

Zoning of mutual influence of approach rock tunnels based on a safety factor

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ABSTRACT: For multiple approach tunnels, the construction of the new tunnel near the existing tunnel modifies the state of stresses and movements around the existing tunnel in an area called the "influence zone". In this study, new method is developed to optimize the relative position of approach tunnels and guide their design via the zoning of mutual influence of the approach tunnels based on a safety factor. The strength reduction method is applied to calculate the global safety factor of multiple approach tunnels in limit state. According to quantitative laws of variation in the safety factor from single tunnel to multiple tunnels, the strong influence, weak influence, and no influence have been zoning to guide the design and construction of approach tunnels. By changing the relative position of the new tunnel complex, the safety factor and failure shapes of the tunnel complex in several cases are obtained through numerical simulation. The results are based on dividing the influence zone of approach construction of the new tunnel complex. Combining the influence zone with failure shapes, the relative position of the tunnel complex is optimized. Special support countermeasures for weak tunnel complex parts and parameters of support are initially determined.

Keywords: approach tunnels; safety factor; strengthen reduction method; influence zone.

1 INTRODUCTION

Due to gradual improvement and expansion as well as modification of the traffic network, lots of new lines are being built near existing ones, and the new railway is parallel to the road, all of which make it inevitable for the new tunnel to be built near existing one. Underground tunnels are susceptible to disturbance, when the distance of the new one from the existing ones is relatively close. In terms of the existing ones, negative impact will be apparent if no countermeasure is taken; for example, its bearing capacity decreases with increase in damage and deformation. Owing to a lack of knowledge and sufficient attention, this type of problem occasionally triggers engineering accidents in China (QIU, 2003).

The construction wherein a new tunnel is built near the existing one while causing negative mutual impact is called tunnel approaching construction and the tunnels are called approach tunnels (QIU, 2003; LI and QIU et al, 2010; ZHENG and QIU, 2006;). In the process of tunnel approach construction, the existing tunnel has disturbed the initial stress field, and the new tunnel is constructed exactly in the disturbed stress field. So, the stressed state of the new tunnel in this situation is different from that in semiinfinite body or infinite body. The construction of new tunnel in turn changes the stressed state of the existing tunnel and brings negative impact to it. The initial stress field tends to be disturbed for many times which make it bear an asymmetric stress, thereby presenting a large variability. It is the disturbance of the stress field that changes the stressed state of both the existing and new tunnels, which contributes to a safety problem of the existing tunnel and complex stressed state of new tunnel. Owing to the differences in the size of the new tunnel, position of the existing tunnel in relation to the new tunnel, geological condition, health state of existing tunnels, and construction method of the new and existing tunnels, the influence of the tunnel approaching construction differs and their stressed mechanisms are complicated during the tunnel approaching construction (Soliman and Duddeck et al., 1993; Perri, 1994;

Chehade and Shahrour, 2008; Chen and Lee et al., 2009;).

Studies on stress redistribution of surrounding rocks, which is caused by tunnel excavation, have shown that the redistribution happens only in a certain range. Its stress concentration factor tends to be higher when the place is much closer to the tunnel and vice versa (Brady B H G, Brown E T., 1999; Kolymbas. 2005; QIU and KONG et al, 2015). The construction of the new tunnel near existing one modifies the state of stresses and movements around the existing tunnel in an area called the "influence zone". The influence zone is mainly influenced by in-situ stress, tunnels span, and the width of the pillar separating the tunnels, the relative position of tunnels, and the construction sequence of the tunnels. The mutual influence of multiple approach tunnels has been studied by many scholars, who have evaluated the mutual influence among the tunnels by based on the additional displacement, stress, and plastic area from numerical simulation or measurement in the field (Barla and Ottoviani, 1974; Ghaboussi, and Ranken, 1977; Gercek, 2005; Zhao and Ma, 2009; Mortazavi et al., 2009; Karademir, 2010; Esterhuizen et al., 2011; Goh and ZHANG, 2012; ZHANG and Goh. 2012; Usmani and Nanda et al., 2014; ZHANG and Goh, 2015). Due to the complexity of underground tunnels, it is difficult to achieve a uniform standard of deciding the influence zone of approach tunnels by the variation of displacement, stressed state, and the even plastic zone. And there is not an integrated and quantitative method for evaluating mutual influence of tunnel approach constructions.

The strength reduction method is introduced in this paper to calculate the safety factor of tunnel approaching construction in the limit state. The safety factor can be used to quantitatively evaluate the mutual influence among approach tunnels. Through the analysis of the influence of the new tunnel on the existing one, the criteria for dividing the influence zone of tunnel approaching construction are put forward on the basis of variation laws of safety factor. And according to the above criterion, the mutual influence zone of the new tunnel complex in Chongqing is divided.

2 ENGINEERING BACKGROUND

The large-scale tunnel complex project, which mainly includes three tunnels with a large crosssection, is located in the Hongyan Village in Chongqing, China. Specifically, the three tunnels are (Fig. 1) a subway station (SS) and two parallel doublelane highway tunnels (HT-L and HT-R); additionally, the minimum distance between the SS and HT-L is only 2.0 m.

Chongqing is located at the southeast edge of the Sichuan basin in China. Strata in Chongqing urban operations belong to the Jurassic middle shaximiao formation, and rock strata are alternate layers of sand and shale. Moreover, rock strata have a flat occurrence, and their dip angle is less than 20°. Detailed field investigations revealed the vertical geological profile of the complex tunnel project: the sandstone and argillaceous sandstone strata are overlaid by a miscellaneous fill with an approximate 2.31 m thickness, as can be seen in Figure 1 and Table 1. Table 2 summarizes the engineering parameters of each individual tunnel.



Figure 1. Cross-sectional layout of the tunnels (unit: m)

Table 1. Geological profile of the ground

Table 1. Geological profile of the ground						
Depth (m)	Description		General Properties			
0~2.13	gray an miscella fill	d brown neous	heterogeneity and low strength			
2.13~28.8	fuchsia brown ceous sa	and argilla- strength and wat		reathered, high ater-sensitive		
28.8~34.8	gray sandstone		slightly weathered and high strength			
34.8~	brown ceous sa	argilla- ndstone	slightly weathered, hig strength and water-sensitive			
Table 2. Engineering parameters of each individual tunnel.						
Tunnel name	Depth (m)	Excavation width (m	on Excavation) height (m)	Cross- sectional ar- ea (m ²)		
SS	26.5	21.3	14.1	389.3		
HT-L	16.0	16.5	11.4	152.9		
HT-R	16.0	16.5	11.4	152.9		
28.8~34.8 34.8~ Table 2. Tunnel name SS HT-L HT-R	ceous sa gray san brown ceous sa Engineeri Depth (m) 26.5 16.0 16.0	ndstone dstone argilla- <u>ndstone</u> <u>ng paramet</u> Excavativ width (m 21.3 16.5 16.5	strength and w slightly weath strength slightly wea strength and w ters of each indiv on Excavation) height (m) 14.1 11.4 11.4	ater-sensitive athered and hig athered, hig ater-sensitive vidual tunnel. Cross- sectional a ea (m ²) 389.3 152.9 152.9		

3 AFETY OF LIMIT-STATE-BASED UNDERGROUND ROCK TUNNELS USING THE SHEAR STRENGTH REDUCTION (SSR) METHOD

In this study, the global stability and FoS for both the single and multiple rock tunnels are accessed using the shear strength reduction method (SSR). The method has been used by various authors to analyze the stability of underground rock tunnels (Matsui and san, 1992; Dawson and Roth, 1999; Dawson and Motamed et al., 2000; LI and QI et al., 2012; Goh and ZHANG, 2012; ZHANG and Goh. 2012; ZHANG and Goh, 2015) and is now available in many commercial finite element (FEM) and finite difference element (FDM) programs.

The SSR method is best described with the Mohr–Coulomb strength criterion, due to the simplicity and linearity of the criterion. For the criterion, the shear strength envelope (defined by the cohesion, c, and friction angle φ) can be reduced by a factor, F, and a new cohesion c and friction angel φ determined for the factored shear envelope:

$$\frac{\tau}{F} = \frac{c}{F} + \sigma \frac{\tan \varphi}{F}$$
(1)
$$F = \frac{\tau}{c^* + \sigma \tan \varphi^*}$$
(2)

where τ is the shear strength, σ is the normal

 $c^* = \frac{c}{F}$ and $\tan \varphi^* = \frac{\tan \varphi}{F}$ are the new Mohr–Coulomb shear strength parameters, and F is the reduction factor and also the factor of safety.

In the computation of underground rock tunnels, F gradually increases, the c and φ decrease as a response according to Equations (1) and (2). Until the tunnel collapses, the current value of F is a safety factor of evaluating tunnel stability.

There are two methods of deciding global safety factors of tunnels in strength reduction: the finite element and finite difference models do not converge to a solution (Goh and ZHANG, 2012; ZHANG and Goh. 2012; ZHANG and Goh, 2015) and the displacement around the tunnel increases sharply (Hammah and Yacoub et al, 2007). The stability of the surrounding rock and displacement speed (the ratio of the displacement increment to time increment) around the tunnel will change with a gradual decrease in its strength; when its strength decreases to a critical state, the surrounding rock will come into an unstable state suddenly. At the moment, the displacement speed will increase sharply, whereas the computation in a software converges to a solution. Therefore, the computed safety factor may be higher

than the actual one if computation converging to a solution is regarded as a criterion for judging tunnel collapse. Naturally, the mutation of displacement around the tunnel is regarded as the principal criterion for judging tunnel collapse.

The strength reduction method can be used to not only analyze the stability of underground rock tunnels, but also get the failure shape which is visible on the contours of the maximum shear strain. Zheng confirmed via experimental analyses that the failure shape in reality is basically consistent with that on the basis of the strength reduction method. (Hammah and Yacoub et al., 2007; ZHENG, 2012)

4 CRITERIA FOR DIVIDING THE INFLUENCE ZONE OF APPROACH TUNNELS BASED ON FOS

4.1 Numerical model

The FLAC^{3D} code (Itasca) was used to conduct the stability analyses of the construction of approach tunnels using the shear strength reduction technique. The numerical model is shown in figure 2. It is 300 m wide, and 136 m high and the longitudinal length is 1 m to simulate plane strain conditions. Figure 1 is presented for reference of the tunnel's shape, and table 2 for reference of the tunnel's dimensions. The model contains 31896 elements and 64963 nodes. In terms of boundary conditions, No vertical displacements were allowed along the base of the model, and no lateral displacements were allowed along the vertical surface of the model, and the top surface of the model was allowed to be deformed freely. The method of excavation is assumed as full-face and no support is applied in all cases. A solid element is applied to simulate the rock mass, complying with the Mohr-Coulomb failure criterion that follows the elastic perfectly in a plastic stress-strain relationship (Usman and Galler 2013). The mechanical parameters of the rock mass is shown in Table 3.



Figure 2. Simulation model for some cases

Table 3 Physical mechanics parameters of surrounding rock

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Material	Cohesion (kPa)	Internal friction angle (°)	Elastic modulus (MPa)	Poisson ratio	Unit weight (kN/m ³)	
Sandy mudstone	400.00	28.5	800.00	0.3 5	25. 9	

For underground rock tunnels, the mutual influence between the new and existing tunnels may change the stressed state of the surrounding rock several times, and then form a stress field which is correlative to the tunnels' positional distribution. Especially, for the multiple approach tunnels, the excavation sequence, and the relative position can influence different failure shapes and the final stability of the surrounding rock, and the stability is evaluated by the global safety factor. Considering HT-L and HT-R for example, the buried depth of the tunnel is 16 m. The corresponding variation of the safety factor is calculated when the horizontal distance between two tunnels changes. The computation operation conditions are shown in figure 3.



Figure 3. Diagram of computation cases (unit: m)

4.2 Analysis of results

When there is only one tunnel without support (called the existing tunnel), its safety factor is primarily calculated and when a new tunnel without support approaches it at different distances of 18 m, 19.5 m, 22.5 m, 32.5 m, 42.5 m, 52.5 m, 62.5 m and 112.5 m, its safety factors are then calculated respectively. By comparing the safety factors of two tunnels with that of a single tunnel, respectively, the mutual influence between the new and existing tunnel is then evaluated.

When a new tunnel approaches the existing one at a distance of 22.5 m, the variation of the existing tunnel's maximum displacement happening at its crown with a strength reduction factor is shown in figure 5. The criterion of uncontrolled displacement at a certain characteristic node was considered to explain the tunnel failure. The displacement mutation characteristics of monitoring points were chosen as the failure criteria of materials in the shear strength reduction method. And when there is only one tunnel, the corresponding variation is shown in figure 4. As shown in figure 4 and 5, the safety factor is 2.85 when there is only one tunnel. The factor drops to 1.65 when the new tunnel approaches it. Moreover, the variation of the new tunnel's maximum displacement at the crown with a strength reduction factor is basically equal to that of the existing one. So, the factor 1.65 can also be regarded as the global safety factor of tunnels.



Figure 4. Plot of strength reduction factor against Crown Settlement when there is only one tunnel



Figure 5. Plot of strength reduction factor against Crown Settlement when there is only one tunnel

As shown in figure 6, the safety factors in different cases are calculated and the corresponding failure shape is gotten. The safety factor is 2.85 when there is only one tunnel. When a new tunnel approaches it at different distances, the corresponding safety factors are listed in figure 6 blow. As the minimum width of the pillar rock between the two tunnels increases, the safety factor of the existing tunnel gradually goes up to the value 2.85 and then keeps stable, it can also be seen from the failure shape of the two tunnels in the limit state. As the minimum width of the pillar rock between the two tunnels increases, their failure shape changes from integrated failure of the two tunnels to separate failure of the existing tunnel and the eventual failure shape is consistent with that of a single tunnel. As the minimum width of the pillar rock between the two tunnels increases, stress concentration effect of pillar rock caused by excavation also weakens and eventually, one tunnel's excavation does not influence another any longer. So, the mutual influence between the two tunnels can be quantitatively evaluated by the variation of their safety factors.

The tunnel can be considered in a state of collapse when its safety factor is less than 1, whereas it can be considered stable when the safety factor is more than 1.

1. When the global safety factor of multiple tunnels (defined as FoS_G) is equal to the value of a single tunnel (defined as FoS_{ST}), $FoS_G = FoS_{ST}$, there is no mutual influence between the tunnels. To go a step further, the new tunnel is considered and there is no influence on the existing tunnel.

2. When $1 < FoS_G < FoS_{ST}$, there is some negative influence between the tunnels and the weak influence does not causes the existing tunnel and the new tunnels to collapse. To go a step further, the influence of the new tunnel on the existing tunnel is considered weak.

3. When $FoS_G < FoS_{ST} < 1$, there is much negative influence between the tunnels and the strong influence causes the existing tunnel and the new tunnels to collapse. To go a step further, the new tunnels are considered and there is a strong influence on the existing tunnel.

By the variation of the safety factor and the traditional criterion for the influence zone, criteria for dividing the influence zone because of the tunnel approaching the construction site are put forward as shown in table 4.



Figure 6. Results in different cases.

Table 4. Criterion for dividing influence zone because of approaching construction of tunnels

i	F8				
	Type of influ- ence zone	Criterion on the basis of FoS (without support)	Fea- tures	Countermeasures	
	Strong- influ- ence zone	FoS _G < FoS _{ST} <1 FoS _G <1< FoS _{ST}	New tunnel has de- struc- tive ef- fects on the ex- isting tunnel	Countermeasures must be taken for the design and con- struction method. According to the strength and dis- placement of sup- port on the existing tunnel, the influ- ence is studied and then countermeas- ures are taken. Meanwhile, meas- urement manage- ment is taken on both the new and the existing tun- nels.	
	Weak- influ- ence zone	1< FoS _G < FoS _{ST}	New tunnel has slight effects on the existing tunnel	Measurements are in general taken in the available meth- od of construction. Allowed displace- ment is deduced according to strength and cur- rent displacement of support on the existing tunnel, and whether other measures are need- ed or not is then decided. In order to ensure the safety of construction, measurement man- agement is ob- served on both the new and existing tunnel.	
	No- influ- ence zone	FoS _G =FoS _{ST}	The in- fluence of new tunnel on the existing tunnel needs no con- sidera- tion	Extra measures are not needed.	

5 INFLUENCE ZONE OF APPROACHING CONSTRUCT OF TUNNEL COMPLEX IN CHONGQING

5.1 Numerical model

For the actual engineering in this study, three of the tunnel complexes are newly built; however, they are applicable to the tunnel approaching construction. In order to carry out research in the mutual influence degree and global safety factor of Chongqing's tunnel complex when the relative positions change, numerical simulations are performed. If the positions of three tunnels are changed together, the cases will increase sharply and the analysis will be rather difficult, so the distance between HT-L and HT-R is kept fixed for the reason of simplifying simulation (by consulting the initial design, the minimum width of the pillar rock is 11.8 m). As shown in figure 7, the SS moves along the direction perpendicular to the connecting line of HT-L and HT-R's crowns at prior periods, and the SS then moves along the direction, between which in the above direction, the included angles are 0° (horizontal direction), 30° , 45° , 60° , 90° (vertical direction), respectively; 4-5 points of each direction are selected as the computation position of the SS (the pink points in figure 7 mark the positions of the SS's crown). The simulation method and parameters can be consulted in the fourth chapter. The numerical model is shown in figure 8.

By comparing the results of the numerical simulation, the global safety factor of three unsupported tunnels when excavated in different sequences is basically equal to that when excavated simultaneously. Therefore, the global safety factor of the three tunnels can be regarded as the value when they are excavated simultaneously.



Figure 7. Diagram of cases for numeral simulation



Figure 8. One of numeral simulation model (case for 90°)

5.2 *Zoning of mutual influence of the tunnel complex*

The points where maximum displacement happens around the tunnel of SS, HT-L and HT-R are recorded during computation, the global safety factor of the three tunnels are gotten respectively, where FoS_G is equal to the minimum among FoS_{SS} , FoS_{HT-L} , and FoS_{HT-R} . The results show that FoS_G is equal to FoS_{SS} in the case of this paper, meaning that the safety factor of the SS decides the global safety factor of the tunnel complex.

According to the results of numerical simulation, the distribution laws of the global safety factor of the tunnel complex in the direction of 0° , 30° , 45° , 60° , 90° , etc. (five angles) are shown in figure 9.



Figure 9. Distribution laws of safety factor (FoS)

As shown in figure 9, the safety factor in the direction of 0° (horizontal direction) increase as the SS moves toward the left, and then its value tends toward stability, which indicates that the mutual influence of the tunnel approaching construction is disappearing. As the SS moves toward the lower left in other directions, the safety factor increases primarily and then basically decreases, which is mainly because the mutual influence between tunnels are becoming weaker as the SS departs from HT-L and HT-R. When the distance of the SS from HT-L and HT-R is far enough only SS becomes unstable among the elements of the tunnel complex. The reason its safety factor continues to decrease is that the initial stress field increases as its buried depth increase.

The main factors which influences the safety factor of the tunnel complex are the in-situ stress, tunnel span, and the width of the pillar separating the tunnels. The main factor which influences in-situ stress is the buried depth. In this paper, tunnel span that keeps steady and the buried depth of the SS and the distance of the SS from the HT-L and HT-R change. What this paper focuses on is the mutual influence of tunnel approaching the construction site, so the influence of buried depth on the safety factor should be excluded. In order to achieve the goal above, the safety factor of the unsupported SS alone in different buried depths (FoS_{A-SS}) and in the same geological condition is calculated and the results is shown in table 5.

Table 5. Safety factors of SS alone (FoS_{A-SS}) in different buried depths

buried depuis						
Buried depth	27 m	41.8 m	56.8 m	71.8 m	86.5 m	
Safety factor	1.85	1.64	1.43	1.28	1.14	

Through contrast of the safety factors in figure 9 and table 5 and consulting the criterion for the distributing influence zone on the basis of FoS in the fourth chapter, the boundary of the influence zone when SS approaches HT-L and HT-R can be primitively decided: the positions where FoS_G is equal to 1 comprises the separator between stronger and weaker influence zones and the positions where FoS_G is equal to FoS_{A-SS} comprises the separator between weaker and no-influence zones. The figure of the mutual influence between the SS and HT-R as well as HT-L can be gotten by connecting contour points of safety factor. In order to facilitate its applicability to actual engineering, the contour lines of the safety factor are assumed to be an elliptic curve. As shown in figure 10, the pink area represents a stronger influence zone; the yellow area represents a weaker influence zone; green area represents a noinfluence zone.



Figure 10. Zoning of mutual influence of new tunnel complex

The failure shapes of some cases in limit state are shown in figure 11. The failure shape of the tunnel complex changes with variation of SS's position. The relative position of the tunnel's influence is not only an integrated safety factor but also an integrated failure shape. When the mutual influence among SS, HT-L, and HT-R disappears, the failure shape is basically consistent with that of the single tunnel. It can also be seen from figure 11 that mutual influence does not arise between any two of the three tunnels. Maybe, the mutual influence only arises between SS and HT-L, and there is no mutual influence between SS and HT-R yet (as shown in figure 11(f)). According to the failure shape, countermeasures for support can be put forward in the period of preliminary design so that the stability of the tunnel complex can be ensured more efficiently.

The divided influence zone can be referred to in the design of the new tunnel complex. As to the determination of the relative position of the tunnel complex, the distance of SS from HT-L and HT-R will be as far as possible so that the SS is in the weak or no-influence zone. if impossible for SS could be located in those areas due to the constraints, optimum position of the SS can be found by its failure shape. Special support countermeasures for the weak part and the parameters of support can be initially decided by a comprehensive analysis of FoS and failure shapes. Moreover, important monitoring positions during construction can be also decided.



Figure 11. Failure shapes of tunnel complex in some cases

6 DISCUSSION AND CONCLUSION

The strength reduction method is introduced in this paper to calculate the safety factor of tunnel approaching construction in limit state. The safety factor can be used to quantitatively evaluate the mutual influence among approach tunnels. Through the analysis of the influence of the new tunnel on the existing one, the criterion for distributing influence zones of the tunnel approaching construction are put forward on the basis of variation laws of safety factor. According to the above criterion, the mutual influence zone of new tunnel complex in Chongqing is divided. Theoretical support is provided for the research into mutual influence between multiple tunnels approaching construction and theoretical basis is supplied to decide the initial risk degree of this type of engineering. This paper presents important meaning for actual engineering and theoretical studies.

The surrounding rock is assumed to be a continuous medium. To take into consideration the crack and joint in it, the surrounding rock's parameter is properly decreased according to the investigation results in the field so that their attenuation to the surrounding rock can be monitored. The distribution of the crack and joint in the surrounding rock can influence and even the change failure shape of the surrounding rock during tunnel approach construction. In numerical simulation, the influence of manual interventions (such as the support method, support strength, and countermeasures of construction) on approach tunnels during actual construction is not taken into consideration, and only the most negative case, namely the case of no support, is considered. This paper is aimed at providing a point of view for fellows and inspiring others.

The main factors affecting the safety factor of the tunnel are the distance between the tunnels, buried depth, geological context and support parameters. This paper does not consider the influence of support parameters. The main reasons are as follows: support parameters should be determined according to the distance between the tunnels, buried depth and geological context. At the initial stage of the design, only when the stability of the tunnel without support is clear, can the support parameters be designed. Therefore, the influence zone proposed in this paper provides a basis for the rapid determination of tunnels stability in the initial stage of design. It provides reference for tunnel planning and design.

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