eJSE International
Electronic Journal of Structural Engineering 18(2) 2018

Column Analogy in Multi-Span Hinged Frames

A. Badir*
Department of Environmental & Civil Engineering, U.A. Whitaker College of Engineering, Florida Gulf Coast University, Florida, US,

M. Badir
Late Professor, Department of Structural Engineering, University of Alexandria, Egypt

*Email: abadir@fgcu.edu

ABSTRACT: A procedure is presented in which the method of column analogy, normally applicable to the analysis of single-span and closed frames, is extended for the analysis of multi-span frames with columns hinged to the ground. The extension involves consideration of conditions of rotations at the hinges. For illustration, a two-span hinged frame is solved in details. Results are in excellent agreement with values obtained using classical structural methods.

KEYWORD: indeterminate structures, multi-cell frames, moment coefficients, superposition.

1 INTRODUCTION

The objective of this paper is to extend the use of the method of column analogy (Cross, 1930, Cross & Morgan 1945, Sözen 2002) to multi-span hinged frames. In a recent paper (Badir & Badir 2012) a column analogy procedure was presented for the analysis of multi-cell structures with fixed columns. The analysis presented herein involves consideration of conditions of rotations at the hinges to replace conditions of moments at the fixed ends. Together with the previous published work (Badir & Badir 2012), it constitutes a generalization of the column analogy method for the analysis of multi-cell structures.

2 METHOD OF ANALYSIS

For a description of the suggested method of analysis, consider the two-span hinged frame of Fig. 1(a) with moments of inertia of the columns: *I* in exterior columns and 2*I* in interior column. This is a simplifying but not a necessary assumption.

2.1 *Division of Multi-Span Frame: Case "0"*

In Fig. 1(b) the multi-span hinged frame is divided into two isolated spans a and b with their inertia and loads by slicing column 1-2 into two halves; Case "0". The moments M_{io} at the various sections i of each frame including M_{k0} and M_{k0}' at column sections $k = 1$ and 1' are computed by column analogy as usual. At hinges $k = 2$ and 2' the rotations r_{ko} and r'_{ko} are also computed. These are simply the reactions at hinges 2 and 2' of the elastic load M_{i0}/EI, where E is Young's Modulus.

2.2 *Correction Forces and Couples*

Each isolated frame will now deform independently under the action of its external forces. In order to restore continuity of the multi-span frame, it is necessary to add corrections. These corrections are taken as the moments resulting from: two equal and opposite forces X_1, and two equal and opposite couples X_2, acting at the top of columns 1-2 and 1'-2' as shown in Fig. 1(c).

2.3 *Continuity Restoration*

In the multi-span frame of Fig. 1(a), the two halves of the sliced column 1-2 must undergo identical deformations in order that they fit in together forming the original column. This situation will be satisfied only when the rotations at the two hinges and the bending moments in the two halves of the sliced column are identical. These two conditions of continuity may be stated as follows: (1) a condition dealing with moments at the top section *k* of the column, Eq. (5) in the work of Badir & Badir (2012), namely:

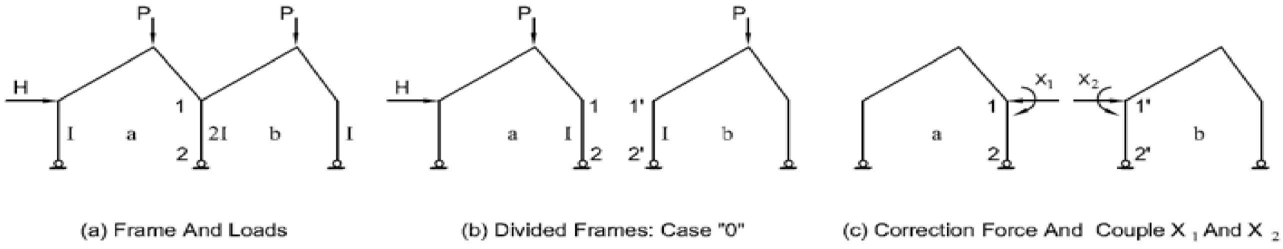

Figure 1 Analysis of two-span hinged frame

$$M_k^* = M_{k0}^* + \sum_n m_{kn}^* \cdot X_n = 0 \quad (1)$$

in which $M^*_k = M_k + M'_k$, $M^*_{k0} = M_{k0} + M'_{k0}$, and $m^*_{kn} = m_{kn} + m'_{kn}$ is the continuity moment-coefficient of case $X_n = 1$, and (2) another condition dealing with rotations at the bottom hinge k of the column, namely:

$$r_k^* = r_{k0}^* + \sum_n r_{kn}^* \cdot X_n = 0 \quad (2)$$

in which $r^*_k = r_k + r'_k$, $r^*_{k0} = r_{k0} + r'_{k0}$, and $r^*_{kn} = r_{kn} + r'_{kn}$ is the continuity rotation-coefficient of case $X_n = 1$. The subscripts of the rotation r have the same meaning as those in moments (Badir & Badir 2012). In general, for every column there are two unknowns and two conditions of continuity, giving as many equations as the number of unknown forces and couples X_n. In the frame of Fig. 1(a) there are two equations and two unknowns X_1 and X_2. Solving Eqs. (1) and (2) simultaneously, the corrections X_n are obtained.

2.4 *Bending Moment in Multi-Span Frame*

In general, the moment at any section of the multi-span frame is determined by superposition as the sum of moment due to external loads in Case "0" and the moment due to correction forces and couples in Cases "n". The final bending moment in the multi-span frame is given by Eqs. (8) and (9) of the work of Badir & Badir (2012); which are restated here for convenience

$$M_i = M_{i0} + \sum_n m_{in} \cdot X_n \quad (3)$$

at sections i not in columns; and

$$\overline{M}_k = \overline{M}_{k0} + \sum_n \overline{m}_{kn} \cdot X_n \quad \text{in columns sections } k \quad (4)$$

in which $\overline{M}_{k0} = M_{k0} - M'_{k0}$, m_{in} and $\overline{m}_{kn} = m_{kn} - m'_{kn}$ are the correction moment-coefficients at sections i and k, respectively.

3 SOLVED EXAMPLE

The dimensions, inertia and loads of a two-span hinged frame are given in Fig. 2(a). The frame is divided into two identical frames: a and b of Fig. 2(b). The analogous column section and its properties are given in Fig. 2(c). In Figs. 2(d) and (e) are sketched the statical moments, on the tension side, in frames a and b, respectively. The straining actions and equations of indeterminate moments are given beside each frame. The computed moments M_{i0}, M_{10}, and M'_{k0} are registered in column (3) of Table 1. The resulting rotations r_{20} and r'_{20} are recorded in column (4) of the table, but the calculations are not shown.

In this frame, one unknown correction force X_1 and one couple X_2 as shown in Fig. 1(c) are needed for continuity restoration. In Figs. 2(f) and (g) are given the statical moments in frames a and b in Case "1", i.e. $X_1 = 1$. The straining actions and the indeterminate moments are given in the figure. The resulting moment-coefficients and rotation-coefficients are shown in columns (5), (6) and (7) of Table 1. A similar treatment of Case "2", i.e. $X_2 = 1$ is shown in Figs. 2(h) and (i). The corresponding coefficients are entered in columns (8), (9) and (10) of Table 1. In columns (11) to (16) of Table 2 are found the values of M_{10}^*, r_{20}^* and the continuity moment and rotation-coefficients m_{11}^*, r_{21}^*, m_{12}^*, and r_{22}^* readily obtained from Table 1. Now the two conditions of continuity (1) and (2) are easily written with the aid of Table 2 as follows

$$-119.67992 + 12.524007 X_1 - 0.576131 X_2 = 0 \quad (5)$$

and

$$-472.3423+86.368463X_1-1.414632X_2=0 \quad (6)$$

Therefore, X_1 = 6.5420506 and X_2 = -65.518481.

Finally, Eq. (3) is used to find the final bending moments M_i with the help of Table 1. The results are

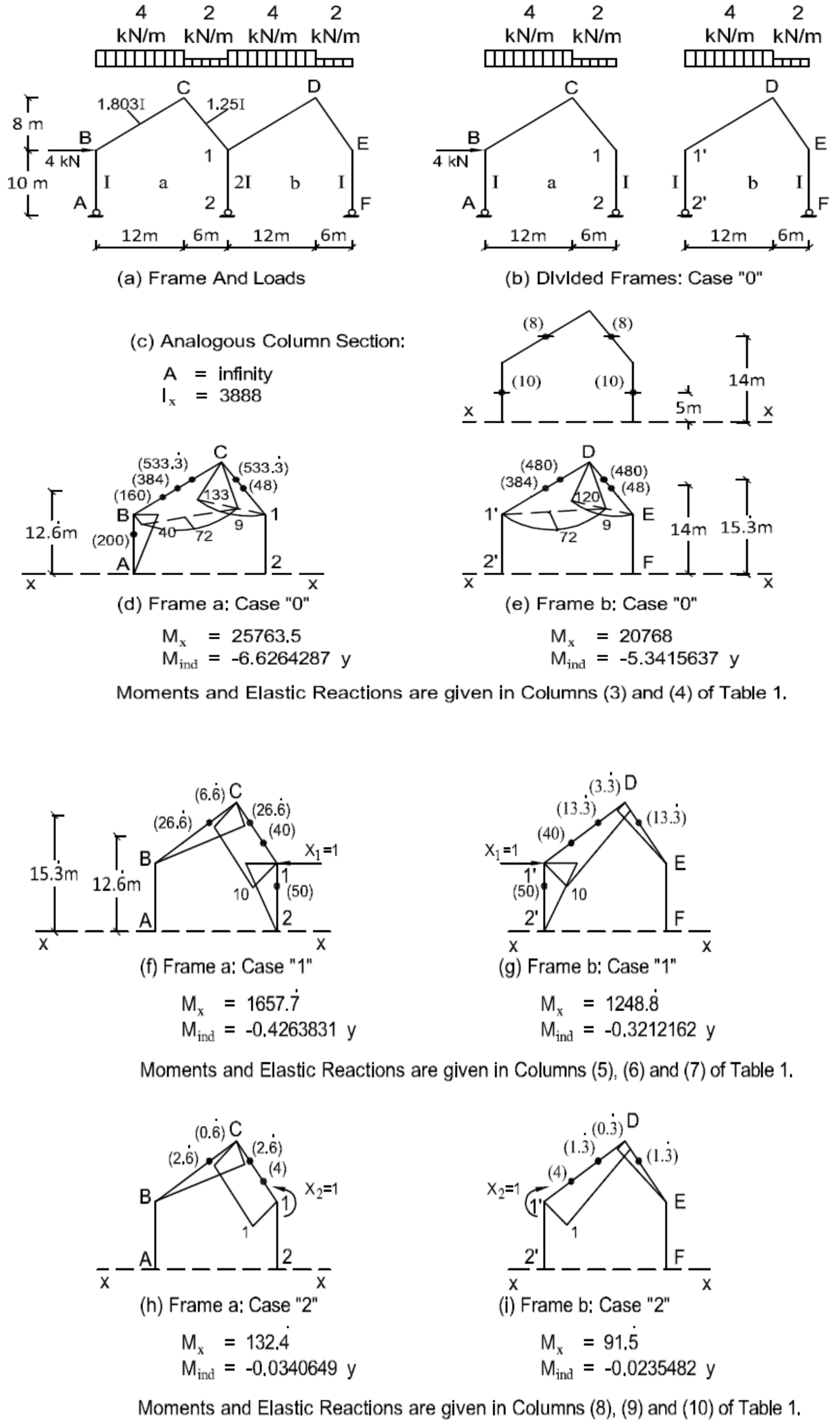

Figure 2. Details of solved example

Table 1. Case “0”, and Moment and Rotation Coefficients; Cases “1 and 2”.

Frame	Section i, k, k’	Case ”0”		Case “1”			Case “2”		
		M_{i0}, M_{k0} M'_{k0}	r_{k0}, r'_{k0}	m_{i1}	m_{k1}, m'_{k1}	r_{k1}, r'_{k1}	m_{i2}	m_{k2}, m'_{k2}	r_{k2}, r'_{k2}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
a	B	-26.264287		-4.263831			-0.340649		
	C	14.05762		-1.0082292			0.0534984		
	1	-66.264287		5.736169	5.736169		0.659351	-0.340649	
	2	0	-353.54766	0		40.35692	0		0.5998627
b	1’	-53.415637		6.787838	6.787838		0.764518	-0.235482	
	2’	0	-118.79464	0		46.011543	0		0.8147695
	D	23.85186		-2.4485583			-0.0905343		
	E	-53.415639		-3.212162			-0.235482		

Table 2. Continuity Moment and Rotation Coefficients; m^*_{kn} and r^*_{kn}

Frame	Section i, k, k’	Case “0”		Case “1”		Case “2”	
		M^*_{k0}	r^*_{k0}	m^*_{k1}	r^*_{k1}	m^*_{k2}	r^*_{k2}
(1)	(2)	(11)	(12)	(13)	(14)	(15)	(16)
a	1	-119.67992		12.524007		-0.576131	
	2		-472.3423		86.358463		1.4146322

Table 3. Correction Moment Coefficients and Final Bending Moments

Frame (1)	Section i, k, k' (2)	$\overline{M}_{k0}$ (17)	$\overline{m}_{k1}$ (18)	$\overline{m}_{k2}$ (19)	M_i (20)	$\overline{M}_k$ (21)
a	B				-31.83968	
	C				3.9566	
	1	-12.84865	-1.061669	-0.105167	-71.937655	-12.838339
b	1'				-59.099316	
	D				13.764937	
	E				-59.001341	

given in column (20) of Table 3. Eq. (4) gives the final bending moment at section 1 of column 1-2 with the aid of columns (17), (18) and (19) of Table 3. The moment $\overline{M}_1$ is given in column (21). The final bending moment diagram is drawn in Fig. 3 on the tension side. Results are in excellent agreement with values obtained using classical structural methods.

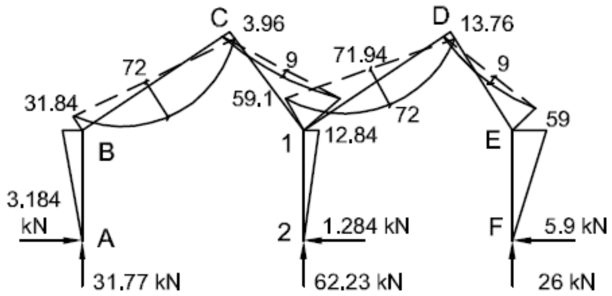

Figure 3. Bending moment diagram (kN ·m) and reactions

4 CONCLUSION

The forgotten method of column analogy used to analyze statically indeterminate single span and closed frames is extended to the analysis of multi-span frames with columns hinged to the ground. The procedure presented here, together with the previously published paper (Badir & Badir 2012), constitute a generalization to Professor Hardy Cross's method of column analogy (Cross, 1930, Cross & Morgan 1945) commonly applied to "one cell" frames, arches, and curved beams.

5 REFERENCES

Cross H., "The Column Analogy", Bulletin 215, Engineering Experiment Station, University of Illinois, 1930.

Cross H. & Morgan N. D., "Continuous Frames of Reinforced Concrete", 7th printing, John Wiley and Sons, U.S.A. 1945.

Sözen M.A., "Remembering the Column Analogy" in S. M. Uzumeri Symposium: Behavior and Design of Concrete Structures for Seismic Performance (ACI SP-197) (S.A. Sheikh, O. Bayrak Editors), Farmington Hills, Mich., ACI International, 2002, pp. 23-44.

Badir, A. & Badir, M., "Column Analogy in Multi-Cell Structures with Fixed Columns," *Electronic Journal of Structural Engineering*, 12(1), 2012, pp. 95-107