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Use of Modal Flexibility Method to Detect Damage in Suspended Cables and the Effects of Cable Parameters

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ABSTRACT: Modal flexibility is a widely accepted technique to detect structural damage using vibration characteristics. Its application to detect damage in long span large diameter cables such as those used in suspension bridge main cables has not received much attention. This paper uses the modal flexibility method incorporating two damage indices (DIs) based on lateral and vertical modes to localize damage in such cables. The competency of those DIs in damage detection is tested by the numerically obtained vibration characteristics of a suspended cable in both intact and damaged states. Three single damage cases and one multiple damage case are considered. The impact of random measurement noise in the modal data on the damage localization capability of these two DIs is next examined. Long span large diameter cables are characterized by the two critical cable parameters named bending stiffness and sag-extensibility. The influence of these parameters in the damage localization capability of the two DIs is evaluated by a parametric study with two single damage cases. Results confirm that the damage index based on lateral vibration modes has the ability to successfully detect and locate damage in suspended cables with 5% noise in modal data for a range of cable parameters. This simple approach therefore can be extended for timely damage detection in cables of suspension bridges and thereby enhance their service during their life spans.

Keywords: Modal flexibility, damage detection, lateral modes, vertical modes, damage indices, noise, bending stiffness, sag-extensibility

1 INTRODUCTION

Advances in civil engineering and material technology have resulted in increasing applications of large diameter and long span cables as key structural components in cable supported structures including suspension and cable stayed bridges, overhead transmission lines, cable supported roofs, and guyed towers. Among those structures, suspension bridges are increasingly used in today's infrastructure systems due to their cost effective structural form (longer main spans), lightness (less material requirement, flexibility) and aesthetics. However, they accumulate damage over the time of their life-cycle due to environmental influences (weather changes, temperature fluctuations and moisture levels), changes in load characteristics (heavier and faster moving traffic) and random actions. Consequently, these factors cause imperfections in the cable protection system allowing moist air and water to enter into the interior of the cable and exposing it to wetness for long periods of time. This can cause reductions in cable diameter up to 30% [1] due to severe corrosion. Main cables are therefore, critical elements for the overall structural performance and safety of the suspension bridges. They also susceptible to long term fatigue damage after many years in service. Damage detection is therefore a priority in terms of optimal allocation of public resources for retrofitting and maintenance of such structures. Among different damage diagnosis approaches, vibration based damage detection (VBDD) techniques have received increased attention particularly due to their relative simplicity and the moderate cost of dynamic measurements [2] with many applications in beam and plate elements, trusses and simple structures in reinforced concrete and steel. Very limited research has been carried out to develop formulae for the estimation of the cable tension force in cable supported structures using measured natural frequencies [3, 4].

The modal flexibility method is a VBDD technique based on mode shapes and natural frequencies to detect damage structures. It has been used by a number of researchers for damage localization in beam and plate like structures (Pandy and Biswas, (1994, 1995) [5, 6]; Toksoy and Aktan (1994) [7];

Farrar and Jauregui (1998) [8]; Shih et al.(2009) [9]). Wang et al. (2000) [10] identified that modal flexibility was a sensitive diagnostic indicator compared with other modal indices while Choi et al. (2008) [11] combined changes in flexibility and damage index method to develop a new hybrid algorithm to estimate damage severity in timber structures. Relative flexibility change (RFC) between intact and damaged states of the cable stayed bridge was studied by Ni et al. (2008) [12], whose RFC index was successful in locating damage in single damage scenarios in the absence of ambient effects. However difficulties were encountered in detecting and locating damage in cross girders. Moragaspitiya et al. (2013) [13] predicted the axial shortening of vertical load bearing elements of reinforced concrete buildings using the modal flexibility method. Recently, Sung et al. (2014) [14] developed a method based on modal flexibility to detect damage in cantilever beam type structures. It was successfully applied to identify damage in a 10-storey building by both numerically and experimentally for single and multiple damage cases. The literature confirms that the modal flexibility method has a wide variety of applications in damage diagnosis studies but not in detecting and localizing damage in large diameter and long span cables. This paper will treat the damage detection in such cable structures using the modal flexibility method along with two damage indices based on the vertical and lateral modes of vibration.

Suspension bridges vibrate in lateral, vertical, torsional and coupled modes [15]. Torsional and coupled modes are complex and measurements associated with rotational coordinates are very difficult to obtain practically [16]. However, modal flexibility method requires first few modes to detect damage successfully and also vibration of cable does not include torsional or coupled modes in its first few modes. Therefore, the approach used in the present paper used two DIs, defined by decomposing modal flexibility into two components. One DI is computed from the modal flexibility associated with the vertical modes of vibration and the other is associated with the lateral modes of vibration. Damage detection potential of the two DIs under the effect of random measurement noise is also discussed. Flexural rigidity and sag extensibility are critical parameters for large diameter sagged cables [17] and their influence on the damage detection capability using modal flexibility is examined.

In order to achieve the objective of this study, vibration characteristics of a suspended cable need to be obtained at both damaged and undamaged state, is not available in literature. However, dynamic characteristics of the main span cable of the Tsing Ma Suspension Bridge measured at their erection stage were available and those were used to validate the FE model of a suspended cable. Four damage scenarios were then, simulated in the FE model by varying the stiffness at the particular locations of interest. Modal data (vibration characteristics) generated with damaged and undamaged states of the FE model were used to examine the DIs for large diameter and long span cable. Results confirm that the DI based on lateral modes is better capable of detecting and locating damage successfully in the cable structure compared to the DI based on vertical modes, even in the presence of 5% noise in the modal data. Next, the parametric study evaluated the behavior of the two DIs with different bending stiffness and sag extensibility of suspended cables. In this study, three different sag extensibility values (based on the sag to main span ratio of suspension bridges) and three different bending stiffness values with two single simulated damage cases in the cable are considered. Results reveal that the lateral damage index is applicable for the range of parameters considered to locate damage in the long span suspended cables. Therefore application of this particular modal flexibility based damage index can be extended for damage detection in the main cables of suspension bridges to provide timely retrofitting.

2 MODAL FLEXIBILITY DECOMPOSITION METHOD

Modal flexibility method is a widely accepted technique in damage detection which associates vibration characteristics of a structure including natural frequencies and mass normalized mode shapes. It does not require any analytical model of a structure to evaluate the flexibility and can be used with only the experimental data collected from the structure [5]. Alternatively, modal flexibility of a structure converges rapidly with increasing frequency and can be therefore computed using few lower natural frequencies and mass normalized mode shape vectors [5]. However, online monitoring systems instrumented in large scale structures an only measure ambient vibration response which means mass normalized mode shape data are not available. In order to improve the application of modal flexibility method in large scale structures, many researchers (Doebling and Farrar (1996) [18], Gao and Spencer (2002) [19], Parlooet al. (2005) [20], Yan and Golinval (2005) [21]) developed various methods to calculate the modal flexibility with ambient vibra-

tion measurements with and without use of FEM. The modal flexibility method is widely used in structural health monitoring applications due to its accuracy, convenient computation and ease of application.
Modal flexibility, F_x at a location *x* of a structure can be obtained from [13], as

$$F_x = \sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T \tag{1}$$

Where *i* (*i*=1, 2, 3 …m) is the mode number considered. In complex structures measured modes are less than the analytical modes available, with only a limited number of lower order modes being measured practically [17]. m and ω_i are the total number of modes considered and the natural frequency of the structure at mode *i*, respectively.

When a structure is subjected to damage or deterioration, it reduces the stiffness which alters the vibration characteristics and increases its flexibility. The resulting changes in the modal flexibility of the structure are represented in Eqs. (2) and (3) at damaged and undamaged state of a structure.

$$F_{xD} = \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_D \tag{2}$$

$$F_{xH} = \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_H \tag{3}$$

Here subscripts D and H denote the damaged and undamaged states of the structure respectively. Eq. (4) below captures the change in modal flexibility of the structure due to damage.

$$F_{xD} - F_{xH} = \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_D - \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_H \tag{4}$$

In this study, $F_{xD} - F_{xH}$ is normalized by the F_H and hence the damage index for localizing damage in a structure is written as in Eq. (5).

$$DI = \frac{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_D - \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_H}{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_H} \tag{5}$$

In general, cable structures vibrate in lateral, vertical, torsional and longitudinal modes [15]. However, as it is complex to measure torsional and coupled modes practically, most damage detection methods therefore, incorporate mode shapes that include translational coordinates. This paper presents two damage indices, which are derived by decomposing the modal flexibility of a structure into two parts. One index is based on the structure's vertical mode shapes and the other is based on its lateral mode shapes. Eq. (5) is rewritten according to the mode of vibration considered, as demonstrated in Eqs. (6) and (7). Where subscripts V and L denote the vertical and lateral modes, respectively.

$$DI_V = \frac{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{DV} - \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{HV}}{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{HV}} \tag{6}$$

$$DI_L = \frac{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{DL} - \left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{HL}}{\left[\sum_{i=1}^{m} \frac{1}{\omega_i^2} \phi_{xi} \phi_{xi}^T\right]_{HL}} \tag{7}$$

Since damage alters the stiffness of the structure, theoretically, peaks should appear in the damage index curves defined in Eqs. (6) and (7) corresponding to the damage location. However, mass contribution of the structure to the mode of vibration is different from lateral modes to vertical modes. Modes which have more mass contribution predict damage accurately. Hence it is worthy to study the competency of the DIs based on mode of vibration for damage detection in suspended cables. This paper therefore, evaluates the potential of the DIs defined in Eqs. (6) and (7) for localizing damage in long span large diameter cables, even with few mode shapes identified from damaged and undamaged states.

3 NUMERICAL STUDY

FE models of real structures validated with the measured vibration responses have been extensively used in damage detection studies. It reduces the cost and time associated with experimental testing and difficulties in studying different damage scenarios in real structures. The next section of this paper presents the validation of the FE model of a suspended cable by comparing measured frequencies in the field. That FE model is used to extract the vibration characteristics of a cable at both damaged and intact states for damage detection studies.

3.1 Finite Element Model Validation

In this study, the main span cable of the Tsing Ma Suspension Bridge is simulated numerically in ANSYS FE code. The geometric and mechanical properties of the cable were obtained from Xu et al. [22] and Bouaanani [23]. The main span length and the cable sag are 1397.8m and 112.5m respectively. Other parameters of the cable are modulus of elasticity E=200GPa, horizontal component of the tension force H_s=122.64MN, mass per unit length m=5832 kg/m and cross sectional area A=0.759m^2.

Since this is a large diameter sagged cable, flexural rigidity was taken into account by simulating it using BEAM 188 elements in ANSYS code. Also this element type includes stress stiffness term which is supported for the large deflection effects in cables. It was consider pinned as two ends which are placed at the same vertical elevation. Displacements at ends were therefore fixed in all three directions and rotations were released. The cable was divided into 100 elements of each 13.978m long in the analysis. In order to consider the geometric nonlinearity of cables, the analysis was conducted in two steps. First the nonlinear static analysis under self-weight was performed and the subsequent modal analysis was conducted using the last solution obtained at the end of the static step (pre stressed modal analysis was conducted). Measured frequencies (f_{mea}) of the cable from a series of ambient vibration tests [22] with the numerical (f_{num}) results were compared by equation; $(f_{mea}- f_{num})*100/(f_{mea})$.

Those frequencies were measured during construction when the cable has just been erected between two towers. Table 1 and 2 compares the results and shows that the percentage difference of the FE analysis results as less than 4.5%, confirming the accuracy of the FE model developed and validating it for using in damage detection studies.

Table 1 Comparison of out-of-plane (lateral modes) frequencies of the main span cable of the Tsing Ma Suspension Bridge

Modes	Measurements[23] Hz	FE Analysis Hz	% Error
1(Lateral)	0.0530	0.0535	-0.95
2(Lateral)	0.1050	0.1067	-1.60
3(Lateral)	0.1560	0.1601	-2.60

Table 2 Comparison of in-plane (vertical modes) frequencies of the main span cable of the Tsing Ma Suspension Bridge

Modes	Measurements [23] Hz	FE Analysis Hz	% Error
1(Vertical)	0.1020	0.1030	-0.98
2(Vertical)	0.1430	0.1490	-4.19
3(Vertical)	0.2070	0.2100	-1.44

4 DAMAGE DIAGNOSIS

4.1 Damage Scenarios

The FE model validated in previous section of this paper is considered as the undamaged baseline model. Four damage cases were introduced in the cable model to synthesize the various damage scenarios. Three damage cases are single damage scenarios with 25% stiffness reduction at the mid span, quarter span and near the support of the cable. One damage case has a 10% stiffness reduction in two different locations of the cable to cater for multiple damage scenarios.

Damage in FE models can be simulated by changing the Young's modulus or changing the cross section area or removing elements at damage location. This study simulated damage in the cable model by reducing Young's modulus of the specified elements. Figure 1 and Table 3 represent the exact locations of the damage cases considered in this study. In order to calculate the modal flexibility based DIs defined previously, the first three in-plane (vertical) and out-of-plane (lateral) natural frequencies and mode shapes were extracted from the FE analysis of both damaged and undamaged state of the cable.

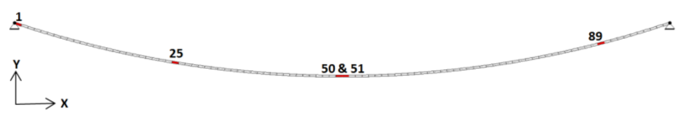

Figure 1. Cable Structure with Damaged Scenarios

Table 3. Damage Cases Considered

Damage Case	Location	Severity of Damage
Single damage scenario		
Damage Case 1	Element 50,51 (X=684.710 to X=713.09)	25%
Damage Case 2	Element 25 (X=332.41 to X=346.36)	25%
Damage Case 3	Element 1 (X=0 to X=13.797)	25%
Multiple damage scenario		
Damage Case 4	Element 25 and Element 89 (X=332.41 to X=346.36) and (X=1245.9 to X=1259.7)	10%

4.2 Damage Detection without Noise in Modal Data

Damage Case 1

The first damage case studied is that in the middle of the cable with a 25% stiffness reduction of the elements 50 and 51. Numerical results of the vertical damage index and lateral damage index are shown in Figure 2 and Figure 3 respectively. Both graphs show the damage index curves reach their maximum values at the nodes of the damaged location. In this case, both damage indices based on the vertical and lateral modes detect the damage successfully at the middle of the cable, and confirm the actual location of the damage considered.

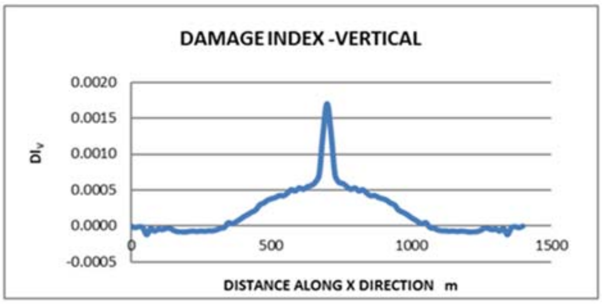

Figure 2. Vertical damage index - Damage Case 1

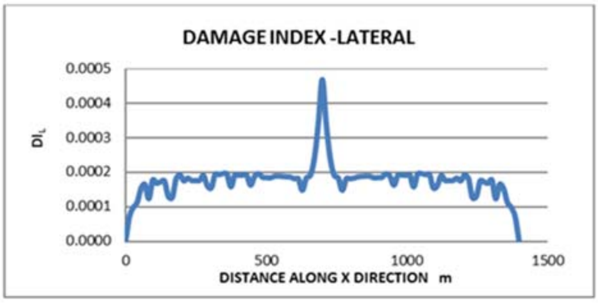

Figure 3. Lateral damage index - Damage Case 1

Damage Case 2

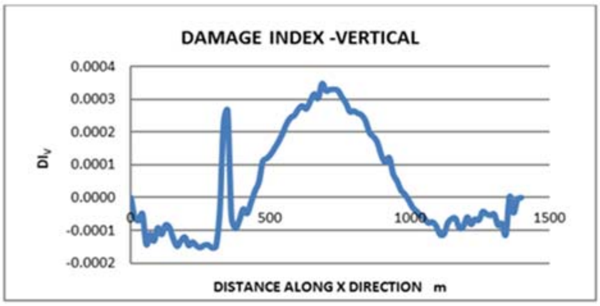

Figure 4. Vertical damage index - Damage Case 2

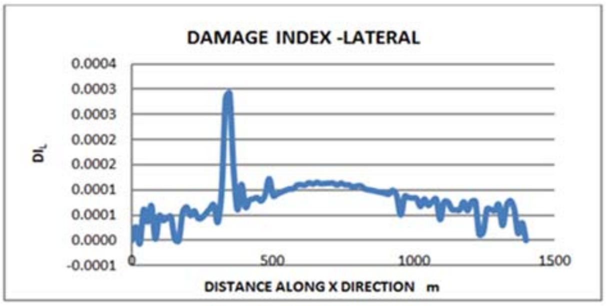

Figure 5. Lateral damage index - Damage Case 2

Figure 4 and Figure 5 show the damage index values for the second damage case with a 25% stiffness reduction in element 25 of the cable. The curve related to the vertical damage indicator peaks not only at the damage location but also at the mid span of the cable. However, the lateral damage index peaks at the exact damage location being considered (element 25). Based on the examination of the two graphs, it can be concluded that incorporating lateral vibration modes for detecting damage in a suspended cable is a successful approach. Therefore, competency of lateral modes in damage detection of suspended cable is further evaluated through Damage Case3 considered in this paper.

Damage Case 3

Two curves of the flexibility indicators for damage case 3 are shown in Figure 6 and Figure 7. Damage Case 3 is simulated in element 1 of the cable model with 25% stiffness reduction. The vertical damage index shows two peaks representing the damage location. However, a plot of the lateral damage index demonstrates only one peak, as in Figure 7 for the exact damage location considered. This again verifies the lateral modes of vibration of a suspended cable have ability to detect damage more accurately than vertical modes.

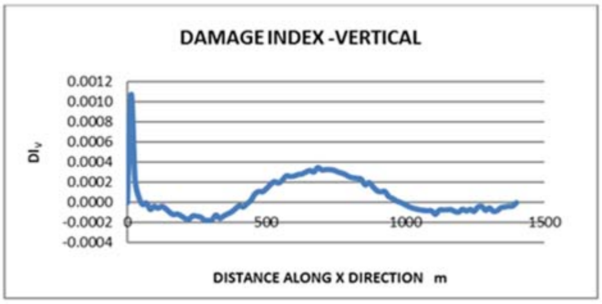

Figure 6. Vertical damage index - Damage Case 3

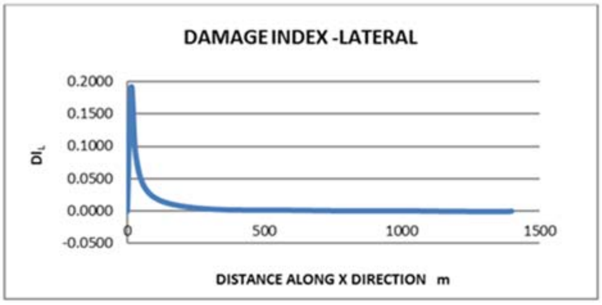

Figure 7. Lateral damage index - Damage Case 3

Damage Case 4

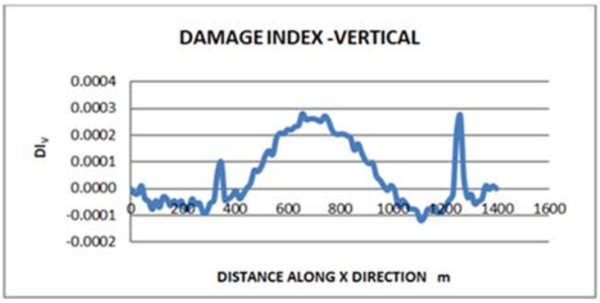

Figure 8. Vertical damage index - Damage Case 4

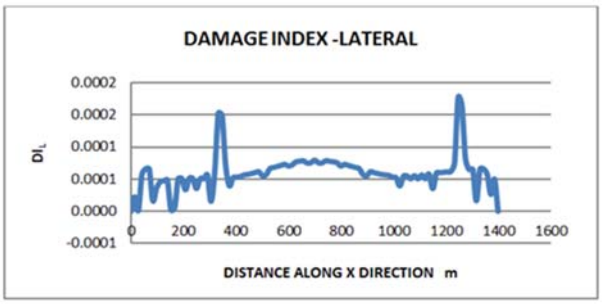

Figure 9. Lateral damage index - Damage Case 4

Damage case 4 is set up to study the damage localization capability of flexibility indicators in multiple damage cases. Element 25 and 89 are subjected to a 10% stiffness reduction in the cable model to simulate damage. The behavior of the vertical and lateral DIs are illustrated in Figure 8 and Figure 9, respectively. Vertical damage index shows three maximum points (peaks) in the graph, of which two represent the damage location and the other is a false alarm. The graph of the lateral damage index has two sharp peaks which correspond to the actual damage locations considered in the simulations. An analysis of the two graphs identified that the lateral vibration modes have better damage detection capability than vertical modes in cable structures.

It is clearly evident that the damage index based on the lateral modes is capable of detecting and locating damage under single and multiple damage scenarios. The use of this damage also enables to reduce the false alarms. The next section examines the competency of both damage indicators under the influence of measurement noise.

4.3 Influence of Measurement Noise in Damage Detection

In reality, measured vibration responses are associated with uncertainties such as measurement noise and computational errors in modal frequencies and mode shapes, respectively [24]. It is therefore necessary to examine the performance of the two DIs in the presence of noise in the modal data. In this study, vibration responses are generated using a validated FE model and there is no noise associated with numerical simulation. Also the measurement noise associated with frequency is very low; therefore 5% random noise is introduced to mode shapes which are generated from the FE model. The contaminated signal for mode shape can be represented as [25];

$$\overline{\phi_{xi}} = \phi_{xi}\left(1 + \gamma_x^{\phi}\rho_x^{\phi}\left|\phi_{max,i}\right|\right) \quad (8)$$

Where $\overline{\phi_{xi}}$ and ϕ_{xi} are mode shape components of the ith mode at location *x* with and without noise, respectively; γ_x^{ϕ} is a random number with a mean equal to zero and a variance equals to 1; ρ_x^{ϕ} refers to the random noise level considered and $\phi_{max,i}$ is the largest component in the ith mode shape.

Figures 10 -17 illustrates the damage localization results of the two damage indicators with and without noise. In Damage Case 1 both damage indicators detect the exact damage location successfully. However, all other cases have similar features for the vertical damage indicator as observed earlier (noise free condition). Damage indicator based on lateral vibration modes has very clear sharp peaks at the damage locations in both single and multiple damage cases considered in the cable. Therefore it can be concluded that the damage indicator based on lateral vibration modes performs well for damage localization even in the presence of 5% noise in the mode shape data. This implies the lateral modes are more suitable in detecting damage in a large diameter long span cable.

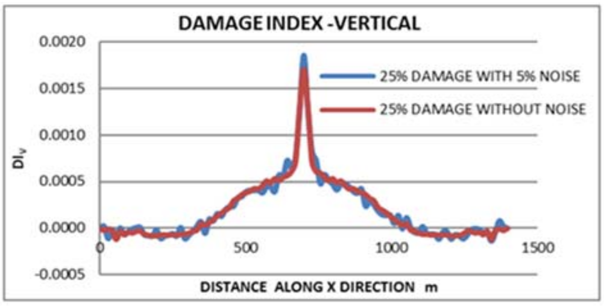

Figure 10. Vertical damage index - Damage Case 1

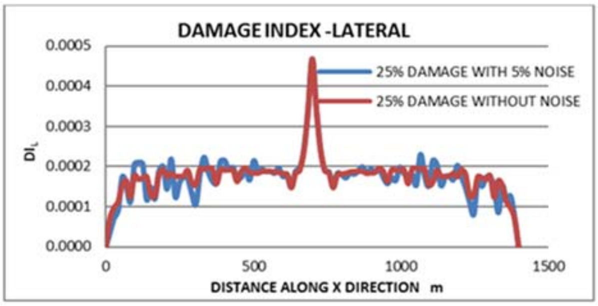

Figure 11. Lateral damage index - Damage Case 1

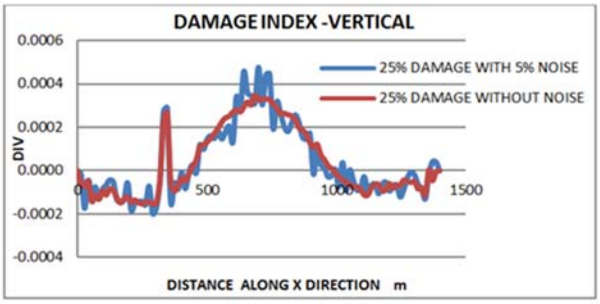

Figure 12. Vertical damage index -Damage Case 2

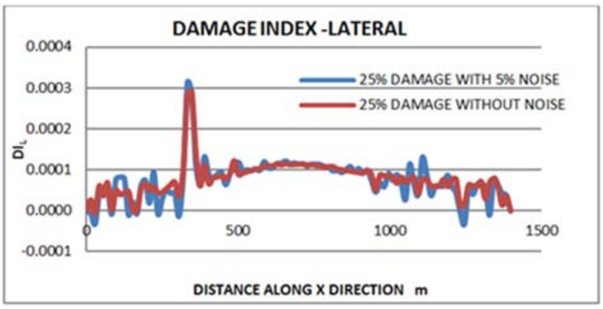

Figure 13. Lateral damage index - Damage Case 2

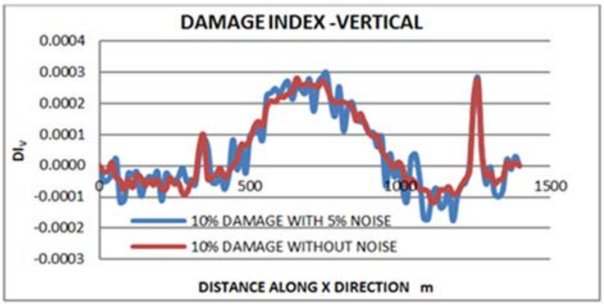

Figure 14. Vertical damage index - Damage Case 3

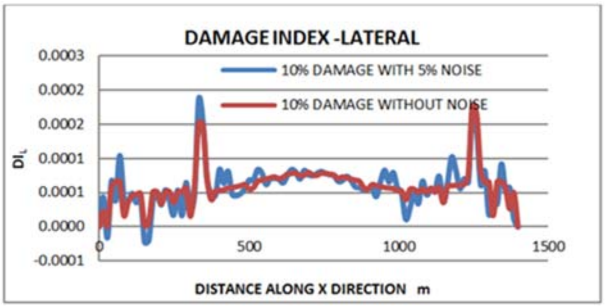

Figure 15. Lateral damage index - Damage Case 3

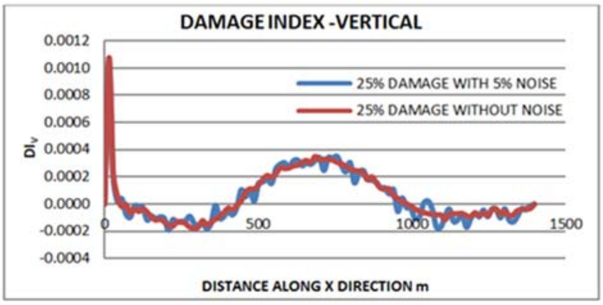

Figure 16. Vertical damage index - Damage Case 4

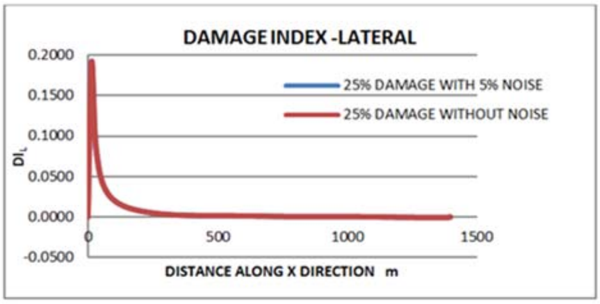

Figure 17. Lateral damage index - Damage Case 4

5 EFFECT OF CABLE PARAMETERS ON DAMAGE DETECTION

Various applications of large diameter suspended cables can be found in civil structures with considerably large sags and bending stiffness, all of which contribute to noticeable changes in the dynamic characteristics. It is therefore worthwhile to examine the interaction of sag and bending stiffness in damage detection of large diameter sagged cables. These two factors are examined by defining two dimensionless parameters ξ and λ^2 characterizing the bending stiffness and sag extensibility respectively.

$$\xi = L\sqrt{\frac{H_s}{EI}} \tag{9}$$

$$\lambda^2 = \frac{EAL}{H_s L_e}\left(\frac{mgL}{H_s}\right)^2 \tag{10}$$

Where *L, E, A,* H_s*, I, m, g* and L_e are the span, Young's modulus, cross section area, horizontal tension, second moment of area, mass per unit length, acceleration of gravity and length of the cable respectively. The bending stiffness parameter (ξ), differentiates the cable and beam actions: when ξ is very small, the beam action predominates, and when ξ is very large the cable action predominates. The sag-extensibility parameter λ^2, accounts for combined axial and geometric stiffness effects [23].

5.1 Cable Model for Parametric Study

To study the influence of different cable parameters on both lateral and vertical damage indicators based on the modal flexibility, cable models with span (1300m) and cross section area (0.75m^2) along with different combinations of sag-extensibility and bending stiffness are considered. Modulus of elasticity of the cable considered is 200GPa. Cables are numerically simulated in the ANSYS FE code as pinned at two ends at the same vertical elevation and are divided into 97 elements. Pre-stressed modal analysis was conducted as previously to consider the effect of geometric nonlinearity of cables.

Two damage scenarios are simulated in the FE model at the middle (Damage Case 1, x=643.22m to x =656.78 m) and quarter span (Damage Case 2, x=319.48m to x =332.87 m) of the cable as shown in Figure 18. For the purpose of comparison, all the cables are subjected to the same intensity of damage by reducing the stiffness 25%.

Figure 18: Cable Structure with Damaged Scenarios for Parametric Study

5.2 Damage Localization and Sag-Extensibility

It is expected that the modal flexibility based damage indicators are influenced by the sag- extensibility of the cables. In this section therefore, interaction of sag-extensibility on damage detection by incorporating lateral and vertical modes of vibration is studied by considering three different sag extensibilities for a suspended cable. Those values are selected according to the wide range of sag to main span length ratios for suspension bridges. The literature identifies the ratio of sag to main span length for suspension bridges as ranging from 0.083 (Tacoma Narrows bridge) to 0.113 (Golden Gate bridge) [26]. Table 4 represents the cable parameters used in this parametric study.

Table 4. Properties of the investigated cables

Cable no.	I (m^4)	H (N)	Sag/Span	λ^2	ξ
1	0.0448	1.3450×10^8	0.070	335	159
2	0.0448	1.0455×10^8	0.090	695	140
3	0.0448	8.5000×10^7	0.110	1255	127

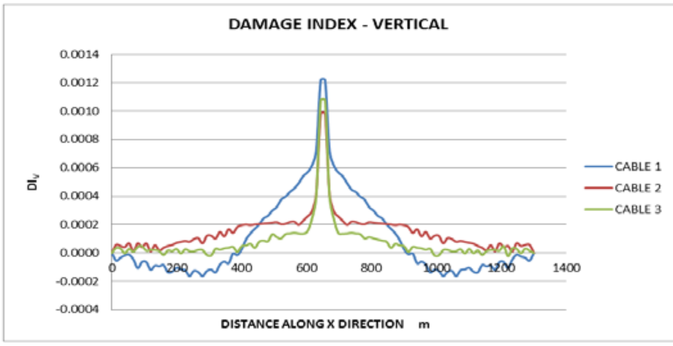

Figure 19. Vertical damage index – Damage Case 1 – Cable 1, 2 & 3

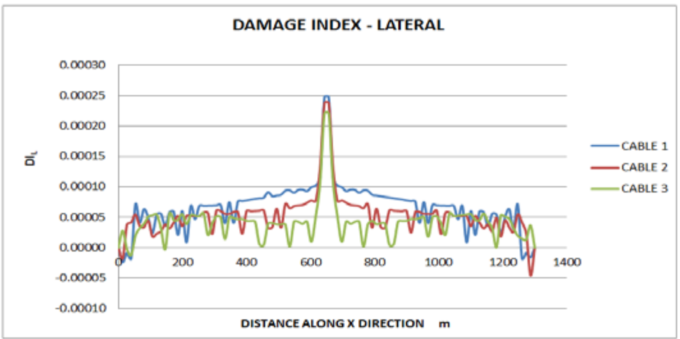

Figure 20. Lateral damage index - Damage Case 1 – Cable 1, 2 & 3

Figure 19 and Figure 20 illustrate the variation of vertical and lateral damage index in the three different cables (different sag-extensibility values) for the first damage case (damage at middle of cable). One common peak at the middle of the both graphs successfully localizes the damage at the mid span of the cable. However, slight variation in the values of the peak can be observed in both the vertical and lateral damage index curves correspond to three different sag-extensibility values. Also it is observed, cables with small sag to span ratio yield larger peak in both cases.

Two graphs of the DIs related to the second damage case (damage at quarter span of cable) are shown in Figure 21 and Figure 22 below. The vertical damage index shows two peaks representing the damage location. However, plot of the lateral damage index demonstrates only one sharp peak, as in Figure 22 which distinct the actual damage considered correctly. For the damage case 2 it is evident that the cables with large sag to span ratio show the highest peaks in both DIs which is contrary to what was observed with mid span damage. It is also observed that peak appearing at the middle of the vertical damage index decrease with increase of sag-extensibility. In other terms, a false alarm in vertical damage index for the cables with high sag-extensibility is becoming small. Results of this parametric study pertains to the comparison of the damage detection capabilities of the damage indicators with different sag-extensibilities demonstrates that the lateral modes of vibration have a very good potential in localizing the damage in a suspended cable using modal flexibility.

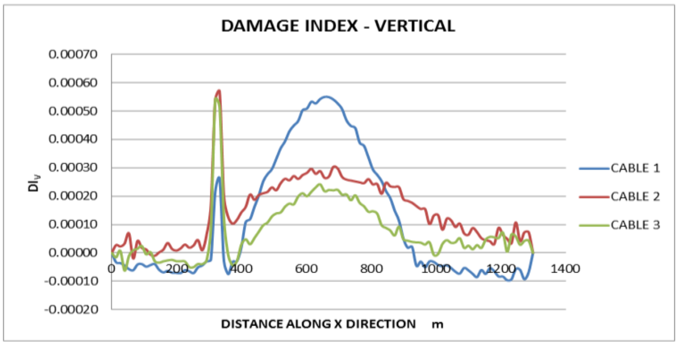

Figure 21. Vertical damage index - Damage Case 2 – Cable 1, 2 & 3

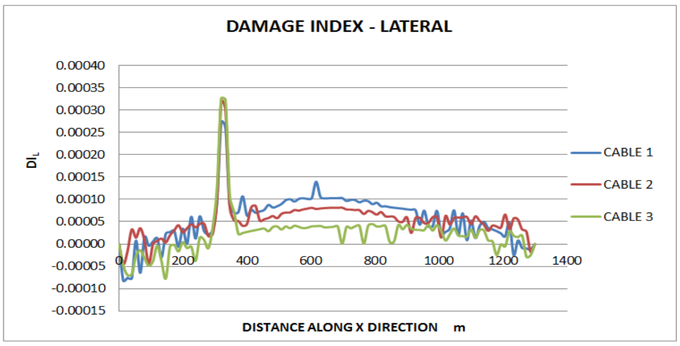

Figure 22. Lateral damage index - Damage Case 2 – Cable 1, 2 & 3

5.3 *Damage Localization and Bending Stiffness*

Flexural rigidity has commonly been ignored in most large diameter sagged cable analysis. However, cables with moderate to large diameters as used in suspension bridges are categorized by small to moderate bending stiffness parameter, which are critical especially when accurate modelling is needed for their structural identification. It is therefore, important to investigate the interaction of bending stiffness of cables and damage detection capability of damage indicators based on lateral and vertical modes of vibration. Different bending stiffness values for a specific cable are illustrated in Table 5.

Table 5. Properties of the investigated cables

Cable no.	I (m^4)	H (N)	Sag/Span	λ^2	ξ
A	0.0448	1.3450×10^8	0.070	335	159
B	0.0336	1.3450×10^8	0.070	335	184
C	0.0224	1.3450×10^8	0.070	335	225

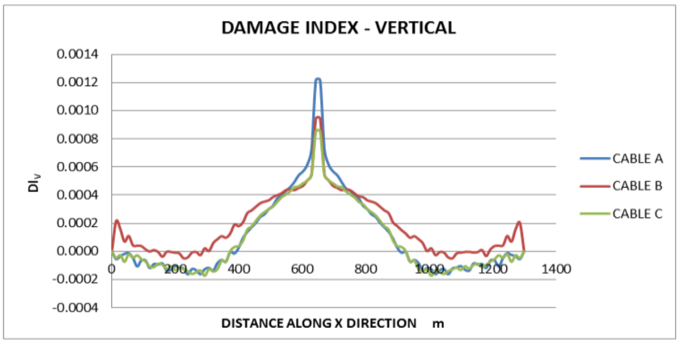

Figure 23. Vertical damage index - Damage Case 1 – Cable A, B & C

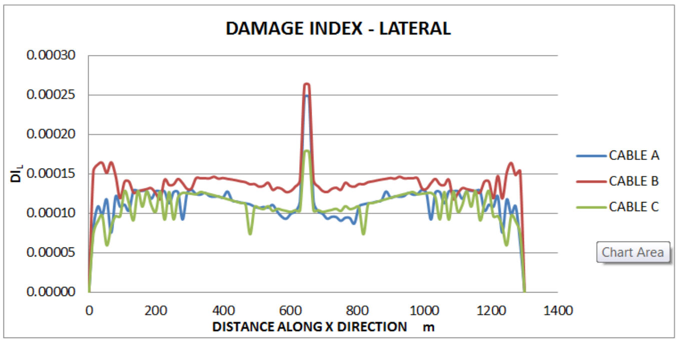

Figure 24. Lateral damage index - Damage Case 1 – Cable A, B & C

Figure 23 and Figure 24 demonstrate the variation of vertical and lateral damage index curves for the first damage scenario considered. Both indices are able to detect the damage correctly in this case and also highlighting that the damage detection capability is reduced when bending stiffness parameter increases.

That is, damage in the cables with higher flexural stiffness can be successfully localized by modal flexibility damage indices. However, it is noted that the damage indicator curves are sharp enough to identify the damage location in all three different stiffness values.

Next, the damage localization capability of the damage indicators for the second damage scenario is illustrated in Figure 25 and Figure 26. Again, the vertical damage index shows false alarms in damage localization. That false alarm can be eliminated by considering the damage index curve from lateral vibration, as discussed before. Therefore vertical damage indicator has an ability to capture the damage only at mid span correctly. However, damage indicator based on lateral modes of vibration is still valid for cables with a wide range of bending stiffness values.

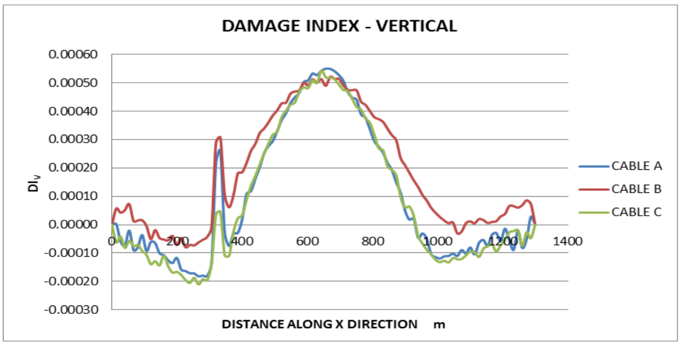

Figure 25. Vertical damage index - Damage Case 2 – Cable A, B & C

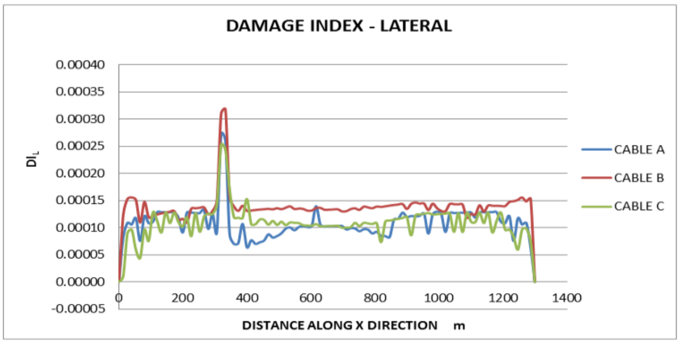

Figure 26. Lateral damage index - Damage Case 2 – Cable A, B & C

6 CONCLUSION

Large diameter and long span suspended cables are used as main structural components in cable supported structures such as suspension and cable stayed bridges, overhead transmission lines, cable supported roofs, and guyed towers. Damage in such cables needs to be detected at an early stage and there is no any attempt has been made in recently using vibration characteristics. This paper therefore, examined the capability of modal flexibility to detect damage in large diameter and long span suspended cables. In order to do that, two damage indices were derived based on modal flexibility incorporating vertical and lateral modes of vibration. The damage detection capability of the two damage indices was evaluated with and without noise in numerically generated modal data. Lateral damage index performs well in localizing damage in the cable model with single and multiple damage scenarios even with presence of 5% measurement noise in mode shapes.

Bending stiffness and sag-extensibility are two critical parameters of a long span large diameter suspended cable. Hence, influence of those parameters on the damage detection capability of the two DIs was evaluated. It is observed that the damage detection capability of the DIs reduce (indicated by reduced peaks) with the increase of sag-extensibility and bending stiffness parameters. However it is important to note that the damage indicator curves based on the lateral mode of vibration are sharp enough to identify the damage location in all the different cases studied. Subsequently, it can be concluded that the lateral modes are more sensitive to damage and lateral damage index derived from modal flexibility is competent in detecting and localizing damage in suspended cables. The research can be extended to detect damage in main cables of full suspension bridges to enhance the optimal service during their life span.

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