Prediction of Railway Vehicles' Dynamic Behavior with Machine Learning Algorithms

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ABSTRACT: The dynamic performance of railway vehicles needs to be carefully monitored to ensure their safe operation. Presently a number of systems such as the Vehicle Track Interaction Monitor and the Instrumented Revenue Vehicles, utilize a number of on-board inertial sensors to obtain near-real time information on the dynamic performance of railway vehicles. These systems provide rich data sets that give an indication of the underlying track condition and the corresponding dynamic response. This paper outlines the use of Machine learning to develop dynamic behavior predictive models for railway vehicles from measured data. This study worked on the development of 2 types of predictive models, viz. regression and classification model. The regression model predicted the time series dynamic response amplitude and the classification model classified the track sections based on the response distribution over it. Train speed and parameters estimated from the unsprung mass were used as predictors in the model. After the trial of a number of predictive models the Ensemble Tree Bagger method was found to have highest overall prediction accuracy. These predictive models can be utilized as a decision making tool to determine safe operational limits and prioritize maintenance interventions.

1 INTRODUCTION

Maintenance expenses constitute a significant fraction of the operational costs in managing railway assets. A conventional approach to maintenance planning is the use of a Track Recording Vehicle (TRV) (Gikas 2005). The instruments on board measure a range of track parameters such as gauge, cross-level and twist as the train runs over the track. These measured parameters are statistically analyzed and compared with thresholds dictated in standards or operational manuals, to decide on the required maintenance priority (Soleimanmeigouni, Ahmadi et al. 2016).

A drawback of this method lies in the use of a passive threshold in making decisions on maintenance interventions. These limits do not completely expose the risk in operation (Liu 2009). It has been shown that a number of maintenance interventions based on geometry measurements have not resulted in any improvement in performance (Dingqing Li 2005). Further, even when individual defects are within the set threshold limits, some combination of defects still results in poor performance and in some cases derailment (Lupton 2003). A more appropriate parameter for the assessment of risk in railway oper-

ations is the dynamic response of the track vehicle (Kraft, Causse et al. 2017). The dynamic response of the track vehicle is a function of the operational speed, loading condition, underlying track features and the wheel rail interface. Thus the dynamic response, gives a direct indication of the operational risks pertaining to the track and operational condition (Zhu, Ahmed et al. 2010).

Instrumented Revenue Vehicles (IRVs), developed by Institute of Railway Technology at Monash University, can be used to achieve a performance based assessment of the track (Glenn Hardie 2015), (Thompson C 2016). An alternative to the direct measurement of the dynamic response of the rolling stock is the use of multibody dynamic simulation (Luber, Haigermoser et al. 2010). The challenge in developing such models is the lack of information on various parameters of the rolling stock. Recent advances in Machine Learning make it an ideal tool for the development of dynamic behavior predictive modes (G.M. Shafiullah 2008, Jordan and Mitchell 2015). This paper explores the development of a model that can predict various dynamic responses of the train from the measured track parameters and operational conditions such as speed and loading state.

The paper is organized as follows: Section 2 clarifies the various notations used in this paper. Section 3 outlines the instrumentation setup to collect data for the model development. Section 4 presents the data conditioning for the model. Development of the machine learning models and the comparison of the predicted results with the measured response are made in Section 5. Finally the conclusion is presented in Section 6.

2 NOTATION

$x(t)$: Track vertical displacement profile
$\hat{x}(f)$: Fourier transform of $x(t)$
$\ddot{x}(t)$: Axle box vertical acceleration
$\hat{\ddot{x}}(f)$: Fourier transform of $\ddot{x}(t)$
f : Frequency of signal
C : Track Curvature
r : Radius of curvature
$\dot{\psi}$: Yaw rate
v : Longitudinal Velocity of train
L_a: Lateral alignment
s : Distance along track chainage
$SND\ Raw_{(Left/Right)}$: Measurement from the Spring Nest Displacement sensor affixed on the left/right axle box.
CL: Cross Level
φ : Bogie roll
w : Distance from the bogie center to SND sensor attachment point

3 INSTRUMENTATION

For the purpose of this project a passenger wagon was instrumented in Indonesia (Chong, Awad et al. 2017, Lingamanaik, Thompson et al. 2017, Ravitharan, LaBrooy et al. 2017). The mainline speed was around 60 Km/h. Inertial sensors were mounted on the unsprung and sprung mass (Lingamanaik, Thompson et al. 2017) to determine the track condition and the dynamic response of the wagon respectively. A differential Global Positioning System (GPS) was affixed to the wagon to determine the position of the train and its' speed. All instruments were powered from the on-board power supply.

Figure 1 shows the instrumented passenger wagon. Dynamic behavior of the wagon was measured using accelerometers, roll, yaw and pitch rate sensors mounted to the undercarriage of the wagon at its center. The accelerometers on the axle box were used to measure the underlying vertical profile of the track.

The volume of the data and the power consumed was managed by programming the data acquisition system to switch off when the train was idle for a prolonged period of time. All recorded data were time aligned with the readings from the differential Global Positioning System which allowed geospatial information to be mapped to locations on the track.

The instrumentation was in operation for 10 normal service runs between Surabaya and Lamongan. Figure 2 shows the track along which the instrumented passenger wagon was in operation. The trip
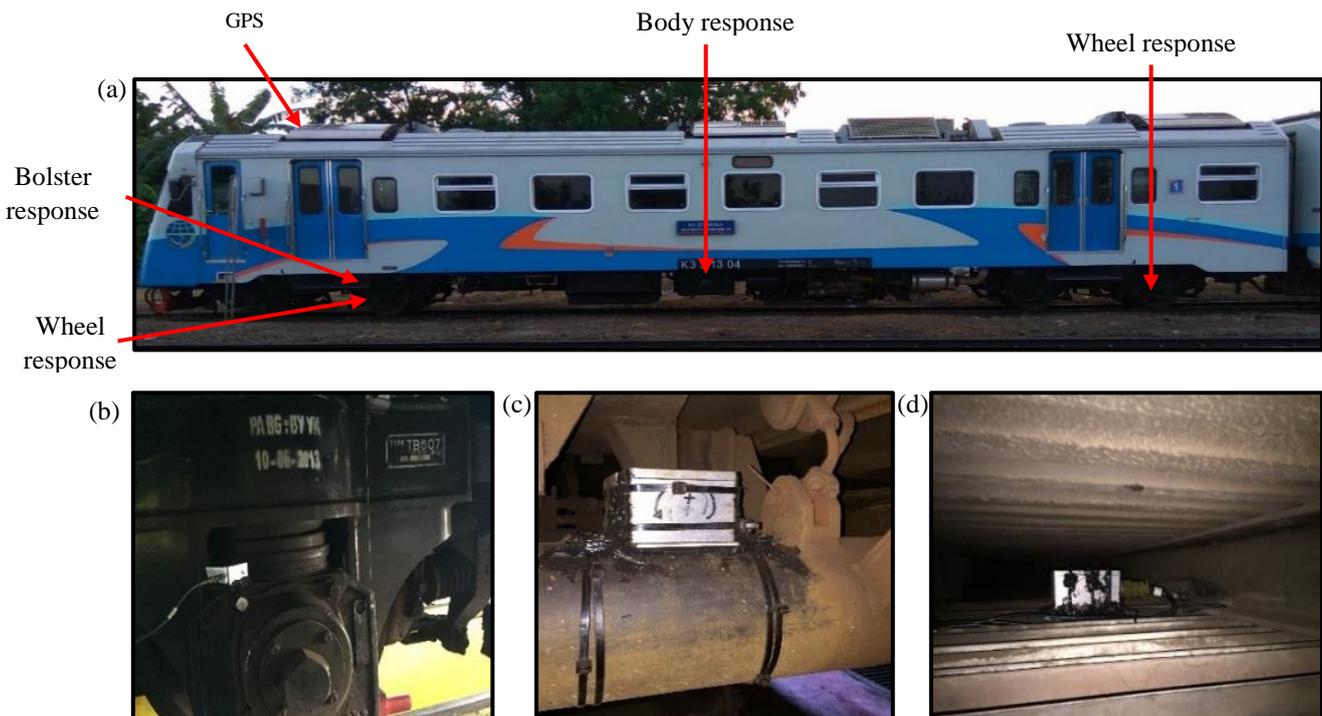
Figure 1. (a) Instrumentation of the passenger wagon to measure track condition and dynamic response. (b) Detailed view of the Spring Nest Displacement Sensor measuring primary suspension behavior.(c) Roll rate sensor to measure the bogie roll. (d) Tri-axial accelerometers and roll rate sensors to measure the dynamic response of the wagon body.

to Surabaya from Lamongan was along the south track and the trip to Lamongan from Surabaya was along the north track.

The instruments were sampling at a rate of 100 Hz. Most dynamic events of interest are only in the order of a few Hz including the natural frequency of the carriages. Thus, a sampling frequency of 100 Hz was sufficient (Ju, Lin et al. 2009). The recorded data from the sensors were wirelessly transmitted to the Institute of Railway Technology facility at Monash University through 4G telecommunication networks. This gave a near real time assessment of the rolling stock and rail performance.
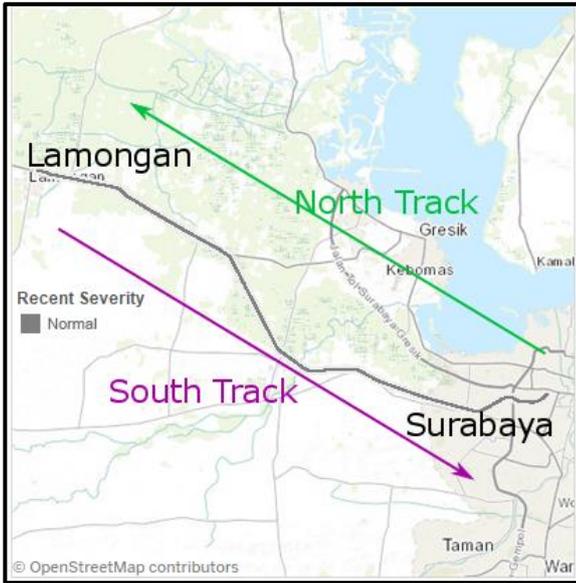
Figure 2. Track Map showing the service line of the instrumented passenger wagon.

4 DATA PROCESSING

The raw data from the sensors were processed to be used for machine learning purposes. A number of filtering and smoothing operations were performed

model were track vertical profile, curvature, track centerline alignment and speed.

The track vertical profile is determined from the accelerometer readings on the axle box. Due to the high relative stiffness of the rail and wheel, the axle box accelerometer measurements can be seen as a direct response to the track vertical profile. Thus the track vertical profile can be recovered from the accelerometer measurement through a double integration scheme as shown in Equation 1(Glenn Hardie 2015).

$$x(t) = \iint \ddot{x}(t)dtdt \tag{1}$$

$x(t)$ and $\ddot{x}(t)$ in Eq. 1 refer to the track vertical profile and the vertical acceleration measurement of the axle box respectively. The vertical profile of the track can be calculated in the frequency domain using Eq. 2.

$$\hat{x}(f) = \hat{\ddot{x}}(f)/(i2\pi f)^2 \tag{2}$$

f in Equation 2 refers to frequency and $\hat{x}(f)$ and $\hat{\ddot{x}}(f)$ refer to the Fourier transform of the track vertical profile and the acceleration measured at the axle box respectively. In theory both Equation 1 and Equation 2 should yield the same result for the track vertical displacement profile. However, the standard integration schemes employed to solve Equation 1 introduces inaccuracies that accumulates (Han 2003). On the other hand evaluation in the frequency domain has been shown to have greater accuracy and flexibility in control.

Another practical consideration was the sensitivity of the accelerometer. At low speeds the magnitude of the axle box acceleration measurements was small and submerged in the noise floor of the measurement. For this reason, the track vertical profile

Figure 3:   Track vertical profile calculated from the axle box accelerometer readings

to remove drifts in measurements and to focus on frequency content of interest. Most dynamic events of importance for the wagon are under 10 Hz, and this guided the frequency choice (Wolfs, Bleakley et al. 2006). Input parameters to the machine learning

was not evaluated over sections where the train speed was less than 30km/h (Glenn Hardie 2016). Additionally, given the length of the carriages (~17m) and the typical speeds of operation, extremely long wavelength features (>50m) are left out of

the calculation (Glenn Hardie 2016). Long wavelength features are typically part of the track design and not indicative of a defected condition of the track. The evaluated track vertical profile for a section of a rail is shown in Figure 3.

The track curvature was calculated from the yaw rate sensors and the speed measurement as shown in Equation 3. Yaw rate sensors were mounted on the bolster as shown in Figure 1 (c). C in Equation 3 indicates the track curvature which is the reciprocal of the radius of curvature ($1/r$). $\dot{\psi}$ indicates the yaw rate and v the speed of the train. Figure 4 shows the curvature over a section of the track.

$$C = \frac{1}{r} = \frac{\dot{\psi}}{v} \tag{3}$$

The lateral alignment is another crucial parameter to be included in the model. Lateral alignment defects can result in adverse dynamic response of the wagon body. Conventionally this parameter is measured using lateral accelerometers attached to the axle box. However, literature sources indicated that the use of a yaw rate sensors to calculate the alignment is a more robust method (Weston, Ling et al. 2007).

This is due to the fact that the yaw rate sensor is not influenced by roll unlike lateral accelerometer sensors. Equation 4 outlines the calculation of the alignment from the yaw rate sensor measurement. L_a in Equation 4 indicates the alignment and s indicates the track chainage length. Figure 5 shows the align-ment over a section of the track.

$$L_a = \iint \frac{\dot{\psi}}{v} ds \, ds \tag{4}$$

Cross level - CL (the height difference between the left and right rails on a track, is another parameter that strongly influences the response of the wagon. There were a number of spring nest displacement (SND) sensors, installed between the bogie and the axle box as shown in Figure 1 (b). The readings of these sensors were a combination of relative extension/compression between the bogie and the wheelset, roll of the bogie and roll of the wheelset. This is illustrated in Figure 6.

The mean measurement of the SND sensor on the left and right wheels of a wheelset gives the measurement of the relative extension/compression between the bogie and the set ($mean(SND_{left}, SND_{right})$). The measurement of the roll rate sensor can be integrated to measure the absolute roll (φ) of the bogie. The roll was then used to determine the displacement contribution at SND from the bogie roll ($SND_{Broll} = w \times \sin(\varphi)$). w is the distance from the center of the bogie to the

attachment point of the SND sensor. Knowing the relative extension/compression between the bogie, the roll of the bogie and the raw measurement of the SND sensor, the roll of the cross level between the left and right rail can be calculated as shown in Equation 5. Figure 7 shows the calculated cross level

Figure 4:   Calculated curvature over a section of the track
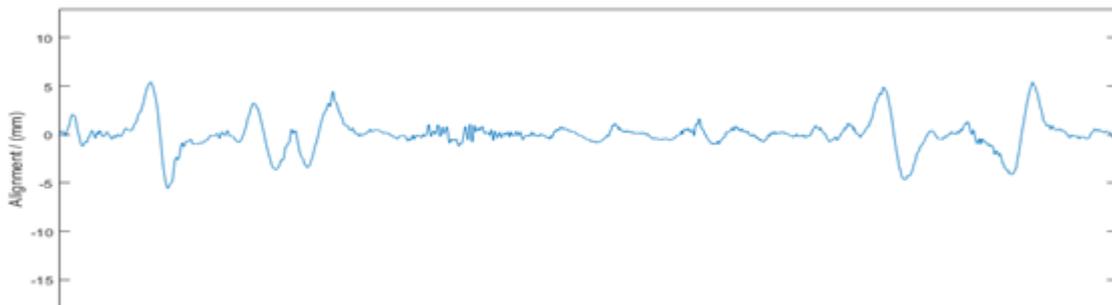
Figure 5:   Calculated alignment over a section of the track

over a section of the track.

$$CL = \left(SND\,Raw_{left} - mean(SND_{left}, SND_{right}) - SND_{Broll}\right) + \left(SND\,Raw_{right} - mean(SND_{left}, SND_{right}) + SND_{Broll}\right) \tag{5}$$

The speed of the train was obtained from the measurement of the differential GPS. The dynamic response of the wagon was measured using accelerometers and roll rate sensors mounted on the undercarriage of the wagon as shown in Figure 1 (c). The measured data was smoothed and filtered to frequency range of interest. Critical dynamic events are often in the low frequency range.

5 DEVELOPMENT OF PREDICTIVE MODEL

Dynamic predictive models can range in complexity and prediction accuracy. Categorically they can be termed as white, grey and black box models. Multibody kinematic models are typical white box models (Blundell and Harty 2004) and provide great insight to the dynamic behavior of the track vehicle. However, these models can be tedious to develop and

some information of the rolling stock and rail that are typically required for the modelling are difficult to source. Additionally they are also often computationally costly to execute. Grey box models do not require the same level of detail for development. Generally they are faster to execute but, they do not provide the same level of insight as the white box models.

Black box models are those that reveal least insight into the relationship between the track condition and dynamic response. These are data driven models in which relationships between inputs and output is determined using Machine Learning techniques (D Li 2006, Guler 2014). The appeal of this modelling process is the fact that an in-depth understanding of the kinetics of the dynamic system is not necessary for the development of the model.

As outlined in the earlier section a number of track and operational parameters can be determined from the instrumented wagon. A significant advantage of being able to obtain all the input and output parameters from the one system is synchronization between the measured signals.

Two different black box machine learning predic-
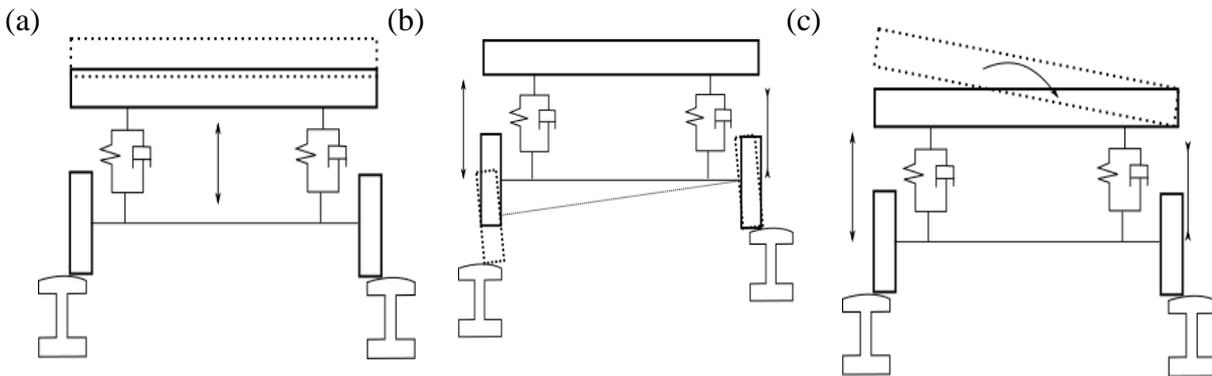
Figure 6: Primary suspension motion represented as a sum of (a) vertical compression/extension (b) Wheelset Roll (c) Bolster Roll
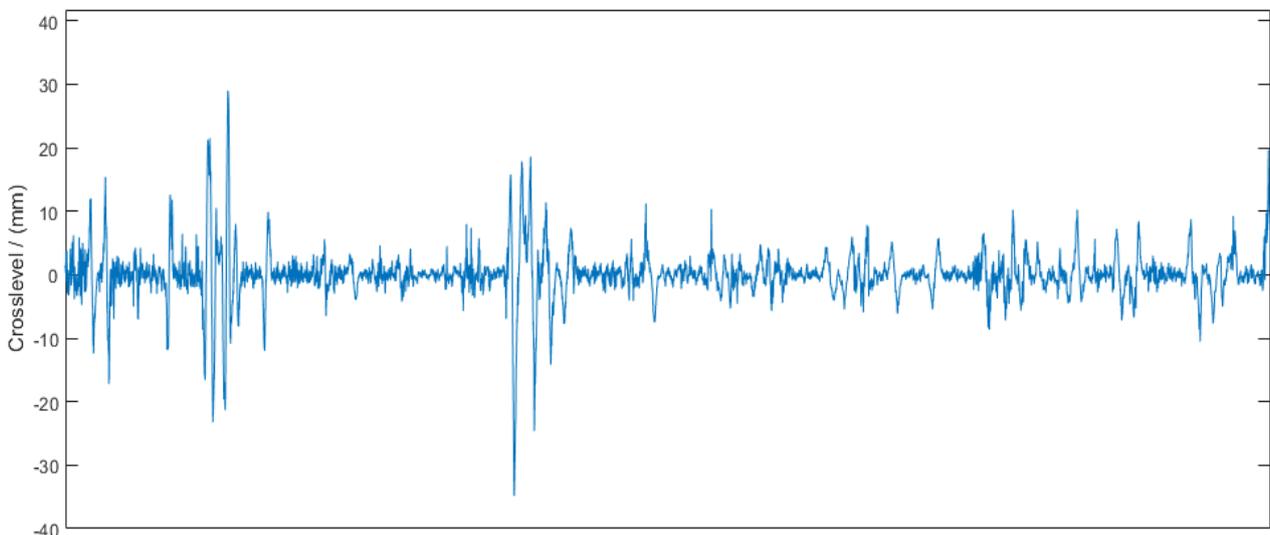
Figure 7: Calculated Cross level over a section of track

tive models were developed as part of the work discussed in this paper. A classification model that categorizes the response over 50 m sections of the track into 4 distinct classes based on peak dynamic response over that section. A regression model that predicts the dynamic response of the rolling stock over the entire length of the track. The models were validated by comparing the dynamic behavior prediction, over a section of the track that was not used in the training process (validation dataset), to the measured responses. Both models utilized a supervised learning scheme where the classes of the classification model and the measured dynamic response of the wagon was supplied for training purposes.

5.1 Classification model

The determination of the maintenance activities on a track, requires the establishment of a good condition indicator value. The indicator value often corresponds to the performance of the train over 50 to 100m sections of track (Soleimanmeigouni, Ahmadi et al. 2016). The model was trained to predict the severity class of the track section from severity 1-4, with severity 4 being the section that induced the lowest amplitude response on the wagon and severity 1 inducing the highest amplitude response. The classification were based on the maximum response over the section. The classification model utilizes the speed, track vertical profile, curvature, alignment and cross level as predictors. Knowing that the frequency content of the predictors plays a significant role in the dynamic behavior of the wagon, the amplitude of the predictors at frequencies 0.5Hz to 10Hz in 0.5Hz increments were also used as inputs to the model. Other statistical parameters such as the maximum, minimum, mean, standard deviation, and root mean square of the track parameters was also used as the input. The model was trained with data from the north track. The south track data was used to validate the model.

The Classification Learner Toolbox in Matlab was used to build the predictive models. A number of different predictive algorithms like Support Vector Machine, Linear Regression, Tree Bagger and Nearest Neighbor were tested. The Tree Bagger model's performance was much better than the other models tested. The Confusion Matrix of the validation dataset was used to quantitatively evaluate the performance of predictive models. A number of optimization techniques were utilized to improve the results of the prediction. Cost matrices were used to minimize the predictions that were off by more than one class. Additionally, it was found that using 70% quantile instead of the maximum response over a section improved the accuracy of the prediction.

Table 1 shows the prediction accuracy of the bounce classification model. The model performs

exceptionally well in predicting the class 4 section of the track. 89% of the time the model was capable of correctly recognizing a severity class 4 section of the track. It is also worth noting that even in the cases where the class was not correctly recognized, most prediction still fall within a single class on either side. For example, 99% of class 3 predictions fall in class 3 or one class on either side of class 3.

Table 1 Confusion Matrix showing the performance of the bounce classification model

Actual Class		Predicted Class		
	4	3	2	1
4	**89%**	7%	3%	1%
3	29%	**38%**	32%	<1%
2	7%	3%	**88%**	2%
1	14%	1%	59%	**26%**

The performance of the model in predicting the Class 1 severity was poorer than the other classes. This is because the training set has limited class 1 data for training. However, here again most prediction are between Class 1 and 2.

Table 2 gives the confusion matrix showing the performance of the roll prediction model developed. Like the earlier model for bounce, this too has high accuracy in predicting all classes but 1. The model only correctly identifies severity 3 15% of the time. However, once again 99% of predictions fall within 1 class on either side of class 3.

Table 2 Confusion Matrix showing the performance of the roll classification model

Actual Class		Predicted Class		
	4	3	2	1
4	**82%**	6%	12%	<1%
3	38%	**15%**	47%	<1%
2	13%	7%	**75%**	5%
1	15%	1.5%	47%	**50%**

The confusion matrices from the prediction of lateral motion and pitch is given in Table 3 and Table 4.

Table 3 Confusion Matrix showing the performance of the lateral motion classification model

Actual Class		Predicted Class		
	4	3	2	1
4	**95%**	5%	<1%	<1%
3	64%	**27%**	8%	1%
2	14%	10%	**64%**	11%
1	1%	1%	20%	**78%**

Table 4 Confusion Matrix showing the performance of the pitch classification model

Actual Class		Predicted Class			
		4	3	2	1
4		**87%**	9%	3%	1%
3		51%	**30%**	13%	5%
2		9%	7%	**61%**	22%
1		2%	1%	19%	**78%**

5.2 Regression Model

A supervised regression predictive model was trained to predict the dynamic response of the passenger wagon. After the trial of a number of machine learning models, the Tree Bagger method was chosen as the best model for the purpose. As in the classification learner, a number of statistical parameters such as the mean, standard deviation, root mean square, minimum and maximum were used for the model. In addition the measured track parameters and the speed over the section of the track were also used as predictors in the model. The Principal Component Analysis (PCA) coefficients of inputs were also used as predictors. PCA redistributes the variance in the data into a number of components, with the first component accounting for most of the variance.

Another key consideration in the development of a regression predictive model was the length of the input data that has to be considered to predict the response at a particular point. There are 3 factors that influence this choice:

1 The time for the dynamic response of the train to decay.
2 The total length of the wagon.
3 Position of the wagon in consideration within the train.

The length of the instrumented passenger carriage was approximately 17m with 4 wagons in the train. Other studies have shown that impact of the irregularity excitement on the train wagon vanished after 40 m (Guler 2014). Thus, the track data condition 40 m ahead of/ behind the instrumented wagon was used for the prediction.

Further, optimization studies were undertaken to determine the optimal size of the track section to be used in the prediction. The results showed that for the purposes of this study a window size of 500 data points (at 100 Hz sampling) gave the best prediction results. The best choice of the number of branches in the tree bagger model was also explored. The mean square error did not show significant improvement when the branch number was increased above 25 (Table 5.). The results obtained through these models were seen to be able to predict major dynamic events. Figure 8 and Figure 9 compare the prediction
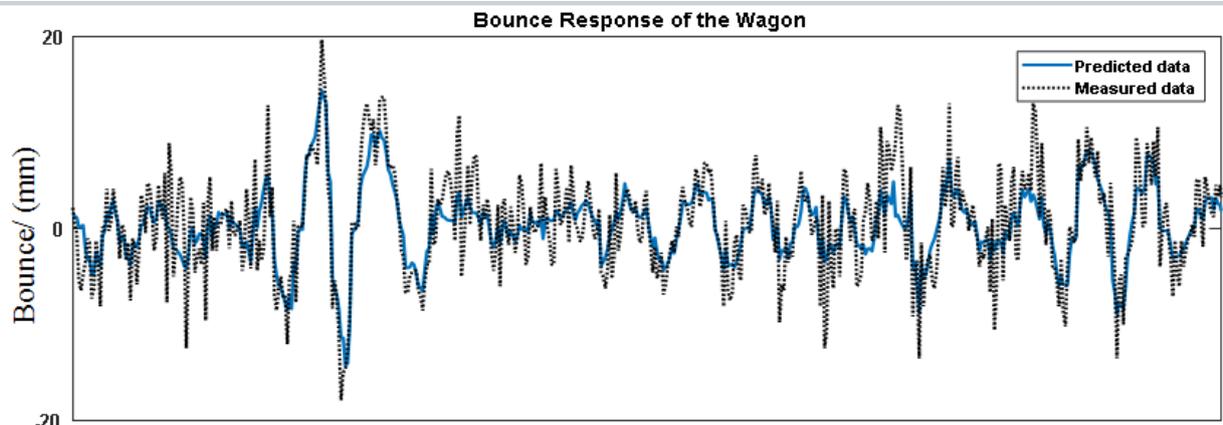
Figure 8: Comparison of the regression model's bounce prediction with the validation data
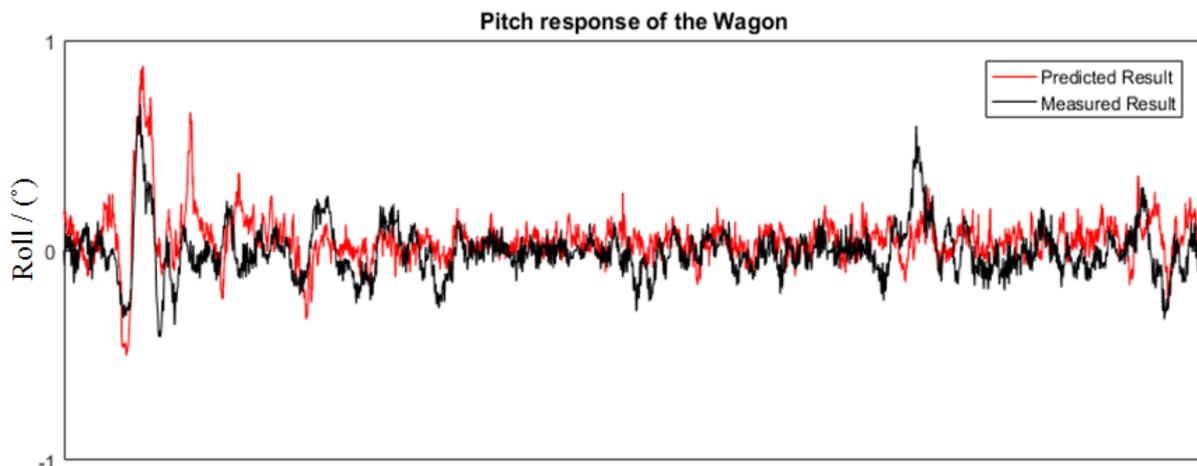
Figure 9: Comparison of the regression model's pitch prediction with the validation data

of bounce and pitch response with the validation data set.

Table 5 Optimization study to determine the number of branches to minimize MSE of regression prediction

No. of branches							
10	25	50	100	250	500	1000	2500
mse 5.42	4.82	5.48	5.42	5.18	5.13	5.14	5.19

The comparison of the predicted response with the measured response shows the ability of the model to predict the high amplitude low frequency events. In practice these are the events of high relevance when evaluating the risks in operations.

6 CONCLUSION

This paper explores the use of machine learning in developing dynamic behavior predictive models. A passenger wagon in Indonesia was temporarily instrumented for the purpose of this study. Data from 10 in service runs were utilized for the development of the models. Two distinct model types were developed; a classification and a regression model. The classification model assigns a class for every 50 m section of the track based on the maximum predicted response in the section. The regression model predicts the dynamic response of the wagon over the entire length of the track. The models were validated by comparing the predicted response, over a section of the track not utilized in training, with the measured response.

Confusion matrices were used to determine the effectiveness of the classification model and the mean squared error was used to determine the accuracy of the regression model. A number of machine learning algorithms were tested and the ensemble tree baggers method gave the most accurate predictions for both the classification and the regression models. Grid search was performed to optimize the meta-parameters of the machine learning algorithm.

The classification model predictions were within one class of the correct class which gives a qualitative indication of the status of the track. The regression model were able to adequately predict low frequency high amplitude events of interest. The natural frequencies of the system are less than 5 Hz, thus the model has high practical relevance.

Work is in progress to further enhance these models. The models are extremely useful in predicting ride comfort and safety in railway lines. They can also be used to evaluate the quantitative effect of temporary speed restriction on the dynamic response of the train.

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