	538	1079	64	160	3	0.5
M4-10-1	0.25	640	533	1071	64	160	3	1.0
M4-10-1.5	0.25	640	529	1062	64	160	3	1.5

In mix designation M1 to M4, silica fume replacement is 10 percent, after hyphen denotes fiber volume fraction (%)

SP (%) -Superplasticizer in percent by weight of binder material

V<sub>f</sub> (%) denote Steel fiber volume fraction in percent in total volume of concrete

f<sub>cf</sub> = cylinder compressive strength of HPSFRC, (MPa)

f<sub>fr</sub> = flexural strength of HPSFRC (MPa)

## 2.2 Testing for strength

Compressive strength tests were performed according to ASTM C 39-92 [3] standards using 150 mm diameter cylinders loaded uniaxially. The tests were

done in a servo- controlled compression testing machine by applying load at the rate of 14 MPa/min. Minimum of three specimens were tested to compute the average compressive strength.

Flexural strength (Modulus of rupture) tests were conducted as per the specification of ASTM C 78-92 [4] using 100 x 100 x 500 mm beams under third-point loading on a simply supported span of 400 mm. The tests were conducted in a 100 kN closed loop hydraulically operated UTM at a deformation rate of 0.1 mm/min.

## 3 ANALYSIS OF TEST RESULTS AND DISCUSSIONS

### 3.1 Mechanical properties

. Average compressive strength values for high performance concrete (HPC) and steel fiber reinforced concrete with w/cm ratio = 0.40- 0.25, obtained in the ranges from 52.7 to 74.9 MPa and 54.8 to 80.4 MPa, respectively, are given in Table 3. The moderate improvement in compressive strength of 11 % was observed for the high performance steel fiber reinforced concrete; for PP fiber reinforced concrete, the improvement obtained is 5.5%. The variation of the compressive strength, f<sub>cf</sub>, as obtained for concrete cylindrical specimens on the effect of steel fiber content with aspect ratio = 80, and the strength ratios between high performance steel fiber reinforced concrete (HPSFRC) and HPC, (f<sub>cf</sub>/f<sub>c</sub>) are presented in Table 3. These ratios can be utilized for the development of the generalized expression irrespective of the influence of varying w/cm ratios and specimen parameters, and the expression can be used for the prediction of 28-day compressive strength of any type of specimens. The effect of fiber content as fiber volume fraction on compression strength of HPSFRC in w/cm = 0.35 is shown in Fig. 1. Similar trend was obtained for other fiber reinforced concrete mixes. An empirical expression for predicting the compressive strength, (f<sub>cf</sub>) of HPS

Table 3. Mechanical properties and other results of HPSFRC

Mix	w/cm	Steel fiber $V_f$ (%)	Experimental values		$f_{rf}/f'_{cf}$	$f_{rf}/f'_{cf}{}^{0.5}$	$f'_{cf}/f_c$	$f_{rf}/f_r$
			$f'_{cf}$ (MPa)	$f_{rf}$				
M1-10-0	0.4	0	52.56	6.21	0.118	0.86	1.00	1.000
M1-10-0.5	0.4	0.5	54.77	7.15	0.131	0.97	1.04	1.151
M1-10-1	0.4	1.0	56.01	7.73	0.138	1.03	1.07	1.245
M1-10-1.5	0.4	1.5	57.40	8.19	0.143	1.08	1.09	1.319
M2-10-0	0.35	0	55.85	6.75	0.121	0.90	1.00	1.000
M2-10-.5	0.35	0.5	59.65	8.06	0.135	1.04	1.07	1.194
M2-10-1	0.35	1.0	61.05	8.54	0.140	1.09	1.09	1.265
M2-10-1.5	0.35	1.5	61.44	9.15	0.149	1.17	1.10	1.356
M3-10-0	0.3	0	63.86	7.40	0.116	0.93	1.00	1.000
M3-10-0.5	0.3	0.5	67.12	8.76	0.131	1.07	1.05	1.184
M3-10-1	0.3	1.0	68.91	9.32	0.135	1.12	1.08	1.259
M3-10-1.5	0.3	1.5	69.67	10.13	0.145	1.21	1.09	1.369
M4-10-0	0.25	0	74.87	8.02	0.107	0.93	1.00	1.000
M4-10-0.5	0.25	0.5	77.42	9.58	0.124	1.09	1.03	1.195
M4-10-1	0.25	1.0	79.96	10.36	0.130	1.16	1.07	1.292
M4-10-1.5	0.25	1.5	80.41	11.01	0.137	1.23	1.07	1.373

$f'_{cf}$  = 150 Ø x 300 mm cylinder compressive strength of HPSFRC, (MPa)

$f_{rf}$  = flexural strength of HPSFRC (MPa)

$f_{rf} / f'_{cf}$  = ratio of flexural strength to cylinder compressive strength of HPSFRCs (MPa)

$f'_{cf} / f_c$  = ratio of compressive strength of HPSFRC to compressive strength of HPC (MPa)

$f_{rf} / f_r$  = ratio of flexural strength of HPSFRC to flexural strength of HPC (MPa)

FRC as a function of fiber volume fraction,  $V_f$  (%) for w/cm ratio = 0.35 using regression analysis has been obtained with  $R^2 = 0.86$ , is shown in Fig. 1(a). Similar trend lines have been observed for other fiber reinforced concretes.

Flexural strength or modulus of rupture,  $f_{rf}$  obtained for HPSFRC (with w/cm ratio = 0.40- 0.25) in the range of 6.21 to 11.01 MPa, and the strength ratios between HPSFRC and HPC, ( $f_{rf} / f_r$ ) and improvement in strength for varying  $V_f$  (%) are given in Table 3. The maximum increase in flexural tensile strength due to the addition of steel fibers ( $V_f = 1.5\%$ ) in HPC was found to be about 37.5%, which indicates significant improvement in strength. An empirical expression for the flexural strength ( $f_{rf}$ ) of HPSFRC as a function of  $V_f$  (%) for w/cm ratio = 0.35 using regression analysis has been obtained with  $R^2 = 0.92$ , is shown in Fig. 1(b). The maximum increase in flexural tensile strength due to the addition of PP fibers ( $V_f = 1\%$ ) in HPC was found to be about 26.5%, which indicates moderate improvement in strength.

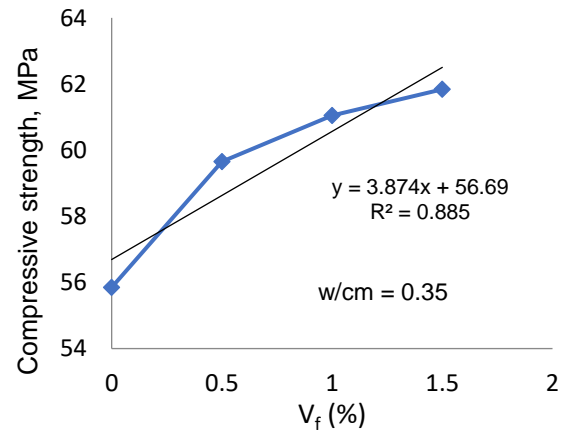


Figure 1 (a). Effect of fiber volume fraction on compressive strength of HPSFRC

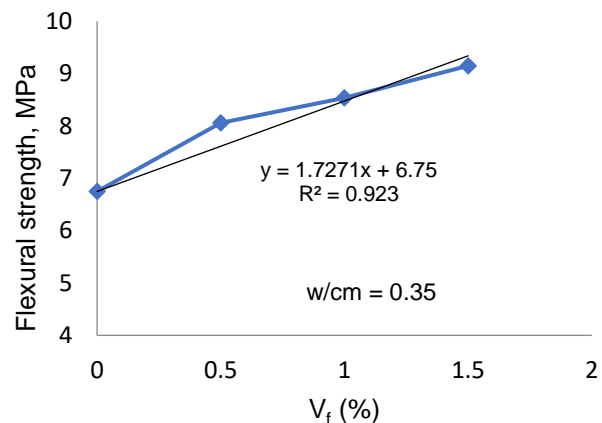


Figure 1 (b). Effect of fiber volume fraction on flexural strength of HPSFRC

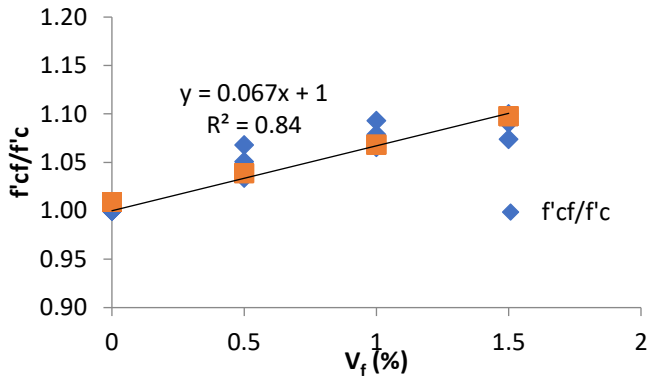


Figure 2. Compressive strength ratios of HPSFRC Vs Fiber volume fraction, V<sub>f</sub> (%)

### 3.2 Relationship between compressive strength ratio and fiber volume fraction (%)

Fig. 2 shows compressive strength ratios, ( $f'_{cf}/f'_c$ ) as a function of the steel fiber volume fraction,  $V_f$  (%). The strength ratio (dimensionless) of axial compressive strengths of HPSFRC has a linear relationship with the fiber volume fraction,  $V_f$  (%). Based on the experimental data, an empirical equation for predicting the compressive strength ratios ( $f'_{cf}/f'_c$ ) of HPSFRC as a function of fiber volume fraction,  $V_f$  (%) for w/cm ratios ranging from 0.25 to 0.40, using regression analysis by least-square method has been obtained with  $R^2 = 0.84$  (refer Fig. 2) as:

$$f'_{cf}/f'_c = 1 + 0.067V_f \quad \dots (1)$$

The coefficient of determination,  $R^2 = 0.84$ , which indicates that 84 % of the variation in strength is explained by the reinforcement parameter, taking in to account the sample size and number of independent variable.

Where,  $f'_c$  = compressive strength of HPC, MPa  
 $f'_{cf}$  = compressive strength of HPSFRC, MPa  
 and

$V_f$  = fiber volume fraction, %.

The values of correlation coefficient (R) and the integral absolute error (IAE) have been obtained as 0.92 and 0.97, respectively. Equation (1), if expanded for  $f'_{cf}$  (the compressive strength of HPSFRC), the second term with coefficient ( $= 0.067 * f'_c * V_f$ ) represents the contribution of matrix strength-fiber

interaction explicitly, which depends on the fiber bond and pullout characteristics of fibers in matrix.

### 3.3 Relationship between flexural strength ratio and fiber volume fraction (%)

The strength ratio (dimensionless) of flexural strengths of HPSFRC, ( $f_{rf}/f_r$ ) has a linear relationship with the fiber volume fraction,  $V_f$  (%). Based on the experimental data, an empirical equation for predicting the flexural strength ratios ( $f_{rf}/f_r$ ), using regression analysis by least-square method has been obtained with  $R^2 = 0.93$  (refer Fig. 3) as:

$$f_{rf}/f_r = 1 + 0.253V_f \quad \dots (2)$$

Where,  $f_r$  = flexural strength of HPC, MPa

$f_{rf}$  = flexural strength of HPSFRC, MPa

$V_f$  = fiber volume fraction, %.

The values of correlation coefficient (R) and the integral absolute error (IAE) have been obtained as 0.964 and 2.06, respectively.

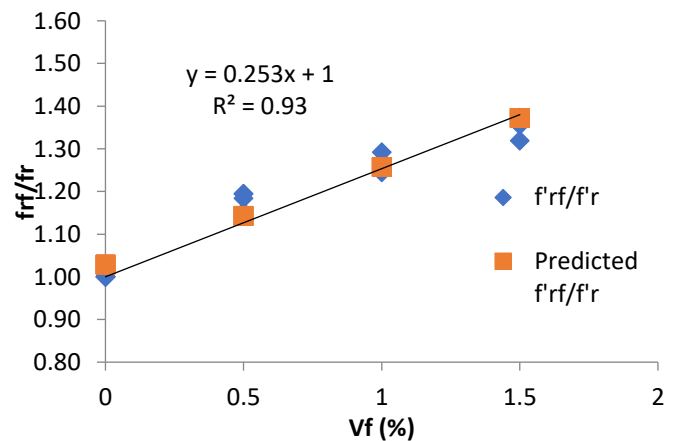


Figure 3. Flexural strength ratios of HPSFRC Vs Fiber volume fraction, V<sub>f</sub> (%)

### 3.4 Relationship between flexural strength and compressive strength

The flexural tension and compressive strength ratio is one of the main indicators to reflect the brittleness of concrete. For concrete, the greater the tension and compression ratio is, the smaller the brittleness, and the greater the toughness and ductility. In this investigation, the flexural tensile and compression ratio of HPSFRC varies from 0.118 to 0.149.

Based on the experimental data, a nonlinear equation for predicting the flexural strength using regression analysis by least-square method has been obtained

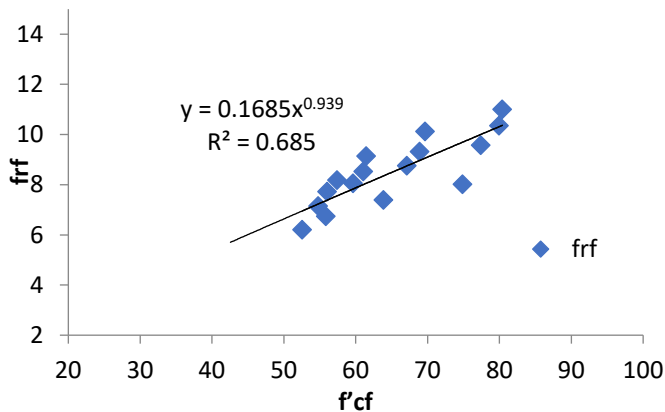


Figure 4. Flexural strength Vs Compressive strength of HPS-FRC

with  $R^2 = 0.685$  (refer Fig. 4) as:

$$f_{rf} = 0.165 f'_{cf}{}^{0.939} \quad \dots(3)$$

Where,  $f_{rf}$  = flexural strength of HPSFRC, MPa

$f'_{cf}$  = compressive strength of HPSFRC, MPa

The values of coefficient of correlation (R) and the integral absolute error (IAE) have been obtained as 0.83 and 6.91, respectively.

#### 4 ARCHITECTURE OF NEURAL NETWORK

The artificial neuron (AN) is an approximately simulated model of a biological neuron. These ANs are used to develop an artificial neural net (ANN) with many inter-connections among different neurons. Each neuron receives weighted inputs from other neurons and communicates its outputs to other neuron by activation function. ANN is a family of massively parallel architecture that is capable of carrying out parallel computations to solve different problems involving complex systems.

A neural network consists of a number of non-linear computational processing elements (PEs), arranged in several layers including an input layer, one or more hidden layers and output layer(s). A PE accepts the input signals and produces one/two output(s), which is a nonlinear function of the weighted

sum of inputs. Most neural network applications are based on the back propagation paradigm which is a gradient descent learning algorithm performed by a delta rule to minimize the error function [7, 15, 33]. In this supervised learning, it back propagates the error signals from the output layer to all the hidden layers, so that their weights can be adjusted accordingly. Back propagation is a generalization of the least square procedure for multilayered feed forward networks with hidden layers. Network is provided with sets of training data, in which network learns by adjusting the connection weights so as to be able to predict the output target for a given set of input samples. Upon successful completion of the training process, a well-trained neural network obtained, should be able to predict the untrained set of input data with an acceptable degree of accuracy.

#### 5 ARTIFICIAL NEURAL NETWORK (ANN) MODEL FOR STRENGTH OF HPSFRC

HPSFRC is a new and highly complex material and thus an attempt to model its behaviors is a great challenging task. The properties of concrete are influenced by a lot of factors. Moreover, a mix is almost never described with all of the important details indicated and thus a strength prediction from the available data is a highly uncertain task [18]. An attempt was made to predict the 28-day compressive strength of HPSFRC mixtures developed by the Authors and earlier researchers. In this NN, feed forward-back propagation algorithm has been used to train and validate the NN model. In this approach the compressive strength of HPSFRC is a function of the following eight input features. For the purpose of analysis, the input elements were transformed into the normalized form and used in the neural network.

1. w/cm ratio
2. Cement (kg)
3. Silica fume (kg)
4. Fine aggregate (kg)
5. Coarse aggregate (kg)
6. Super plasticizer (kg)

7. Fiber volume fraction (kg)

The basic methodology for developing a successful ANN model is to train a neural network for relationship between the influencing factors of concrete mixtures and its mechanical properties. The most commonly used NN model is the multilayered per-

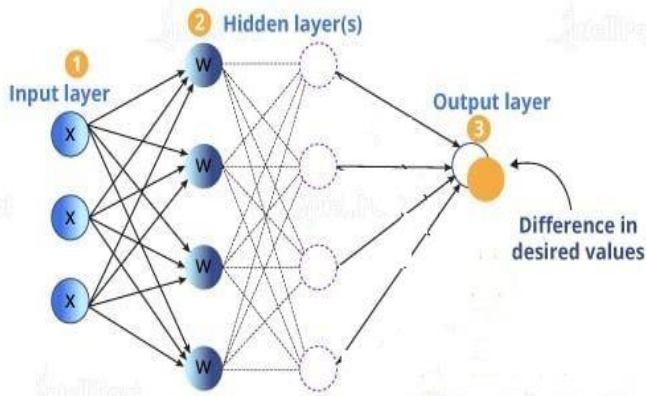


Figure 5. Architecture of Neural Network Model (NN Pattern: 8-8-5-1)

ceptron (MLP) as it has a supervised training process [13, 18, 24]. The goal of MLP is to capture and represent complex input/ output relationships using the data sets. In this study, the 28-day compressive strength of HPSFRC was modeled using multilayer feed forward-back propagation (popular algorithm) neural network, which is commonly used in material modeling. A BP-NN consists of an input layer, hidden layers and an output layer is shown in Fig. 5. The input layer receives the external input neurons, which contains possible influencing factors (variables) and transforms signals to the hidden layers. The hidden layers contain a large number of processing elements (PE). By using activation function, they transform signals to the output layer. The network outputs are compared with the known targets and propagate error back to the networks using delta rule (learning mechanism) that performs a gradient descent on the error space, to adjust weights and biases as optional. Training process of neural networks is summarized as follows:

1. Assign the initial connection weights  $W_{ji}$ , and threshold values  $\theta_j$ , if biases considered.

2 calculate the input values of a hidden layer  $net_{pj}$ . The input of each node which is the activation value for the  $j$ th neuron is defined as:

$$net_{pj} = \sum_i W_{ji} X_{pi} + \theta_j \quad \text{---(4)}$$

3. The output of a hidden layer is derived from  $net_{pj}$  as:

$$O_{pj} = f_j(net_{pj}) \quad \text{---(5)}$$

where,  $W_{ji}$  = connection weight that connects  $i$ th node in the input(preceding) layer to the  $j$ th node in hidden (current) layer,  $X_{pi}$  = input parameter in  $i$ th node,  $O_{pj}$  = output of hidden layer,  $\theta_j$  is the threshold value assigned to neuron  $j$  which is absent in this model.

4. The non-linear sigmoid function is commonly used as an activation function in back-propagation neural networks is expressed by

$$f(net) = 1/(1 + e^{-\lambda net}) \quad \text{---(6)}$$

where  $f(.)$  = activation function, which has to be differentiable, generally taken as a sigmoid function,  $\lambda$  = constant which guides the shape of the sigmoid function.

5. Calculate input value of an output layer  $k$ ,  $net_{pk}$  using output value of hidden layer  $j$   $O_{pj}$ , connection weights  $W_{kj}$  and biases  $\theta_k$  between hidden  $j$  and output layer  $k$ . Then output value of output layer  $O_{pk}$ , is derived from

$$net_{pk} = \sum_j W_{kj} O_{pj} + \theta_k \quad \text{---(7)}$$

$$O_{pk} = f_k(net_{pk}) \quad \text{---(8)}$$

6. Updating of weight vectors

The error function between the calculated output,  $O_{pk}$  and target value,  $T_k$  of an output layer may be expressed as

$$E = \frac{1}{2} \sum_{k=1} (O_{pk} - T_k)^2 \quad \text{---(9)}$$

propagated to the hidden layer neurons and then to the input layer neurons modifying the connection weights and biases by a delta rule to train the network.

From input to hidden neurons

Table 4. Ranges of components of data sets for HPFRC

Components	Minimum	Maximum	Average
Water/cm.	0.21	0.45	0.33
Cement (kg)	372.2	608.0	467.24
Silica fume (kg)	0	115.4	37.22
Fine aggregate	344	870	663.69
Coarse aggregate	881	1243	1060.11
Water (kg)	109.5	210.0	162.58
Superplasticizer (kg)	0.0	29.09	9.90
Fiber (kg)	0.0	120	47.41
Fiber volume fraction	0.0	0.015	0.0054
Strength (MPa)	42.35	100	65.16

$$\Delta W_{ji} = -\mu \partial E / \partial W_{ji} \quad \text{and} \quad \nabla E = \partial E / \partial W_{ji} \quad \text{---(10)}$$

$$W_{ji}(t+1) = W_{ji}(t) + \Delta W_{ji} \quad \text{---(11)}$$

From hidden to output neurons

$$\Delta W_{kj} = -\mu \partial E / \partial W_{kj} \quad \text{and} \quad \nabla E = \partial E / \partial W_{kj};$$

$$\nabla E = -(T_k - O_{pk}) f'_k(\text{net}_{pk}) O_{pj} \quad \text{---(12)}$$

$$W_{kj}(t+1) = W_{kj}(t) + \Delta W_{kj} \quad \text{---(13)}$$

where,  $\mu$  = learning rate parameter which is a positive constant,  $t$  = learning cycle.

Repeat steps 1 to 6 until global error goes below a target error.

## 6 COLLECTION OF DATA

The authors have collected experimental data from 40 different sources by an extensive study, which was used to check the reliability of the strength model. Data sets of concrete mixtures were assembled to have a fairly representative group governing all of the major parameters that influence the strength of HPC/ HPSFRC. In all about 250 mix-

tures from the above investigations were evaluated. During evaluation, some of the concrete samples were deleted from the data due to the large size aggregates, special curing conditions, etc. A database of 219 records each containing the eight independent variables was made. These were 183 pairs of vectors in the training set and 36 pairs of vectors in the validating set. The ranges of components of data sets collected are given in Table 4.

## 7 PROCESSING AND POST PROCESSING OF DATA

Input vector components have the different quantitative limits, so that normalization of data is needed. Different linear translations that can be used to normalize the input vector components to the values ranging from 0 to 1. One of the translations used in this paper is given in equation (14) as:

$$X_i = a X_{io} - b \quad \text{---(11)} \quad \dots(14)$$

$$\text{where, } a = \frac{1}{X_{\max} - X_{\min}}$$

$$b = \frac{X_{\min}}{X_{\max} - X_{\min}}$$

where  $X_{io}$  and  $X_i$  are the  $i^{\text{th}}$  components of the input vector before and after normalization, respectively, and  $X_{\max}$  and  $X_{\min}$  are the maximum and minimum values of all the components of the input vectors before normalization. The components of the output vector required to be translated from values between 0 and 1 by the equation (12).

$$Y_i = Y_{io} (Y_{\max} - Y_{\min}) + Y_{\min} \quad \text{---(15)}$$

Where,  $Y_{io}$  and  $Y_i$  are the  $i^{\text{th}}$  components of the output vector before and after translation, respectively and  $Y_{\max}$  and  $Y_{\min}$  are the maximum and minimum values of all the components of the output vectors, respectively.



## 8 IMPLIMENTATION PROCESS

### 8.1 Training and testing of Back Propagation-Neural Networks

. In order to produce a good quality HPC/ HPSFRC, and to satisfy the requirements of strength, workability, durability and serviceability, mix proportions play vital roles. The compressive strength test is carried out at 28<sup>th</sup> day, and therefore, it is tedious to predict the early strength of mixes at construction sites which will delay the progress of the works. In this study, the NN pattern used is 8-8-5-1 (refer Fig. 5). The neural networks for predicting the 28-day compressive strength of mixes was trained with data

sets of 183 samples for verifying the robustness of the models. During training the NNs, the weights were updated till the error was less than the target error. The neural networks developed (refer Fig. 5) in the investigation has seven nodes in the input layer and 1 node in the output layer; number of hidden layers = 2. To simplify the learning process, input and output elements were normalized between 0 and 1 to be compatible with the limits of activation function, which is generally taken as sigmoid function. The network parameters considered in this approach are: Learning rate = 0.60; Momentum factor = 0.7 (optimizing). Fig. 6 shows the normalized error verses no. of epochs in training the NNs, and Fig. 7

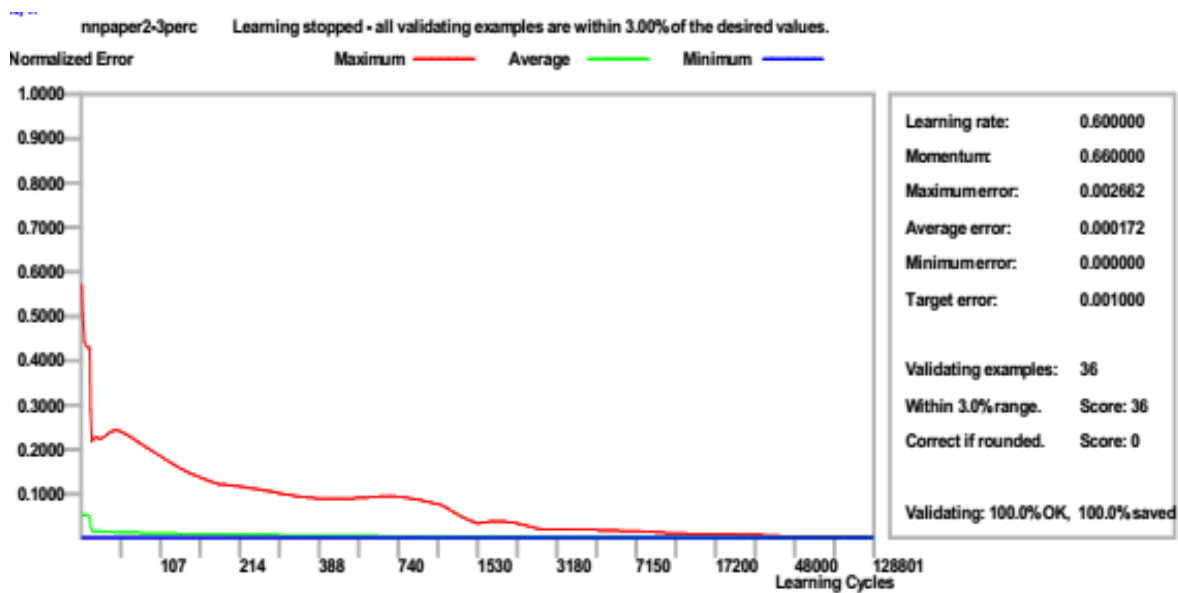


Figure 6. Learning curve - Normalized error Versus No. of Epoch.

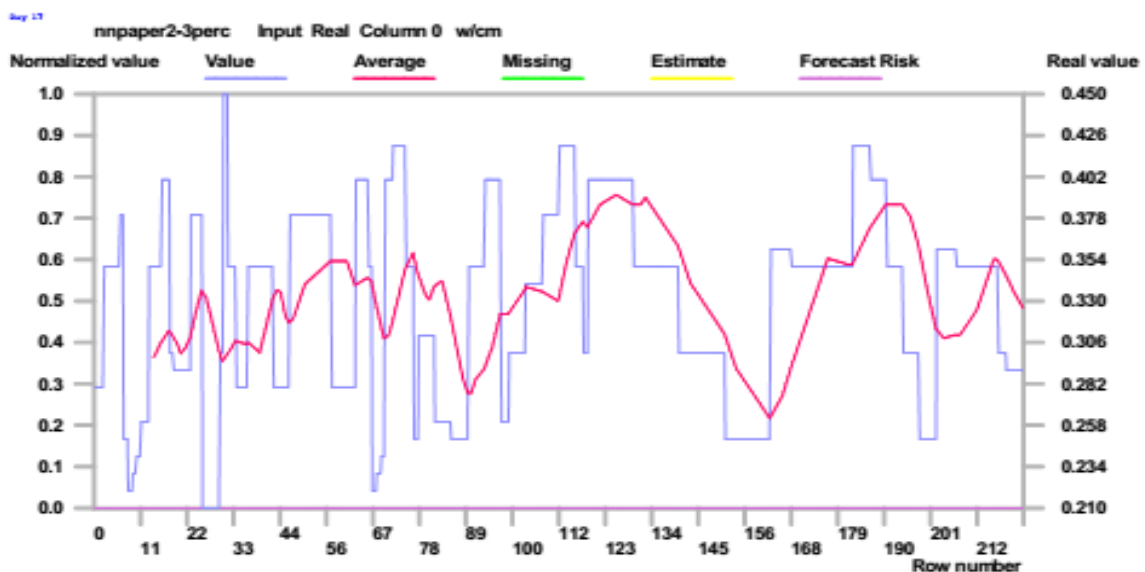
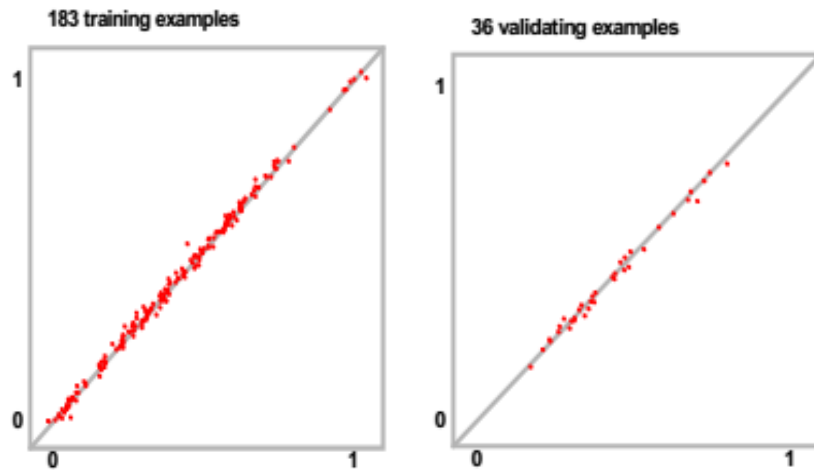


Figure 7. Normalized predicted value versus Row number of data.

shows the normalized predicted values versus row no. of data. Fig. 8 shows the correlation between predicted strengths and normalized actual strengths in training the data sets. It is clearly seen from Fig. 8 that all the data sets are almost on the zero variation

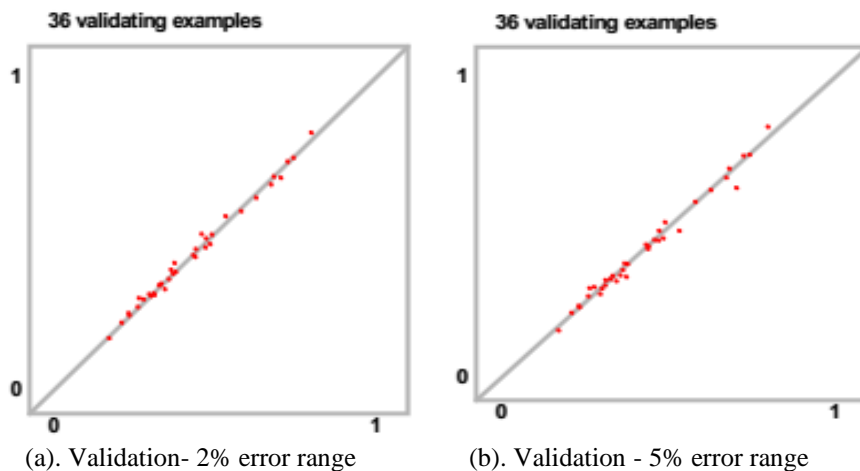
line, which indicates the better prediction capability and reliability of the NN models. Table 5 shows the relation of convergence in training the neural networks and validation of data sets.



X axis: True value after scaling; Y axis: predicted value after scaling  
 (a). Training of data set (b). Validation - 3% error range  
 Figure 8. Correlation between predicted and normalized actual strength

After training the neural networks, the test data sets of authors and earlier researchers were used to evaluate the confidence in the performance of trained networks. The target error in training the NN was fixed as 0.001. The test data sets of 36 mixes used to validate the BP-NN model is given in Table 5. Validating results of the HPSFRC mixes obtained by NN model is summarized in Table 6. The error percentage of the predicted strength compared to the actual

strength is also shown in Table 6. The trained NN model is validated for 36 data sets of authors and earlier researchers for 2, 3, and 5% error. Figs. 8 & 9 show the correlation between predicted strengths and normalized actual strengths for validating examples at 2, 3, and 5% error range. From the analysis of the data, it was noted that 100 % of data is within the testing errors and therefore, the NN model is predicting the strengths with reliability, and the significance of the model is very good.



(a). Validation- 2% error range (b). Validation - 5% error range  
 Figure 9. Correlation between predicted and normalized actual strengths  
 X axis: True value after scaling; Y axis: predicted value after scaling

Table 5. Convergence in training the ANNs

Parameter	No. of epochs	Error range	Training of data Target error: 0.001		Validation of data
			Average Error	Max. Error	Max. error
Compressive Strength	22811	5%	0.000152	0.00244	0.00447
	12881	3%	0.000172	0.00266	0.00165
	27000	2%	0.000121	0.00203	0.00188



Figure 10. Relative sensitivity of 8 input data.

### 9 COMPARISON BETWEEN NN MODELING AND EXPERIMENTAL DATA

To testify the effectiveness of the trained neural networks, the predicted 28-day compressive strength based on the 16 mix proportion parameters were compared with the experimental test results. The results showed that the BP-NN model developed can be used with a very good degree of reliability to predict the compressive strength of concrete mixes. The absolute percent error for the predicted strengths compared to the tested values using this model is

within 2%. Therefore, the proposed back propagation-neural network model proves better, that demonstrates the effectiveness and reliability in predicting the strength of HPSFRC mixes. On examining the validity of the proposed model, there exists a good correlation between the predicted values and the experimental values of different researchers, is shown in Fig. 9. The sensitivity analysis was also carried out to evaluate the sensitivity of the input PE, in which supplementary cementitious materials (silica fume) is having higher sensitivity (relative sensitivity) compared with other elements as shown in Fig. 10.

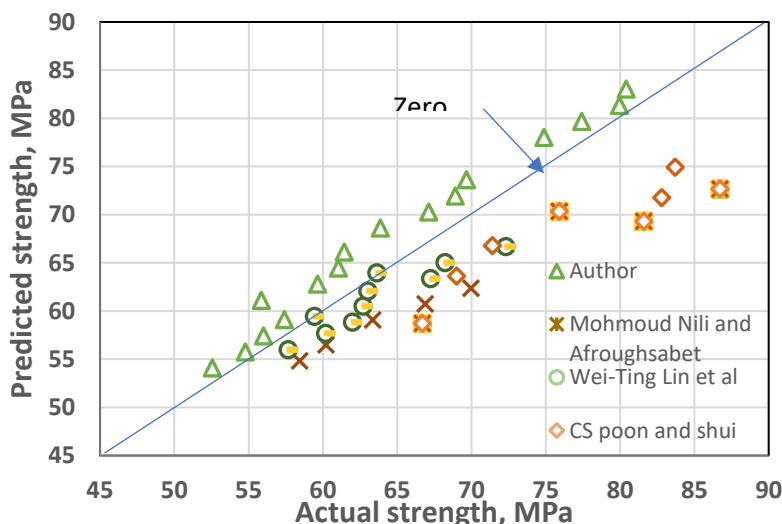


Figure 11. Correlation between actual 28-day compressive strengths and the predicted strengths by the MLR

10 MODELING OF STRENGTH MLR MODEL

Multiple linear regression (MLR) estimates the coefficients of the linear equation, involving more than one independent variable that best predict the value of the dependent variable. MLR model was developed by analyzing the experimental data sets containing 8 parameters by using XLSTAT software. MLR model (Eq. 16) developed for 28-day compressive strength HPSFRC with coefficient of determination ( $R^2$ ) = 0.78, is expressed as:

$$y = 104.1771 - 64.9915 (w/cm) + 0.05054 C - 0.0047 FA - 0.00768 CA + 0.21215 SF - 0.22383W - 0.3302 SP + 0.04047 Fiber$$

$$R=0.883) \quad - (16)$$

where,  $y$  = estimated compressive strength or dependent variable and  $n$  = no. of independent parameters or variables and corresponding 8 regression coefficients.

Fig. 10 shows the correlation of predicted values with the experimental values (compressive strengths) of the authors and earlier researchers [22,

Table 6. Validation of data: Comparison of experimental values with the predicted values by BP-NN models

Author/Researcher	w/cm	C (kg)	SF (kg)	Steel fiber (kg)	Actual Compressive strength, MPa	Predicted values		
						Within 5 % range	Within 3 % range	Within 2% range
Authors	0.4	394.2	43.8	0	52.56	All the Values	All the values	All the values
	0.4	394.2	43.8	39	54.77			
	0.4	394.2	43.8	78	56.01			
	0.4	394.2	43.8	117.5	57.4			
	0.35	437.4	48.6	0	55.85			
	0.35	437.4	48.6	39	59.65			
	0.35	437.4	48.6	78	61.05			
	0.35	437.4	48.6	117.5	61.44			
	0.3	495	55	0	63.86			
	0.3	495	55	39	67.12			
	0.3	495	55	78	68.91			
	0.3	495	55	117.5	69.67			
	0.25	576	64	0	74.87			
	0.25	576	64	39	77.42			
	0.25	576	64	78	79.96			
0.25	576	64	117.5	80.41				
Mahmoud Nili and Afroughsabet	0.36	450	0	39	58.44	All the Values	All the values	All the values
	0.36	450	0	78	60.21			
	0.36	414	36	0	63.34			
	0.36	414	36	39	66.87			
	0.36	414	36	78	69.97			
Wei-Ting Lin et al.	0.35	558	0	39	57.66	All the Values	All the values	All the values
	0.35	558	0	78	60.18			
	0.35	558	0	118	59.43			
	0.35	530.1	27.9	0	61.99			
	0.35	530.1	27.9	39	62.71			
	0.35	530.1	27.9	75	63.03			
	0.35	530.1	27.9	118	63.62			
	0.35	502.2	55.8	0	67.24			
	0.35	502.2	55.8	39	68.21			
0.35	502.2	55.8	78	72.32				
Mansur et al.	0.3	495	55	0	81.6	All the Values	All the values	no
	0.3	495	55	78.5	86.73			yes
Poon and Shui	0.29	500	0	0	69	All the Values	All the values	All the values
	0.29	500	0	78	71.4			
	0.29	450	50	0	82.8			
	0.29	450	50	78	83.7			

23, 26, 34]. It is seen from the graph (Fig. 11) that the higher variability for the data points of the Mansur et al. (1999) [22] and Poon and Shui (2004) [26], indicates the satisfactory performance of the MLR model. Table 7 shows the experimental values and the absolute variations based on the predicted values by the multiple linear regression model (Eq. 16). The applicability of the statistical model was verified with the test data of authors and earlier researchers. It was observed that 100% of the estimated values lie within  $\pm 19.4\%$  of the actual values. The average absolute error (AAE) obtained is 6.18% and the correlation coefficient,  $R = 0.84$ . It was found that the performance of MLR model in predicting the compressive strength of HPSFRC mixes is satisfactory. On comparison with statistical model, the BP-NN models are predicting the strengths with higher accuracy and reliability, and also the trained model is much useful in mix proportion design.

Table 7. Comparison between the experimental results and predicted values by MLR model

Mix designation	Steel fiber content, $V_f$ , (%)	28-day compressive Strength, MPa		Absolute variation (%)
		Experimental	Predicted	
M1-10-0	0	52.56	54.08	2.81
M1-10-0.5	0.5	54.77	55.75	1.75
M1-10-1	1.0	56.01	57.41	2.44
M1-10-1.5	1.5	57.40	59.10	2.87
M2-10-0	0	55.85	61.10	8.60
M2-10-.5	0.5	59.65	62.77	4.97
M2-10-1	1.0	61.05	64.43	5.25
M2-10-1.5	1.5	61.44	66.12	7.08
M3-10-0	0	63.86	68.60	6.91
M3-10-0.5	0.5	67.12	70.26	4.47
M3-10-1	1.0	68.91	71.93	4.19
M3-10-1.5	1.5	69.67	73.61	5.36
M4-10-0	0	74.87	77.99	4.00
M4-10-0.5	0.5	77.42	79.66	2.81
M4-10-1	1.0	79.96	81.32	1.67
M4-10-1.5	1.5	80.41	83.01	3.13

## 11 CONCLUSIONS

Based on the experimental and numerical investigation on HPSFRC with w/cm ratios ranging from 0.40 to 0.25, the following conclusions are drawn.

- Addition of steel fibers in HPC mixes increases the compressive strength moderately and modulus of rupture significantly. The maximum improvement in compressive and flexural strengths for HPSFRC obtained are 10.6% and 38%, respectively at fiber volume fraction,  $V_f = 1.5\%$  compared to HPC and for PPFRC improvement in compressive and flexural strengths are marginal and moderate, respectively.
- Empirical equations developed for the prediction of compressive strength and flexural strength as a function of steel fiber volume fraction, and the IAE values computed are 0.99 and 2.06, respectively.
- Relation between flexural strength and compressive strength of HPSFRC has been developed with correlation coefficient,  $r = 0.83$ .
- BP-NN models can be constructed based on the influencing factors of strength, to predict the 28-day compressive strength of concrete mixes.
- The optimum network configuration was selected from the analyses for various network parameters.
- The strength models based on ANNs attained good prediction accuracy. The accuracy of the model can be improved by increasing the number of training records for various mix design parameters.
- On predicting the 28-day compressive strength of HPSFRC by MLR model, the average absolute error (AAE) obtained for the experimental data is 6.18%. It is observed that the performance of MLR model in predicting the strengths of HPSFRC mixes is satisfactory.
- BP-NN model was validated with the results of different researchers and authors at 2, 3, and 5% error range, in which 100% of data is within the range, indicates the good prediction capabilities and reliability of the models.

• Neural network models are convenient and easy for numerical experiment to review the effects of variables involved in the mix proportions, and its applications to predict the concrete strength is practical.

## NOTATION

The following symbols were used in this paper

$W_{ji}$  = initial connection weights

$net_{pj}$  = input values of a hidden layer

$\lambda$  = constant

$f(\cdot)$  = activation function, which has to be differentiable

$O_{pj}$  = output of a hidden layer

$net_{pk}$  = input values of an output layer

$O_{pk}$  = calculated value of an output layer

$T_k$  = target value of an output layer

$E$  = global error between the calculated output and target value

$\mu$  = learning rate parameter

IAE= Integral absolute error

AAE= Average absolute error

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## 12 REFERENCES

[1] ACI 211.4R-93, *Guide for selecting proportions for High strength concrete with Portland cement and Fly ash*. ACI Manual of concrete practice (Part1), 1999.

[2] ACI 544.3R-93, *Guide for specifying, mixing, placing and finishing steel fiber reinforced concrete*, ACI Manual of Concrete Practice, Farmington Hills, 2006.

[3] ASTM C 39-1992, *Standard test method for compressive strength of fiber reinforced concrete*, ASTM International, American Society for Testing and Materials, 2004.

[4] ASTM C 78-1994, *Standard test method for flexural strength of concrete specimens*, ASTM International, American Society for Testing and Materials, 2004.

[5] Beale, M. H., Hagan, M. T., and Demuth, H. B., *Neural Network Toolbox- User's Guide*, MathWorks Inc., Natick, MA, USA, 2017.

[6] Boukhatem, B., Kenai, S., Hamou, A. T., and Ghrici, M., "Predicting concrete properties using neural networks with principal component analysis technique", *Computers and Concrete*, Vol. 10, No. 6, 2012, pp. 25-32.

[7] Cheng Yeh, I. C., "Design of high-performance concrete mixtures using neural networks and nonlinear programming", *J. Compu Civ Eng., ASCE*, Vol. 13, No.1, 1999, pp.36-42.

[8] Chen, S. S., and Shah, K., "Neural networks dynamic analysis of bridges". *Proc. Int. Conf. Compu in Civil Eng., ASCE*, 1999, pp. 1010-1013.

[9] Ezheldin, A.S. and Balaguru, P.N., "Normal and high strength fiber reinforced concrete under compression", *ASCE, J. Materials in Civil Engineering*, Vol. 4, No. 4, 1992, pp. 415- 429.

[10] Feng, M. Q., and Bahng, E. Y., "Damage assessment of jacketed RC columns using vibration tests", *J. Struct Engineering*, Vol. 125, No. 3, 1999, pp.265-271.

[11] Ghaboussi, J., Garrett, J. H., and Wu, X., "Knowledge based modeling of material behavior with neural networks", *J. Eng Mech., ASCE*, Vol. 117, No. 1, 1991, pp.129-134.

[12] Handong, Y., Wei, S., and Huisu, C., "The effect of silica fume and steel fiber on the dynamic mechanical performance of high-strength concrete", *Cement and Concrete Research*, Vol. 29, No. 2, 1999, pp. 423-426.

[13] Hola, J., and Schabowicz, K., "Application of artificial neural networks to determine Concrete compressive strength based on non-destructive tests", *J. Civ Eng Manage*, Vol. 11, No. 1, 2005a, pp. 23-33.

[14] Hsu, L. S., and Hsu, C. T., "Stress-strain behavior of steel fiber reinforced high- strength concrete under compression", *ACI Struct Journal*, Vol. 91, No. 4, 1994, pp. 448-457.

[15] Hegazy, T. Moselhi, O., and Fazio, P., "Development of practical neural network applications using back propagation", *Microcomputers in Civ Engg.*, Vol. 9, No. 2, 1994, pp. 145-159.

[16] Jain, J. C., Shih-Lin, H, Chi, S.Y, and Chem, C., "Neural network forecast model in deep excavation", *J. Compu Civ Eng., ASCE*, Vol. 16, No. 1, 2002, pp. 59-65.

[17] Jin-Keun Kim, Yun-Young Kim, "Experimental study of the fatigue behavior of high strength concrete". *Cement and Concrete Research*, Vol. 28, No. 10, 1996, pp. 1513-1523.

[18] Kasperkiewicz, J., Racz, J., and Dubrawsk, A., "HPC Strength Prediction Using Artificial Neural Network", *J. Compu Civ Eng.*, Vol. 4, 1995, pp.279-284.

[19] Kim, J. I., Kim, D. K., and Yazdani, Fr., "Application of neural networks for estimation of concrete strength", *J. Mate. Civ. Eng., ASCE*, Vol. 16, No. 3, 2004, pp. 257-264.

[20] Khayat, K. H., and Ghezal, A., "Utility of statistical modeling in proportioning self-consolidation concrete", *Proceedings RILEM Inter. Symposium on self-compacting concrete*, Stockholm, 1999, pp. 345-359.

[21] Luca Lanzi, Chiara Bisagni, and Sergio Ricci, "Neural network systems to reproduce crash behavior of structural components", *Compu Struct*, Vol. 82, No. 1, 2004, pp. 93-108.

[22] Mansur, M. A., Chin, M. S., and Wee, Y. H., "Stress-strain relationship of high strength fiber concrete in com-

- pression”, *ASCE, J. Mate Civil Eng.*, Vol. 13, No. 1, 1999, pp. 21-29.
- [23] Mahmoud Nili, and Afrouhsabet, V., “Combined effect of silica fume and steel fibers on the impact resistance and mechanical properties of concrete”, *Int. j. Impact Eng.*, Vol. 37, 2010, pp. 879- 886.
- [24] Moncef Nehdi, Hassan, E. I. C., et al., “Predicting performance of self-compacting concrete mixtures using artificial neural networks”, *ACI Materials Journal*, Vol. 98, No. 5, 2004, pp. 394-401.
- [25] Ni Hong-Guang, and Wang Ji-Zong, “Predicting of compressive strength of concrete by artificial neural networks”, *Cement and Concrete Research*, Vol. 30, 2000, pp. 1245-1250.
- [26] Poon, C. S., Shui, Z. H., and Lam, L., “Compressive behavior of fiber reinforced high-performance concrete subjected to elevated temperature”, *Cement and Concrete Research*, Vol. 34, No. 12, 2004, pp. 2215-2222.
- [27] Ramadoss Perumal, “Correlation of compressive strength and other engineering properties of high-performance steel fiber reinforced concrete”, *ASCE, J. Mate Civil Eng.*, Vol. 27, No. 1, 2015, pp. 1-7.
- [28] Ramadoss, P., and Nagamani, K., “Statistical methods of investigation on the compressive strength of high-performance steel fiber reinforced concrete”, *Computers and Concrete*, Vol. 9, No. 2, 2011, pp. 153-169.
- [29] Ramadoss, P., and Nagamani, K., “Mechanical properties of steel fiber reinforced silica fume concrete”, *J. Civil Engg Research and Practice*, Vol. 4, No. 1, 2007, pp. 27-44.
- [30] Ramadoss, P., and Nagamani, K., “A new strength model for the high-performance fiber reinforced concrete”, *Computers and Concrete*, Vol. 5, No. 1, 2008, pp. 21-36.
- [31] Ramadoss Perumal, “Modeling for the evaluation of strength and toughness of high-performance fiber reinforced concrete”, *J. Eng. Science and Technology*, Vol. 2, 2012, pp. 153-169.
- [32] Savino, V., Lanzoni, L., Taratino, A.M. and Vivani, M., “Simple and effective models to predict the compressive and tensile strength of HPFRC as steel fiber content and type changes”, *Composites Part B*, 2017, doi:10.1016/j.compositesb.2017.11.003.
- [33] Wang, J. Z., Ni, H. G., and He, J. Y., “The application of automatic acquisition of knowledge to mix design of concrete”, *Cement and Concrete Research*, Vol. 29, No. 7, 1999, pp. 1875-1880.
- [34] Wei-Ting, L., Ran Huang, Chin-Lai, L., and Hui-Mi, H., “Effect of steel fibers on the mechanical properties of cement based composites containing silica fume”, *J. Marine Science and Tech.*, Vol. 16, No. 2, 2008, pp. 214- 221.
- [35] Yeh, I. C., “Modeling of strength of high-performance concrete using artificial neural networks”, *Cement and Concrete Research*, Vol. 28, No.12, 1998, pp. 1797-1808.