ABSTRACT: The strength reduction finite element method (SRFEM) needs to modify the input shear strength material parameters manually based on various reduction factors, which is trivial. In order to simplify the computation process, the SRFEM can be achieved by using field variables provided by ABAQUS; therefore, it can be done automatically in software. By deriving the relationship between field variables and reduction factor and getting the corresponding step time $t_1$ when the calculation cannot converge, then it can quantitatively calculate the stability coefficient of the slope. According to the different decay extent and speed of shear strength parameters $c$ and $\phi$ in the process of slope failure, this paper adopted the dual reduction factors finite element method based on field variables for stability analysis of slope, and verified the feasibility of this method in the stability analysis by combining with classic examples, and found that this method not only improves computational efficiency but also the calculation accuracy.

2 BASIC PRINCIPLE OF DUAL REDUCTION FACTORS METHOD

According to the different decay extent and speed of shear strength parameters cohesion and frictional resistance, it should use different reduction coefficient in shear strength reduction method; it means that by dividing the shear strength parameters $c$ and $\phi$ by different reduction factors $F_1, F_2$, it can get new values of $c$ and $\phi$ which are substituted into the calculation as new material parameters until reaching the critical state (Figure 1). Then the corresponding $F_1, F_2$ are the slope stability factors.
3 FIELD VARIABLES BASED ON ABAQUS

The field variables based on ABAQUS is an environment variable. With the field variables material parameters can automatically change with the calculation in the same analysis step, or have different values in different analysis steps. In order to achieve dual reduction factors method, it needs to establish the relationships between cohesion and field variables, friction resistance and field variable. Assume that the field variables were \( f_1, f_2 \), and the value of \( F(f_1) \) from a to b, the value of \( F(f_2) \) from c to d.

3.1 Relationship between shear strength parameters and field variables

\[ c = c_0 / F(f_1) \]  
\[ \tan \phi = \tan \phi_0 / F(f_2) \]  
Where \( c_0 \) and \( \phi_0 \) are initial cohesion and initial internal friction angle, \( c \) and \( \phi \) are cohesion and internal friction angle changing with field variables. Therefore, the ranges of shear strength parameters in analysis step of finite element method are:

\[ c_0 / b \leq c \leq c_0 / a \quad \tan \phi_0 / d \leq \tan \phi \leq \tan \phi_0 / c \]

3.2 Relationship between shear strength parameters and reduction factors

\[ F_1 = c / c_0 \]  
\[ F_2 = \tan \phi_0 / \tan \phi \]  

3.3 Relationship between field variables and reduction factors

Combine with (1), (2), (3), (4):

\[ F_1 = F(f_1) \quad a \leq F_1 \leq b \]  
\[ F_2 = F(f_2) \quad c \leq F_2 \leq d \]  

3.4 Calculation of reduction factors

Assume that the value of step time is from g to h, and the functions \( \Phi(t), F(f_1), F(f_2) \) are monotone increasing in the range. Therefore, assuming the step time is \( t_1 \) when the calculation cannot converge, the relationships between step time and reduction factors are:

\[ F_1 = [a(h-t_1) + b(t_1 - g)] / (h - g) \]  
\[ F_2 = [c(h-t_1) + d(t_1 - g)] / (h - g) \]

There are countless combinations of \( F_1, F_2 \) in dual reduction factors method, in order to reduce the amount of computation, let \( F_1 = K F_2 \). Based on the above analysis, the value of coefficients \( a \) and \( b \) should include the possible values of reduction factors, let \( a = 0.5, b = 2.0, g = 0, h = 1.0 \), then \( c = 0.5 / K, d = 2.0 / K \). Therefore, the reduction factors are:

\[ F_1 = 1.5t_1 + 0.5 \]  
\[ F_2 = (1.5t_1 + 0.5) / K \]

4 EXAMPLE

In order to verify the correctness of the method, this paper used this method to analyze some classic examples, and compared with the results given in the literature. The examples are adopted by plane strain finite element model; the boundary conditions are vertical rollers on both side’s boundary and full fixity at the base.

4.1 Example 1

Example 1 is a homogeneous slope, slope angle \( \beta = 45^\circ \), \( H = 10 \text{m} \), the length of model base and upper are 20m and 8m. Cohesion is 12.38KPa, friction angle is 20°, and unit weight is 20KN/m3. Use quadrilateral node unit; divide into 1 396 grid cell, as shown in Figure 2.

There are countless combinations of \( F_1, F_2 \) in dual reduction factors method. In order to simplify the computation, let \( K = 0.8, 0.9, 1.1, 1.2 \). Take divergence of calculation as instability criterion in slope stability analysis. The calculation is converge before the point B which is shown in Figure 3, after which the calculation is not converge; the corresponding step time is \( t_1 \). Then calculate the stability factor \( F_1 \) and \( F_2 \) through formula (9) and formula (10); it is similar to the other three situations, which are not listed here, the calculation results are shown in Table 1. The step time equals to 0.3718 when the calculation is not converge in strength reduction finite element method; the stability coefficient calculated is 1.0577 through formula (9) (Figure 4).
4.2 Example 2

Example 2 is examination EX11 from ACADS slope stability analysis program in Australian. As shown in Figure 5, Cohesion is 3.0KPa, friction angle is 19.6°, and unit weight is 20kN/m³. Use quadrilateral node unit, divide into 728 grid cell. Let K=0.8, 0.9, 1.1, 1.2 as same as the example 1. When calculation is not converge, it is shown as point B in Figure 6 (K=0.8); then calculate the stability factors through the corresponding step time at this moment; it is similar to the other three situations, which are not listed them here. The calculation results are shown in Table 2.

The stability factor given is 1.0. Table 1 show that the step time when calculation is not converge increases with value of K. According to formula (9) and formula (10), the value of reduction factor F1 is increasing, the value of reduction factor F2 is reducing. Taking the average of the two values as stability factor of slope, it shows that the final result is similar with recommend answer. It means that the dual reduction factors method based on field variables can apply to the analysis of slope stability.
The step time equals to 0.3274 when the calculation is not converge in strength reduction finite element method, the stability coefficient calculated is 0.9911 through formula (9). The recommended answer is 1.0 of this example. It means that the final result in dual reduction factors method based on field variables is similar with the recommend answer and the result of strength reduction finite element method. In practical engineering, the values of K can be determined by geotechnical test of soil in the slope.

4.3 Example 3

The above examples are homogeneous slopes; in order to verify the applicability of the method in the complex soil slope, Example 3 is the examination EX13 from ACADS slope stability analysis program in Australian, which is an inhomogeneous slope. The material parameters are shown in Table 3. Use quadrilateral node unit, divide into 982 grid cell. Slope geometry and numerical mesh is shown in Figure 7.

Table 3 Material parameters of the soil.

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>c(KPa)</th>
<th>φ</th>
<th>γ(KN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>0</td>
<td>38</td>
<td>19.5</td>
</tr>
<tr>
<td>②</td>
<td>5.3</td>
<td>23</td>
<td>19.5</td>
</tr>
<tr>
<td>③</td>
<td>7.2</td>
<td>20</td>
<td>19.5</td>
</tr>
</tbody>
</table>

Take divergence of calculation as instability criterion in slope stability analysis. The step time equals to 0.5909 when the calculation is not converge in strength reduction finite element method; the stability coefficient calculated is 1.3864. The horizontal displacement change with the strength reduction factor is shown in Figure 8. As shown in Figure8, when the reduction factor is about 0.9, horizontal displacement mutates, however, apparently slope failure does not occur. With the further reduction of the intensity, displacement develops until no stress distribution can be achieved in ABAQUS to satisfy both the yield criterion and global equilibrium, which means that calculation is no longer convergence. Let K=0.8, 0.9, 1.1, 1.2 as same as the above examples. When calculation is not converge, it is shown as point B in Figure9 (K=1.2); then calculate the stability factors according to the corresponding step time at this moment. The calculation results are shown in Table 4.

![Figure 7 Numerical mesh for example 3.](image)

![Figure 8 Horizontal displacement changes with F.](image)
Table 4 Calculation results of different K.

<table>
<thead>
<tr>
<th>K</th>
<th>t₁</th>
<th>F₁</th>
<th>F₂</th>
<th>((F₁+F₂)/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.4903</td>
<td>1.2355</td>
<td>1.5443</td>
<td>1.3899</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5461</td>
<td>1.3192</td>
<td>1.4657</td>
<td>1.3924</td>
</tr>
<tr>
<td>1.1</td>
<td>0.6414</td>
<td>1.4621</td>
<td>1.3292</td>
<td>1.3956</td>
</tr>
<tr>
<td>1.2</td>
<td>0.6852</td>
<td>1.5278</td>
<td>1.2732</td>
<td>1.4005</td>
</tr>
</tbody>
</table>

As shown in table 4, the result of dual reduction factors method based on field variables in inhomogeneous slope is consistent with the result in homogeneous slope. With the increasing of the value of K, the step time when calculation is not converge increases; F1 increases; F2 decreases. The recommended answer is 1.39 of this example, so the dual reduction factors method based on field variables can applied to inhomogeneous slope stability analysis.

5 CONCLUSIONS

This paper simplified the computational process of strength reduction based on field variable in ABAQUS which can make entire strength reduction process be done automatically in the software, and calculated the stability factor of slope quantitatively according to the step time when calculation is not converge.

With Dual Reduction Factors Method Based on Field Variables, this paper analyzed the classic examples of slope stability analysis. The results are similar with recommend answers; it shows that this method can be applied to slope stability analysis.

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