

Active State Earth Pressure based on Dubrova's Method of Redistribution of Pressure

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ABSTRACT: Modes and magnitude of wall movement have great influence on the distribution of earth pressure behind retaining wall. For gravity retaining wall exerted by cohesionless soil rotating outward about the base, based on Dubrova's method of redistribution of pressure and forming mechanism of earth pressure, the distribution pattern of internal friction angle along the wall back was established for different displacements, and then, an improved method of calculating active earth pressure was proposed, which considers the influence of displacement. When the retaining wall is in an at-rest state, the distribution of earth pressure calculated in this paper is defined as initial active state earth pressure, and it is not equal to static earth pressure, because the retaining wall has a tendency of active displacement. As the wall movement develops, the mobilized internal friction angle of the quasi sliding surface increases gradually from the initial internal friction angle ϕ_0 to the internal friction angle ϕ , and accordingly, the distribution of earth pressure at the intersection of the sliding surface and the wall back decrease gradually from the distribution of initial active state earth pressure in static state to that of Coulomb active earth pressure.

KEYWORD: Soil mechanics; Distribution of active earth pressure; Gravity retaining wall; Magnitude of displacement; Internal friction angle of backfill

1 INSTRUCTIONS

Correctly calculating the distribution of earth pressure is the prerequisite for retaining wall design. The main methods of earth pressure calculation in engineering are Rankine earth pressure theory and Coulomb earth pressure theory. Rankine's theory is based on the stress level in the elastic half-space body and the limit equilibrium condition of soil, which requires that the back of the retaining wall is vertical and smooth and the filling surface is horizontal. Coulomb's theory is based on the static equilibrium condition of sliding soil wedge when the soil behind the wall reaches the limit equilibrium state. Coulomb's method is more widely used.

Terzaghi[1], Sherif *et al.*[2], Fang *et al.*[3] and Zhou *et al.*[4] studied the effect of modes and magnitude of wall movement on the earth pressure distribution behind the retaining wall through laboratory tests, which shows that earth pressure distribution behind the wall will be significantly affected by the modes and magnitude of wall movement whether the backfill is sand or clay. Chang put forward the concept of mobilized internal friction angle of backfill and mobilized external friction angle between backfill and the wall and established their displacement-related distribution patterns along the wall back. Based on Coulomb's earth pressure theory, a simplified earth pressure calculation method was proposed which shows the influence of the modes and magnitude of wall movement[5]. Zhang *et al.* established the basic equation of wall active earth pressure under the con-

dition of non-limit state by taking horizontal thin layer of the sliding wedge as the differential element. Based on the moment equilibrium condition of the whole sliding wedge, the theoretical formula of earth pressure distribution and resultant force point was set up[6]. From the stress Mohr's circle of cohesive soil, Xu *et al.* deduced the relationship between the internal friction angle of cohesive soil and the wall displacement when the soil is in a intermediate state between initial state and ultimate active equilibrium state[7]. Applying horizontal-slice analysis method, the formula of earth pressure in non-limit active equilibrium state is obtained. Hu *et al.* studied the non-limit active earth pressure of retaining wall by thin-layer element method and obtained the theoretical formula of lateral coefficient, resultant force and action point of earth pressure[8]. Based on the calculation method of Bang's active earth pressure, Wang *et al.* proposed an improved calculation method of non-limit active earth pressure considering the influence of wall displacement[9]. On the basis of the stress-strain concept, Chen *et al.* studied the non-linear relationship between wall displacement and internal friction angle of soil and soil-wall external friction angle, and a theoretical calculation method of non-limit earth pressure was proposed considering the effect of retaining wall displacement[10]. Wang *et al.* deduced the analytical solution of active earth pressure of cohesionless soil using thin-layer element method[11]. Liu *et al.* put forward an expression of active earth pressure under

translational mode by modified horizontal layer method, which considers the influence of average shear stress between horizontal layers in sliding wedge of backfill[12].

Only when the displacement of retaining wall is large enough, can the backfill behind the wall be able to reach the limit equilibrium state represented in the classical theory of earth pressure. However, in most cases, the wall displacement is not enough, and the backfill behind the retaining wall is often in a non-limit equilibrium state. According to Chang's relationship between the wall displacement and the backfill friction angle[5] and Fang's experimental results[3], an improved distribution model of friction angle along wall back is established for different wall movement. Based on the Dubrova's method of redistribution of pressure[13], an improved calculation method of active earth pressure for gravity retaining wall with cohesionless backfill soil rotating outward about base is put forward, which considers the influence of wall displacement.

2 THEORIES

2.1 Dubrova's redistribution of pressure[13]

Coulomb earth pressure theory shows that the active earth thrust acting on a vertical wall with a horizontal backfill of cohesionless soils is

$$P_a = \frac{\gamma H^2}{2 \cos \delta} \left[\frac{1}{1/\cos \phi + \sqrt{\tan^2 \phi + \tan \phi \tan \delta}} \right]^2 \quad (1)$$

where γ is the soil unit weight, H is the height of the wall, ϕ is the internal friction angle, and δ is the external friction angle between wall and soil.

The active earth pressure can be obtained by Equation 1 only when the sliding wedge reaches the limit equilibrium state, and the friction angle on the sliding surface is ϕ .

Dubrova considers that a series of quasi-Coulomb-sliding surfaces are formed in the backfill which is in the non-limit equilibrium state, and the mobilized internal friction angle on the sliding surface is ψ .

Replacing ϕ in Equation 1 with ψ , the active earth thrust at any depth z after the wall is

$$P = \frac{\gamma z^2}{2 \cos \delta} \left[\frac{1}{1/\cos \psi + \sqrt{\tan^2 \psi + \tan \psi \tan \delta}} \right]^2 \quad (2)$$

Dubrova points out that the external wall friction angle δ is only related to ϕ , and independent of ψ .

Differentiating P in Equation 2 with respect to z , the lateral active earth pressure $p_a(z)$ at any depth z will be expressed as

$$p_a(z) = \frac{dP}{dz} = \frac{\gamma}{\cos \delta} \cdot$$

$$\left[\frac{z \cos^2 \psi}{(1+m \sin \psi)^2} - \frac{z^2 \cos \psi}{(1+m \sin \psi)^3} (\sin \psi + \frac{m^2+1}{2m}) \frac{d\psi}{dz} \right] \quad (3)$$

where $m = (1 + \tan \delta / \tan \psi)^{1/2}$.

Dubrova thinks that m is related to ϕ , and take $m = (1 + \tan \delta / \tan \phi)^{1/2}$. At the same time, taking $(1+m^2)/2m = m$, which will lead to little loss, Equation 3 is further simplified as

$$p_a(z) = \frac{dP}{dz} = \frac{\gamma}{\cos \delta} \cdot \left[\frac{z \cos^2 \psi}{(1+m \sin \psi)^2} - \frac{z^2 \cos \psi}{(1+m \sin \psi)^3} (\sin \psi + m) \frac{d\psi}{dz} \right] \quad (4)$$

For any kind of active movement mode, once the distribution of ψ along the back of the retaining wall is specified, the corresponding distribution of active state earth pressure can be determined by Equation 4.

2.2 Distribution of friction angle along the back of the wall

For the case of a wall rotating outward about the base, according to Dubrova's redistribution of pressure, the mobilized internal friction angle of backfill at the top of the wall is ϕ , and the limiting active state exists only at the very top. The mobilized internal friction angle of backfill at the bottom of the wall is 0, and at any depth z $\psi = \phi(1-z/H)$. That is to say, the distribution of ψ along the back of the wall is linear as shown in Fig.1(a), which takes into account the effect of the mode of wall movement but does not consider the effect of the magnitude.

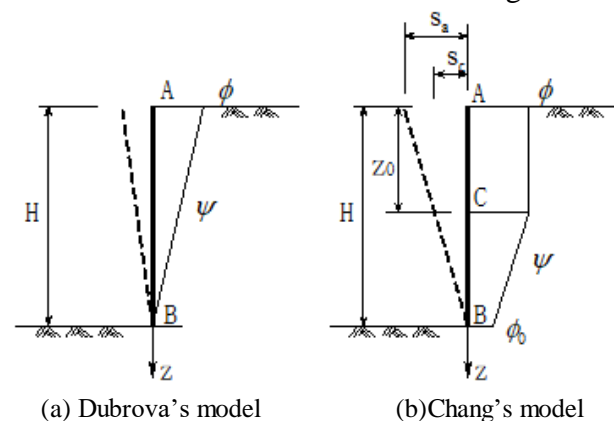


Figure 1. Relationship between internal friction angle and displacement

Chang also assumed that a series of quasi-Coulomb-sliding surfaces are formed in the backfill behind the wall. The mobilized internal friction angle ψ of the sliding surface is related to the displacement, s , of the intersection point of the sliding surface and retaining wall, as shown in Fig. 1 (b), where H is the wall height, the critical displacement, s_c , is needed

for shearing resistance to mobilize fully, uniform along the wall and about $0.0003H$, and s_a is the maximum wall movement at the top of the wall. For the upper portion AC of the wall in Fig.1 (b), the horizontal movement $s \geq s_c$ at any depth z , where $0 \leq z \leq z_0$ and $z_0 = (s_a - s_c) / s_a H$, and the mobilized internal friction angle ψ of the sliding surface intersecting the wall at this depth is ϕ . The horizontal displacement at the bottom of the wall is 0, and the mobilized internal friction angle of the sliding surface intersecting the wall at point B is defined as initial internal friction angle ϕ_0 . For the lower part CB of the wall in Fig.1(b), the horizontal movement $0 \leq s \leq s_c$ at any depth z , where $z_0 \leq z \leq H$, and the variation of the mobilized internal friction angle ψ of the sliding surface with z is linear, which is obtained by linear interpolation between ϕ at point C and ϕ_0 at point B as illustrated in Fig.1(b).

The distribution of internal friction angle along the back of the wall introduced by Chang considers the influence of the mode and magnitude of wall movement[5]. Fang's indoor model test shows that the distribution of earth pressure at the bottom of the wall gradually converges to Coulomb's active earth pressure as the retaining wall rotates outward around the wall base[3]. While Chang considered that the mobilized internal friction angle of the sliding surface intersecting with the wall at point B will always take the initial internal friction angle ϕ_0 as wall rotates outward around the base. Therefore, the corresponding calculated earth pressure distribution acting at point B is always constant, and it can not embody the process of gradual decrease of earth pressure at this point which Fang's experiment shows.

Based on Chang's relationship between the mobilized internal friction angle of the backfill and the wall displacement, for retaining wall rotating outward around the wall base, a modified distribution pattern of the mobilized internal friction angle ψ along the back of the wall considering the influence of displacement is established. Two cases are considered.

(1) $0 \leq s_a \leq s_c$

The mobilized internal friction angle ϕ_A of backfill behind point A at the top of the wall could be calculated by following Equation 5.

$$\phi_A = \phi_0 + \frac{s_a}{s_c} (\phi - \phi_0) \quad (5)$$

The mobilized internal friction angle ϕ_B of the sliding surface behind point B at the bottom of the wall could be calculated by following Equation 6.

$$\phi_B = \phi_0 + \frac{2}{\pi} \arctan\left(\frac{s_a}{s_c}\right) (\phi - \phi_0) \quad (6)$$

Equation 6 can also be used to calculate ϕ_B when $s_a > s_c$. Equation 6 shows that when $s_a = 0$, $\phi_B = \phi_0$, and when $s_a = \infty$, $\phi_B = \phi$. It means that as the displacement of the retaining wall rotating outward around the base develops, the mobilized internal friction angle ϕ_B of the sliding surface intersecting the bottom of the wall gradually increased from ϕ_0 to ϕ .

The mobilized friction angle ψ of the sliding surface at any depth z behind the wall could be obtained by linear interpolation between ϕ_A at the top of the wall and ϕ_B at the bottom of the wall as follows.

$$\psi = \phi_B + \frac{H - z}{H} (\phi_A - \phi_B) \quad (7)$$

The distribution of ψ along the back of the wall is illustrated in Fig.2(a). Equation 5 and Equation 6 show that when s_a is known, ϕ_A and ϕ_B are constant, and the first-order derivative of ψ in Equation 7 on z at any depth along the wall back is

$$\frac{d\psi}{dz} = -\frac{1}{H} (\phi_A - \phi_B) \quad (8)$$

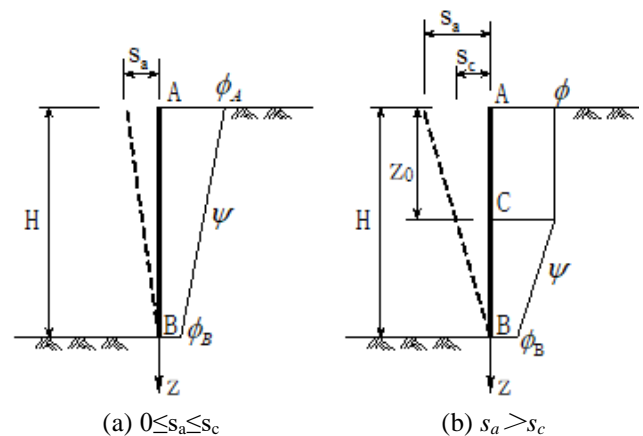


Figure 2. Distribution of internal friction angle along the back of the wall

(2) $s_a > s_c$

As shown in Fig.2(b), the displacement, s , of point C is equal to critical displacement, s_c , and the mobilized internal friction angle of the sliding surface intersecting the wall at this point is ϕ . The displacement, s , of the upper part AC is not less than s_c , and as mentioned above, the mobilized internal friction angle of sliding surface intersecting with this portion is ϕ . The length, z_0 , of AC is determined as follows:

$$z_0 = \left(1 - \frac{s_c}{s_a}\right) H \quad (9)$$

The mobilized internal friction angle ϕ_B at point B is still calculated by above Equation 6. The mobilized friction angle ψ of the sliding surface intersecting with lower portion BC could be obtained by linear

interpolation between ϕ at point C and ϕ_B at the bottom of the wall, which is shown below.

$$\psi = \phi_B + \frac{H - z}{H - z_0}(\phi - \phi_B) \quad (z_0 \leq z \leq H) \quad (10)$$

Similarly, when the wall top movement, s_a , is known, the first-order derivative of ψ on any depth, z , along the wall back is

$$\frac{d\psi}{dz} = 0 \quad (0 \leq z \leq z_0) \quad (11)$$

$$\frac{d\psi}{dz} = -\frac{1}{H - z_0}(\phi - \phi_B) \quad (z_0 < z \leq H) \quad (12)$$

2.3 Modified active state earth pressure theory

Substitution of Equation 8, Equation 11, or Equation 12 into Equation 4 yields the distribution of active state earth pressure for any wall displacement.

(1) $0 \leq s_a \leq s_c$

Substituting Equation 8 into Equation 4, the lateral active earth pressure will be

$$p_a(z) = \frac{\gamma}{\cos \delta} \cdot \left[\frac{z \cos^2 \psi}{(1 + m \sin \psi)^2} + \frac{z^2 \cos \psi}{(1 + m \sin \psi)^3} (\sin \psi + m) \frac{(\phi_A - \phi_B)}{H} \right] \quad (13)$$

where ϕ_A and ϕ_B are calculated by Equation 5 and Equation 6 respectively, and ψ is calculated by Equation 7.

By Equation 13, the active state earth pressure distribution behind the wall can be determined under the condition of $0 \leq s_a \leq s_c$.

(2) $s_a > s_c$

For the upper part AC of the wall in Fig.2(b), substituting Equation 11 into Equation 4, the following Equation 14 will be obtained.

$$p_a(z) = \frac{\gamma}{\cos \delta} \cdot \frac{z \cos^2 \phi}{(1 + \sqrt{1 + \tan \delta / \tan \phi} \sin \phi)^2} \quad (14)$$

For the lower part CB of the wall, substituting Equation 12 into Equation 4, the following Equation 15 will also be obtained.

$$p_a(z) = \frac{\gamma}{\cos \delta} \cdot \left[\frac{z \cos^2 \psi}{(1 + m \sin \psi)^2} + \frac{(z - z_0)^2 \cos \psi}{(1 + m \sin \psi)^3} (\sin \psi + m) \frac{(\phi - \phi_B)}{H - z_0} \right] \quad (15)$$

In Equation 15, ϕ_B is calculated by Equation 6, and ψ is calculated by Equation 10. The first term describes the effect of the magnitude of wall movement on the distribution of earth pressure, while the second term shows the effect of the mode of wall movement.

When $s_a > s_c$, the lateral active earth pressure behind the upper portion AC of the wall can be calculated by Equation 14, and the lateral active earth pressure behind the lower part CB can be calculated by Equation 15.

3 ANALYTICAL RESULTS AND DISCUSSIONS

Fang investigated the distribution of active state earth pressure behind retaining wall by indoor model test[3]. Based on Fang's model test data, comparison of theoretical solutions presented in this paper with model test results was made. The soil used for the model test is air-dried sand. The height of retaining wall with vertical back $H=1.016\text{m}$, the weight of backfill $\gamma = 15.34\text{kN/m}^3$, the internal friction angle $\phi = 33.4^\circ$, the initial internal friction angle $\phi_0 = 7.5^\circ$, and the external friction angle between retaining wall and soil $\delta = 16.7^\circ$. Fig.3 gives the distribution of active state earth pressure calculated by the theoretical formula, and also shows the Fang's indoor model test results.

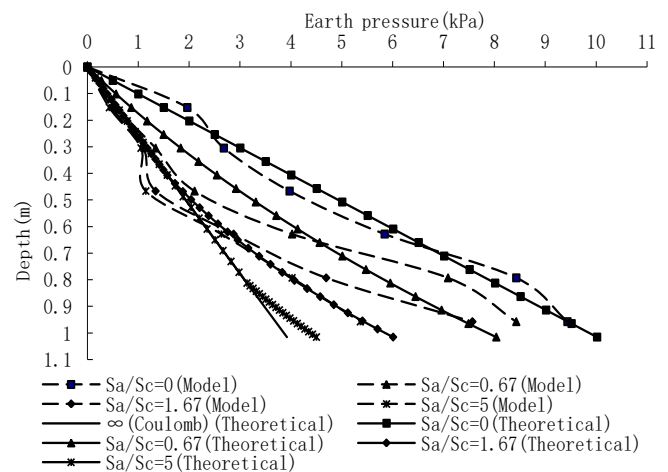


Figure 3. Comparison of theoretical solutions with model test results

Comparisons of the curves are showed in Fig. 3. When $s_a/s_c = 0$, the retaining wall is in a static state, but there is a trend of rotating outward, and the distribution of earth pressure behind the wall is defined as the initial active state earth pressure. When $s_a/s_c = 1$, only the soil behind the very top of the wall reaches the limit equilibrium state, the shearing resistance of which is mobilized fully. As the wall continues its rotation, s_a will increase accordingly, the full mobilization of shearing resistance will propagate downwards, and the earth pressure acting on the Corresponding part of the wall will gradually converge to the Coulomb's active earth pressure. When $s_a / s_c = \infty$, the theoretical analysis shows that the entire soil mass behind the wall reaches the limit equilibrium state, and the earth pressure acting on the wall is equal to the coulomb's active earth pressure. Both theoretical and experimental results show that

with the increase of displacement s_a of the top of the wall, the value of earth pressure at the bottom gradually decreases from the initial earth pressure to Coulomb's active earth pressure. However, Chang's method can not show the process of the gradually decreasing of the active earth pressure at the bottom of the wall as the displacement s_a of the top of the wall increases from 0 to ∞ .

The theoretical and experimental values of the resultant forces and application points of lateral thrusts for different wall displacements are illustrated in Tab.1, in which h_0/H is the ratio of the height, h_0 , of the total lateral force from the base of the wall to the height, H , of the retaining wall. The data in Tab.1 show that as the displacement of retaining wall increases, the resultant force of earth pressure from theoretical analysis or model tests decreases gradually and finally converges to Coulomb's active earth pressure. The height of the application point of the active thrust from the wall base is first decreased and then increased. When the displacement of the retaining wall is large enough, the point of application of the active thrust is located $1/3H$ above the base of the wall. It can be seen that the displacement of retaining wall significantly affects the active thrust behind the wall and the position of the point of application.

Table 1. Theoretical and experimental value of resultant force and application point of lateral thrust

s_a / s_c	h_0 / H		E (kN/m)	
	theory	model test	theory	model test
0	0.334	0.339	5.088	4.336
0.67	0.312	0.272	3.603	3.608
1.67	0.302	0.233	2.458	2.943
5	0.327	0.253	2.044	2.524
∞ (Coulomb)	0.333	-	1.991	-

4 CONCLUSION

For gravity retaining wall with cohesionless backfill soil rotating outward about the base of wall, Dubrova's method of redistribution of pressure was modified, and a improved method of calculating active earth pressure was proposed, which can consider the influence of modes and magnitude of wall movement and the progressive rupture mechanisms. The calculation method presented in this paper can reflect the developing process of the earth pressure and shear strength of the soil behind the wall with the increase of the displacement of the retaining wall, and the earth pressure at the bottom of the wall gradually converges to the Coulomb active earth pressure. The idea of derivation could also be expanded to develop a method of calculating passive

earth pressure taking account of the influence of deformation patterns.

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