

# Structural Optimization for Asymmetric Framed Structures without Shear Walls

Anandu Sreeraj, Vasugi V

*SMBS, VIT Chennai, Tamil Nadu, India.*

Bindu Sunil

*Geostructurals (P) LTD, Kochi, Kerala, India*

**ABSTRACT:** This paper proposes a method for the preliminary arrangement of columns having variable sizes on an asymmetric plan, such that the resulting structure will have minimum torsion. Asymmetry can be due to shape of the plan as well as arrangement of columns having variable sizes. This study shows the influence of first three modes in determining the structural behavior of RCC framed buildings during earthquake. The time periods were approximately calculated by using 3D-shear-torsion beam method. The optimization was done using Genetic algorithm in Matlab. The seismic performance of general asymmetric structures with uniform square columns were compared with optimized asymmetric structures having columns with variable sizes. The results from pushover analysis showed significant increase in strength and ductility along the direction having torsion.

## 1 INTRODUCTION

The advancements in construction industry has led to a decline in space for future developments. This space constraint will give architects no option but to opt for an asymmetric plan. During earthquake, a building having unsymmetrical plan can undergo torsional vibrations. This is due to the difference in locations of center of mass and center of rigidity at stories. Hence translational vibrations on that building will be in coupled nature (Kan and Chopra, 1977). These coupled vibrations can be dangerous, since it will lead to an un-equal distribution of earthquake forces on the lateral load resisting systems.

For a multistoried structure, it is difficult to find the location of center of rigidity exactly. They are load dependent (Cheung and Tso, 1986). Hence during the design of a structure, codes provide an extra safety factor called accidental torsion (Llera and Chopra, 1995).

The earthquake forces acts at the center of mass of the structure. If the forces act at the center of rigidity, the building will only have uncoupled vibrations. Hence the time period of the coupled vibrations should be greater than uncoupled vibration. This study is based on the idea that when the columns are selected based on least first three natural time periods, the coupling of the structure would get reduced. Also, when a structure's Eigen frequency is increased, its base shear capacity will proportionally increase (Arroyo and Guitierrez, 2016).

For calculating the coupled time period Rafezy et al., 2007, proposed an approximate method (3d shear-torsion beam). In this method, the framed

structure has been considered as a cantilever beam, with structural properties as that of the selected structure. It uses continuum approach and D' Alembert's principle for formulating the governing differential equation. Since it uses continuum approach, the mass was considered uniform throughout. The generalized equation for calculating coupled natural frequency was found using Wittrick-Williams algorithm.

This analytical investigation uses 3d shear-torsion beam method (Rafezy et al., 2007) in conjunction with Genetic algorithm for selecting a sample of columns which limits the structural torsion of an asymmetric plan. Matlab programs were used for the optimization and the seismic performance of the optimized models were compared with general asymmetric models with uniform square columns. Etabs 2016 was used for linear dynamic and non-linear static analysis. The cost of materials required for both general and optimized asymmetric models were also calculated in this study.

## 2 OPTIMIZATION USING GENETIC ALGORITHM

Genetic algorithm is simply an optimization method which selects the fittest solution from a population of natural selection (McCall, 2004). This method uses the principle "Select the best, discard the rest". In the present study, a natural selection of column samples constitute the population. In this study, the criteria considered for the fittest solution is the sample of column combination having least first three natu-

ral time periods. Only the fundamental modes were considered for optimization because these modes generally have the highest modal mass participation. This analytical investigation also shows the influence of fundamental modes in determining the structural response of a building during earthquake.

The flow chart for the optimization program using Matlab is shown in Figure 1. The column dimensions used for optimization, co-ordinate location of columns, column combination sample size and all the required structural properties should be given manually. The program will generate a number of random column combinations within the sample size. For any sample size, equal number of random column combinations would be generated. The time periods of the generated column combinations will be calculated using 3d shear-torsion beam method. Equations used for approximate method is shown in Appendix I. The Matlab program, after optimization, will give an output of columns required at different co-ordinate location. This column combination will have the least first three time periods compared to the entire random samples. For a sample of 50 million, the Matlab program took almost 12 minutes in a 3<sup>rd</sup> generation Intel core i5 processor with 8GB RAM.

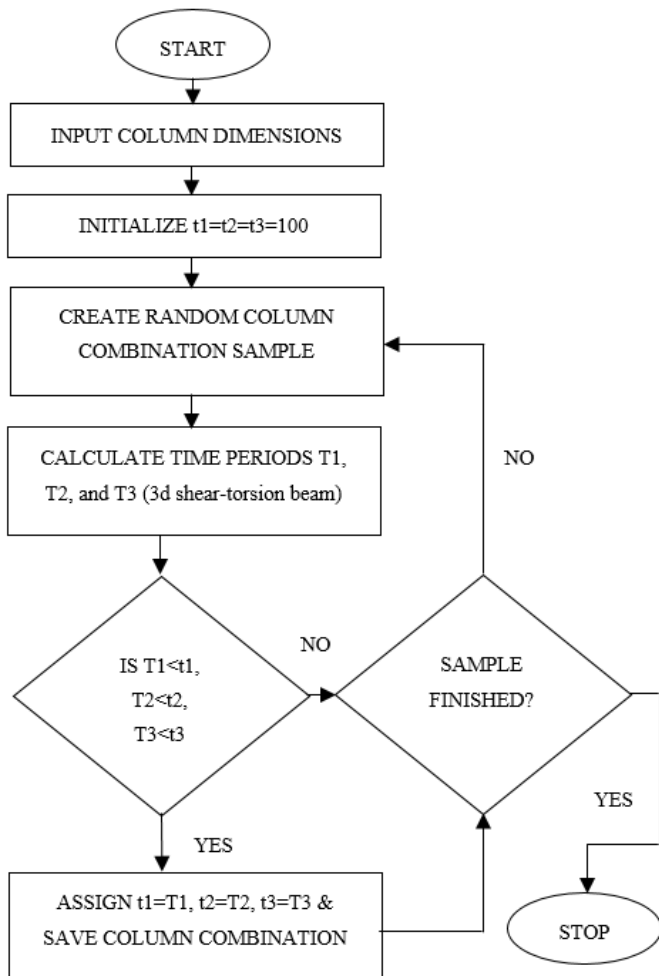


Figure 1. Flow chart for optimization.

### 3 MODALS CONSIDERED FOR ANALYSIS

Asymmetric structures having re-entrant corners are highly vulnerable to seismic forces (Prajwal et al., 2017). In the present study, two models, namely Model 1 and Model 2, having different asymmetric plans were considered. Hence a total of 4 models having two general asymmetric models and two optimized asymmetric models were used in the analysis. Each model was having 10 stories with a storey height of 3m. The bay width along X and Y axis were 6m and 4m respectively.

#### 3.1 General asymmetric models

The plan of General asymmetric model 1 and General asymmetric model 2 are shown in Figure 2 and Figure 3 respectively. In these models the column sizes were all 500×500 mm and all beams had a cross sectional dimension of 250×500 mm.

#### 3.2 Optimized asymmetric models

The plan of Optimized asymmetric model 1 and Optimized asymmetric model 2 are shown in Figure 4 and Figure 5 respectively. The column sizes used for optimization were 500×500, 600×400, 400×600, 300×800 and 800×300 (All dimensions are in millimeters). The rectangular column sizes were selected such that its cross-sectional areas were within the area of square column, 500×500 mm. For the optimization of model 1, all the above column sizes were used, while for the optimization of model 2 only 300×800 mm and 800×300 mm were used. The Matlab output after optimization is given in Appendix II.

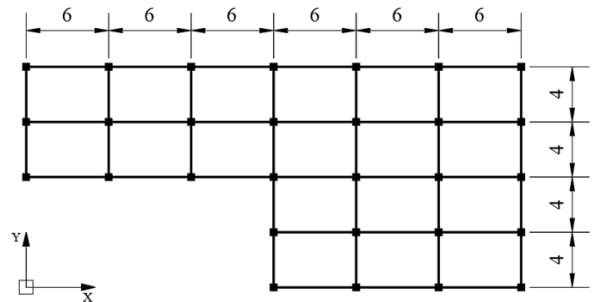


Figure 2. Plan of General asymmetric model 1.

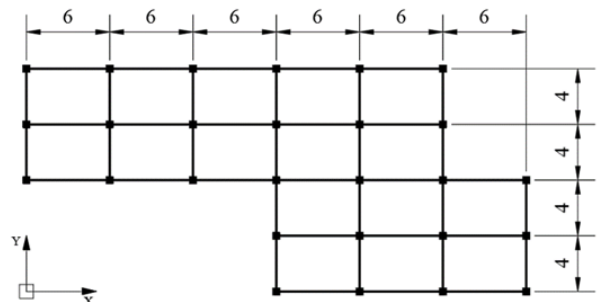


Figure 3. Plan of General asymmetric model 2.

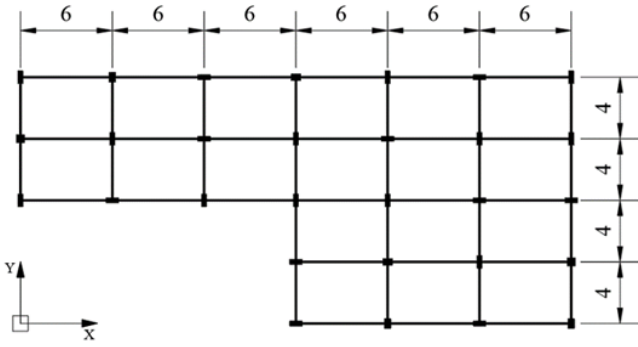


Figure 4. Plan of Optimized asymmetric model 1.

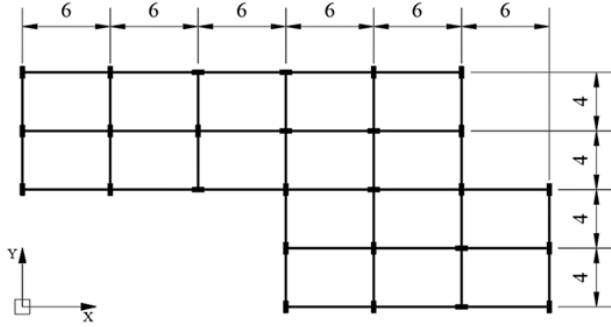


Figure 5. Plan of Optimized asymmetric model 2.

## 4 RESULTS AND DISCUSSIONS

All the above 4 buildings were modeled in Etabs 2016. The accuracy of time period calculated using approximate method was validated using Etabs FEA and these models were subjected to linear dynamic analysis and non-linear static analysis.

### 4.1 Comparison of Time periods

Since time period being a function of mass and stiffness, it is not necessary for the optimized models to have a time period less than the models with square columns. The mass and stiffness of the optimized models are different compared to the general models, since their column sizes are different. Hence a combination having least first three time periods, selected from the sample, is considered as the optimized model. The comparison of time periods for Model 1 and Model 2 are shown in Table 1 and Table 2 respectively. Approximate method showed very good accuracy for 1<sup>st</sup> and 2<sup>nd</sup> modes. 3<sup>rd</sup> mode showed reasonable accuracy.

Table 1. Time periods for Model 1.

Mode number	General model 1		Optimized model 1	
	Approximate method (s)	Etabs (FEA) (s)	Approximate method (s)	Etabs (FEA) (s)
1	1.1704	1.168	1.1422	1.187
2	1.0835	1.048	1.0515	1.045
3	0.8368	0.949	0.8077	0.932

Table 2. Time periods for Model 2.

Mode number	General model 1		Optimized model 1	
	Approximate method (s)	Etabs (FEA) (s)	Approximate method (s)	Etabs (FEA) (s)
1	1.1434	1.158	1.1380	1.205
2	1.0458	1.045	0.9962	0.99
3	0.9019	0.952	0.8573	0.884

### 4.2 Linear dynamic analysis

Response spectrum analysis was done according to IS 1893 (Part 1): 2016. The building and site specifications for seismic analysis are given in Table 3. A total of 12 modes were considered for the dynamic analysis, which ensured a modal mass participation well above 90% for each model at the 12<sup>th</sup> mode.

Table 3. Building and site specifications.

Specifications	
Grade of concrete for columns	M40
Grade of concrete for beams	M25
Grade of steel	Fe500
Diaphragm	Rigid
Dead Load	3KN/m <sup>2</sup>
Live load	3KN/m <sup>2</sup>
Seismic Zone	V
Soil type	II
Importance factor	1
Response reduction factor	3
Damping	5%

#### 4.2.1 Comparison of Torsion

For quantifying the effect of torsion due to dynamic loading, the ratio of maximum displacement to minimum displacement (refer Figure 6) for each storey was calculated. IS 1893 (Part 1): 2016 limits this ratio to 1.5. The values of ratios along X and Y directions for Model 1 and Model 2 are given in Table 4 and Table 5 respectively. The analysis showed effective reduction in torsion for the models along Y direction. Along X direction, both general models and optimized models showed very less torsion.

### 4.2.2 Comparison of Storey drifts

According to IS 1893 (part 1): 2016, the storey drift ratio should be within the limit 0.004. All the 4 models had drift ratios well within the limit. The comparison of storey drifts along X and Y directions are shown in Figure 7 and Figure 8 respectively. In the X direction, both general and optimized models showed comparable drifts while in Y direction, optimized structures showed very less drift comparatively. Refer Table 6 for maximum drift values for each model.

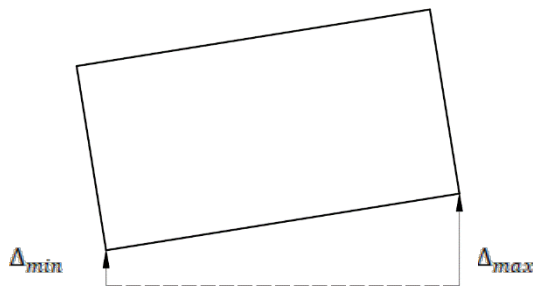


Figure 6. Representation of torsion at a storey.

Table 4. Displacement ratio in X direction

X direction	$\frac{\Delta_{max}}{\Delta_{min}}$			
	General asymmetric		Optimized asymmetric	
Storey	Model 1	Model 2	Model 1	Model 2
10	1.029	1.025	1.002	1.028
9	1.029	1.024	1.001	1.028
8	1.0287	1.023	1	1.027
7	1.0283	1.023	1	1.026
6	1.028	1.022	1	1.026
5	1.027	1.02	1.002	1.025
4	1.027	1.023	1.004	1.025
3	1.026	1.019	1.006	1.024
2	1.025	1.018	1.01	1.024
1	1.023	1.015	1.02	1.024

Table 5. Displacement ratio in Y direction

Y direction	$\frac{\Delta_{max}}{\Delta_{min}}$			
	General asymmetric		Optimized asymmetric	
Storey	Model 1	Model 2	Model 1	Model 2
10	1.248	1.273	1.057	1.007
9	1.238	1.267	1.048	1.013
8	1.231	1.261	1.04	1.019
7	1.224	1.256	1.03	1.025
6	1.215	1.25	1.024	1.032
5	1.205	1.244	1.015	1.04
4	1.195	1.237	1.003	1.048
3	1.182	1.228	1.012	1.059
2	1.163	1.215	1.033	1.0738
1	1.135	1.195	1.065	1.094

### 4.3 Non-linear static analysis

In order to conduct pushover analysis, the above models were designed using IS 456: 2000 after the dynamic analysis. A 5% accidental eccentricity was provided for design. The procedure for pushover analysis is provided in FEMA – 356. Pre-defined hinges according to ASCE 41-13 were used. Transverse steel will increase the member strength and ductility (Mander et al., 1988). For the exact result, user defined hinges calculated after detailing should be provided. Default hinges and user defined hinges shows comparatively similar hinge formation at the yielding (Inel and Ozmen, 2006). Since this being a comparative study, only default hinges were necessary. For beams M3 hinges and for columns P-M2M3 hinges in Etabs 2016 were assigned respectively. The transverse reinforcement was taken as conforming. The program will receive the required data for hinges from the analysis and design. Pushover analysis was done in both orthogonal directions for minimum steel required by each model.

#### 4.3.1 Comparison of Base shear vs Displacement

The Base shear vs displacement graph for models in X and Y directions are shown in Figure 9 and Figure 10 respectively. The results revealed effective increase in strength and ductility along Y direction for optimized models due to reduction in torsion. Along X direction the optimized asymmetric models maintained comparable results with general asymmetric models, since both were having less torsion. Refer Table 6 and Table 7 for ductility ratios.

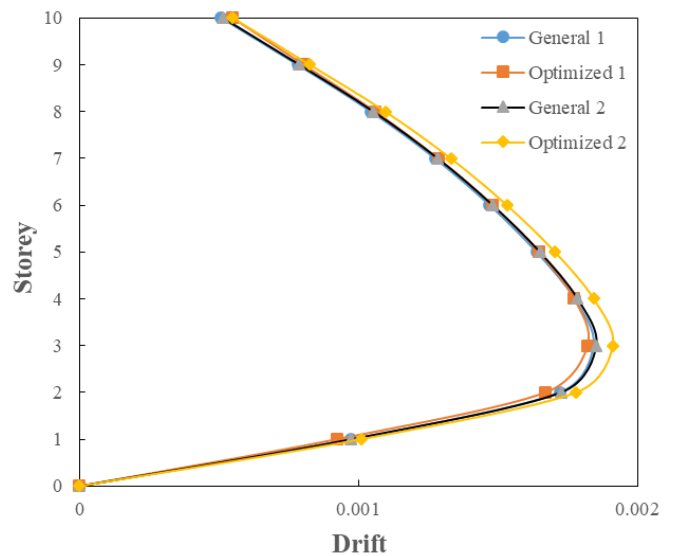


Figure 7. Storey drifts along X direction.

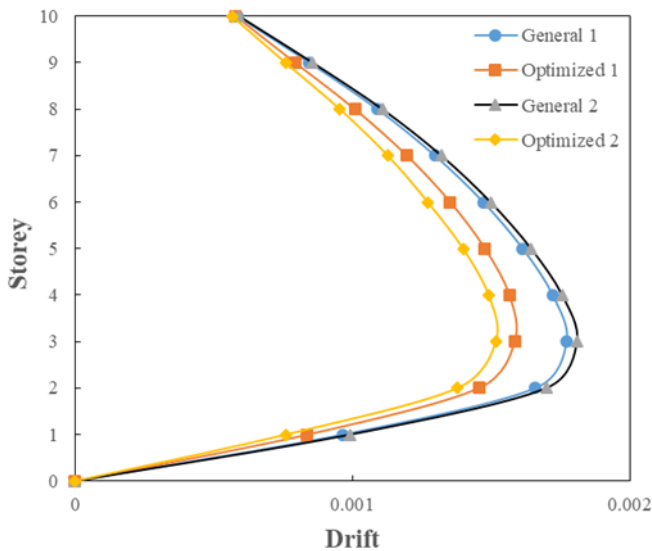


Figure 8. Storey drifts along Y direction.

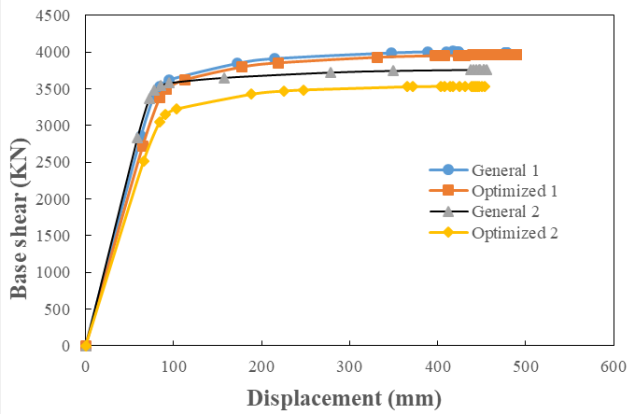


Figure 9. Base shear vs Displacement in X direction.

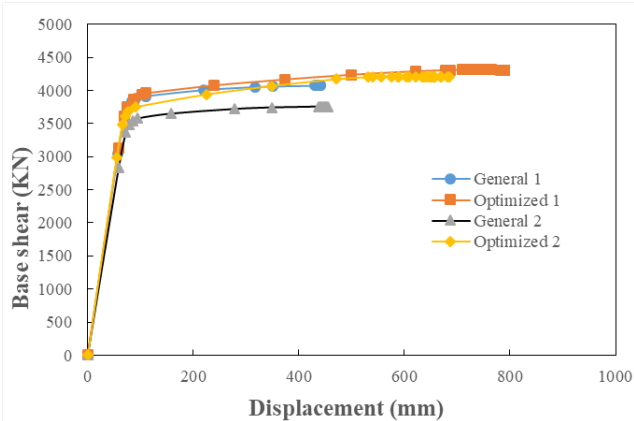


Figure 10. Base shear vs Displacement in Y direction.

Table 6. Maximum storey drift

Model	Maximum storey drift	
	X direction	Y direction
General model 1	0.001841	0.001772
Optimized model 1	0.00182	0.001585
General model 2	0.001851	0.001809
Optimized model 2	0.00191	0.001517

Table 7. Pushover results for Model 1.

Model	General 1		Optimized 1		
	Direction	X	Y	X	Y
Max Base shear (KN)		4009.24	4069.84	3960.05	4312.68
Max Displacement (mm)		478.91	437.09	489.57	789.28
Ductility ratio		5.19	5.53	4.9	8.39

Table 8. Pushover results for Model 2.

Model	General 2		Optimized 2		
	Direction	X	Y	X	Y
Max Base shear (KN)		3714.39	3758.41	3535.93	4210.22
Max Displacement (mm)		435.35	443.5	453.18	684.86
Ductility ratio		4.73	5.56	4.65	7.82

#### 4.4 Cost Analysis

The cost of reinforcing steel and concrete were calculated after taking the quantity. The quantity of materials are shown in Table 9. Steel quantity for optimized models were seen higher due to decrease in sectional area for columns. The basic rate for steel is ₹52/kg while for concrete, it is ₹7000/m<sup>3</sup> at Kerala, India. The cost of materials (refer Table 10) used in the moment resisting frame was calculated and compared. The Optimized asymmetric model 1 showed a 0.35% reduction in overall cost while Optimized asymmetric model 2 showed a 0.84% reduction in overall cost. Hence Optimized asymmetric models were having slightly lesser cost compared to General asymmetric models.

Table 9. Quantity of materials

Model	Vol of concrete (m <sup>3</sup> )		Vol of steel (kg)	
	Column	Beam	Column	Beam
General 1	217.5	261.25	19833.23	23553.53
Optimized 1	209.4	259.06	20941.56	23446.43
General 2	202.5	238.75	19268.51	22193.39
Optimized 2	194.4	235.5	21269.49	20931.23

Table 10. Cost of materials.

Model	Cost of Concrete (₹)	Cost of steel (₹)	Total (₹)
General 1	3351250	2256112	5607362
Optimized 1	3279238	2308175	5587413
General 2	3088750	1552020	4640770
Optimized 2	3009300	1592569	4601869

## 5 CONCLUSIONS

A method for preliminary arrangement of columns with variable sizes on an asymmetric plan was proposed from this study. If columns with variable sizes are not arranged properly, it can result in high torsion. The effect of torsion was evident from the results of linear dynamic and non-linear static analysis.

The general asymmetric models used in this study had very less torsion along X direction and comparatively higher torsion along Y direction. After optimization, 82% of torsion along Y direction was reduced.

The storey drifts along the Y direction, having more torsion than X direction, has been observed to be reduced after optimization. Along the X direction, which is having very less torsion compared to Y direction in general asymmetric models, the optimized models showed comparable results for storey drifts.

The pushover analysis further strengthened the results from response spectrum analysis. The stiffness and ductility of the models along Y direction was effectively increased while, along X direction, comparable results were seen after optimization. The optimized models showed 32% increase in ductility along the Y direction.

Although the cost variation of materials required seemed to be insignificant (refer Table 10), the optimized model showed improved seismic performance.

The approximate method for time period calculation showed 98% accuracy for the first two translational modes while it showed 91% accuracy for the third torsional mode.

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**Appendix I. – Equations used for the approximate calculation of time periods**

$$GA = \frac{12E}{h \left( \frac{1}{\Sigma_1^n (\frac{I_x}{h})} + \frac{1}{\Sigma_1^{n-1} (\frac{I_y}{l})} \right)} \quad (1)$$

(Smith and Crowe, 1986)

*GA* - Shear rigidity of the frame considered.

$$\bar{x}_s = \frac{\sum_j \bar{y}_j GA_{x,j}}{\sum_j GA_{x,j}}, \bar{y}_s = \frac{\sum_j \bar{x}_j GA_{y,j}}{\sum_j GA_{y,j}} \quad (2)$$

(Kuang and Ng, 2000)

$\bar{x}_s$  - X co-ordinate of shear center from origin.

$\bar{y}_s$  - Y co-ordinate of shear center from origin.

$$r_m^2 = \frac{L^2 + B^2}{12} + x_c^2 + y_c^2 \quad (3)$$

(Kuang and Ng, 2000)

$r_m^2$  – Inertial radius of gyration.

$$\begin{vmatrix} b^2 + \lambda_x^2 & 0 & -y_c \lambda_x^2 \\ 0 & b^2 + \lambda_y^2 & x_c \lambda_y^2 \\ -y_c \lambda_\phi^2 & x_c \lambda_\phi^2 & r_m^2 (b^2 + \lambda_\phi^2) \end{vmatrix} = 0 \quad (4)$$

(Rafezy et al., 2007)

Eqn. (4) is a cubic equation in frequency parameter  $b^2$  having 3 negative real roots.

$$\omega_j^{(k)} = \left( k - \frac{1}{2} \right) \frac{\pi}{b_j} \quad (5)$$

(Rafezy et al., 2007)

$\omega_j^{(k)}$  – Natural frequency

$$T_j^{(k)} = \left( \frac{2\pi}{\omega_j^{(k)}} \right) \quad (6)$$

$T_j^{(k)}$  – Natural Time period

**Appendix II. – Matlab output after optimization**

For the optimization of General asymmetric models, 50 million samples were used. The output after the optimization for Model 1 and Model 2 are given in Table A and Table B respectively.

Table A. Matlab output of optimized asymmetric model 1

X co-ordinate (m)	Y co-ordinate (m)	Column size
18	0	800×300
24	0	400×600
30	0	800×300
36	0	400×600
18	4	800×300
24	4	600×400
30	4	300×800
36	4	500×500
0	8	300×800
6	8	800×300
12	8	300×800
18	8	300×800
24	8	300×800
30	8	800×300
36	8	800×300
0	12	500×500
6	12	300×800
12	12	800×300
18	12	300×800
24	12	800×300
30	12	300×800
36	12	300×800
0	16	300×800
6	16	400×600
12	16	800×300
18	16	600×400
24	16	300×800
30	16	800×300
36	16	300×800

Table B. Matlab output of optimized asymmetric model 2

X co-ordinate (m)	Y co-ordinate (m)	Column size
18	0	300×800
24	0	300×800
30	0	800×300
36	0	300×800
18	4	300×800
24	4	300×800
30	4	800×300
36	4	300×800
0	8	300×800
6	8	300×800
12	8	800×300
18	8	300×800
24	8	800×300

30	8	300×800
36	8	300×800
0	12	300×800
6	12	300×800
12	12	300×800
18	12	800×300
24	12	800×300
30	12	300×800
0	16	300×800
6	16	300×800
12	16	800×300
18	16	800×300
24	16	300×800
30	16	300×800

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