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Structural Damage Identification with Extracted Impulse Response Functions and Optimal Sensor Locations

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ABSTRACT: This paper presents a structural damage identification approach based on the time domain impulse response functions, which are extracted from the measured dynamic responses with the input available. The theoretical sensitivity of the impulse response function with respect to the system stiffness parameters considering the damping model is derived. The first-order sensitivity based model updating technique is performed for the iterative model updating. The initial structural finite element model and acceleration measurements from the damaged structure are required. Local damage is identified as a reduction in the elemental stiffness factors. The impulse response function sensitivity based optimal sensor placement strategy is employed to investigate the best sensor locations for identification. Numerical studies on a beam model are conducted to validate the proposed approach for the extraction of time domain impulse response functions and subsequent damage identification. The simulated damage can be identified effectively and accurately.

Keywords: Damage identification, Impulse response function, Time domain, Model updating, Sensitivity, Optimal sensor placement

1 INTRODUCTION

Monitoring the service and health conditions of structures is of great interest for the structure owners in terms of effective management strategy and maintenance planning. Dynamic vibration measurements are generally used with system identification techniques to detect local structural damages. Numerous studies have been conducted in the field by using vibration characteristics for the condition assessment and damage detection. The idea is based on the fact that the damages in the structural systems will change the vibration properties. These vibration characteristics may include frequencies, mode shapes, modal strain energy, strain intensity, flexibility and frequency response functions, etc, which are derived from the measured vibration time histories.

The measured time domain responses can also be used for damage detection directly. Cattarius and Inman [1] developed a non-destructive time domain approach to examine structural damage. Choi and Stubbs [2] proposed a methodology to locate and quantify the damage in a structure via time domain responses. The mean strain energy for a specified time interval for each element of the structure was obtained to build the damage index. The feasibility of the methodology was demonstrated using simulated data from a continuous beam structure. Link and Weiland [3] discussed the methodology of computational model updating techniques for damage identification in the modal and time domain. The modal data as well as time histories from impact tests were used for model updating and identifying the damage of a layered test beam which consisted of two thin aluminum sheets and an adhesive layer. Zhu *et al*. [4] presented a statistical model updating technique for damage detection of underwater pipeline systems via vibration measurements. Different damage scenarios with different damage locations and severities were simulated by removing one or several springs that were used to simulate the bedding conditions. Numerical and experimental results showed that the proposed approach is effective and reliable in identifying the bedding conditions and damages in the pipe structures. Wang *et al*. [5] developed an ARMAX model updating method for structural damage identification in the time domain. The developed model was used to identify structural damage through an updating process.

Many studies such as the works in Ref. [3-5] conducted the damage identification of structures via the vibration measurements by using the model updating techniques. As a system intrinsic vibration property, the impulse response function represents the time domain response of a system under the input of an impulse excitation. The impulse response function has been derived analytically [6] and used for substructural response reconstruction [7]. A subsequent study by using the developed wavelet-based response reconstruction for substructure damage identification has been conducted with experimental verifications [8]. The wavelet-based response reconstruction technique based on impulse response functions has also been extended to the damage identification of a target substructure or a full structure under moving loads [9, 10]. The local damages in the bridge structures can be identified effectively with both numerical and experimental investigations.

This paper presents a structural damage identification approach based on the time domain impulse response functions, which are extracted from the measured dynamic responses with the input available. The theoretical sensitivity of the impulse response function with respect to the system stiffness parameters considering the damping model is derived. The first-order sensitivity based model updating technique is performed for the iterative model updating. The initial structural finite element model and acceleration measurements from the damaged structure are required. Local damage is identified as a reduction in the elemental stiffness factors. The impulse response function sensitivity based optimal sensor placement strategy is employed to investigate the best sensor locations for the proposed damage identification approach. Numerical studies on an Euler-Bernoulli beam structure are conducted to validate the proposed approach for the extraction of time domain impulse response functions and subsequent structural damage identification.

2 SYSTEM IMPULSE RESPONSE FUNCTION AND TIME DOMAIN EXTRACTION

The Fast Fourier Transformation (FFT) based extraction of the impulse response function has been developed based on the frequency domain spectral densities. The impulse response function in the frequency domain can be obtained by dividing the cross power spectral density between the input force and output response with the auto power spectral density of the input force. Inverse FFT is performed to transform the impulse response function in the frequency domain back into the time domain. Both the forward FFT and backward inverse FFT are required in the extraction of impulse response functions. It is noted that leakage, end effects and aliasing occur in the FFT analysis. Filtering, windowing and ensemble-averaging techniques are often employed to alleviate these deficiencies. Nevertheless, these errors in the FFT process still exist which may lead to a reduction in the accuracy of the impulse response estimation [11]. Moreover, the basis functions associated with each frequency component in the Fourier-transformed domain span the entire measured time interval, hence making different signals indistinguishable as long as their spectral densities are the same. The errors in the inverse FFT could also significantly affect the extraction accuracy of the impulse response functions. It has been also reported that the impulse response data can be extracted via the wavelet transform from known measured responses and input excitation information to avoid errors in the Fourier transformation process of both the input and output signals [12, 6].

To avoid the errors induced by performing FFT or wavelet transformation, methods for the extraction of impulse response functions from time domain measured responses and inputs have been developed and will be presented in the following sections. The direct use of time domain inputs and outputs for the extraction will improve the accuracy by avoiding the errors in the forward and inverse FFT analysis, and possibly the errors in the forward and inverse wavelet transforms.

2.1 Analytical impulse response function of a structure

The general equation of motion of a damped structural system with n Degrees-of-Freedom (DOFs) can be written as

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=[D]\{f(t)\} \quad (1)$$

in which $[M]$, $[C]$ and $[K]$ are the $n \times n$ mass, damping and stiffness matrices of the structure respectively; $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are respectively the acceleration, velocity and displacement response vectors of the structure; $\{f(t)\}$ is a vector of applied forces on the associated DOFs of the structure with the mapping matrix $[D]$ relating the excitation force location to the corresponding DOF. Rayleigh damping $[C]=a_1[M]+a_2[K]$ is assumed, where a_1 and a_2 are the Rayleigh damping coefficients. The dynamic responses of the structure can be obtained from Equation (1) using the time integration algorithm, i.e. Newmark-β method [13].

Recently, the impulse response function has been derived analytically from the general equation of motion and it will be introduced briefly below [6]. The equation of motion of the above-mentioned structural system under the unit impulse excitation is

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=D\delta(t) \quad (2)$$

where $\delta(t)$ is the Dirac delta function. The impulse response function can be represented as a free vibration state with the specific initial conditions. Assuming that the system is in static equilibrium initially, the unit impulse response function can be computed from the equation of motion using the Newmark-β method as

$$\begin{cases}[M]\ddot{h}(t)+[C]\dot{h}(t)+[K]h(t)=0 \\ h(0)=0, \quad \dot{h}(0)=M^{-1}D\end{cases} \quad (3)$$

where $h(t)$, $\dot{h}(t)$ and $\ddot{h}(t)$ are the unit impulse response functions for displacement, velocity and acceleration, respectively.

When the structural system is under general excitation $f(t)$ with zero initial conditions, the acceleration response $\ddot{x}_l(k)$ at location l at time instant k is

$$\ddot{x}_l(k)=\int_0^k \ddot{h}_l(\tau)f(t-\tau)d\tau \quad (4)$$

in which $\ddot{h}_l(t)$ is the temporal unit impulse response function at location l. Equation (4) can be expressed in the discrete form as

$$\ddot{x}_l(k)=\sum_{i=0}^{k}\ddot{h}_l(i)f(k-i) \quad (5)$$

The entire time domain response at location l can be obtained in the matrix multiplication form as

$$X=H\cdot F \quad (6)$$

where X and H are the output response and impulse response function vectors, respectively. They are denoted as

$$X=\{\ddot{x}_l(0),\ddot{x}_l(1),\ddot{x}_l(2),\cdots,\ddot{x}_l(n)\}^T \quad (7)$$

$$H=\{\ddot{h}_l(0),\ddot{h}_l(1),\ddot{h}_l(2),\cdots,\ddot{h}_l(n)\}^T \quad (8)$$

F is the input force matrix and is expressed as

$$F=\begin{bmatrix} f(0) & f(1) & \cdots & f(n) \\ 0 & f(0) & \cdots & f(n-1) \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & f(0)\end{bmatrix} \quad (9)$$

2.2 Extraction of impulse response function from measured responses

Providing excitation forces are available, the impulse response function can be derived from measured temporal responses by solving Equation (6).

$$H=X\cdot\left(F^T\cdot F\right)^{-1}\cdot F^T \quad (10)$$

in which F^T is the transpose of matrix F. It is noted that the pseudo-inverse is used to extract the impulse response function. Normally the condition number for the excitation force matrix F is not an extremely large value since the columns are basically independent. However, when the matrix F is badly ill-conditioned, the truncated Singular Value Decomposition (TSVD) can be employed to eliminate those very small singular values and the corresponding vectors to have a better and more stable solution for the pseudo-inverse.

The extracted impulse response function will be used for the damage identification. One main advantage of using impulse response functions instead of measured responses is that the impulse response function is an inherent system property and the ensembling technique can be performed to reduce the errors in the impulse response function with repeated tests.

3 IMPULSE RESPONSE FUNCTION SENSITIVITY BASED DAMAGE IDENTIFICATION

The parametric model updating methods for damage identification are popular because they keep a clear physical understanding of the stiffness matrix. In this study, an impulse response function sensitivity based finite element model updating method is used for the structural damage identification. The local damage is assumed as a stiffness reduction, i.e., a reduction in the elastic modulus of a specific element. The mass matrix is assumed to be unchanged before and after the damage.

3.1 Damage model

Linear damage scenario is assumed in this study. The initial linear-elastic structure is assumed remaining linear-elastic after the minor local damage. The damaged structure stiffness matrix $[K_d]$ is defined as

$$[K_d]=\sum_{i=1}^{n}\alpha_i K_i=\sum_{i=1}^{n}(1-\Delta\alpha_i)K_i \quad (11)$$

where K_i denotes the intact ith elemental stiffness matrix; α_i and $\Delta\alpha_i$ represent the system stiffness parameter and the extent of stiffness reduction in the ith element, respectively.

3.2 Sensitivity of Impulse Response Function

The perturbation equation of motion is obtained by calculating the differentiation of Equation (3) with respect to system parameter α. Rayleigh damping $[C]=a_1[M]+a_2[K]$ is defined, and the perturbation parameter is taken as the stiffness change in structures. The damping model is included and considered in the sensitivity calculation, which improves the accuracy of the computed sensitivity by ignoring the damping effect [6]. The following equation can be derived by differentiating Equation (3)

$$\begin{gathered}[M]\frac{\partial \ddot{h}(t)}{\partial \alpha}+a_1[M]\frac{\partial \dot{h}(t)}{\partial \alpha}+a_2\frac{\partial [K]}{\partial \alpha}\dot{h}(t)+ \\ a_2[K]\frac{\partial \dot{h}(t)}{\partial \alpha}+\frac{\partial [K]}{\partial \alpha}h(t)+[K]\frac{\partial h(t)}{\partial \alpha}=0\end{gathered} \tag{12}$$

Re-arranging Equation (12), we have

$$\begin{gathered}[M]\frac{\partial \ddot{h}(t)}{\partial \alpha}+[C]\frac{\partial \dot{h}(t)}{\partial \alpha}+[K]\frac{\partial h(t)}{\partial \alpha}= \\ -\frac{\partial [K]}{\partial \alpha}h(t)-a_2\frac{\partial [K]}{\partial \alpha}\dot{h}(t)\end{gathered} \tag{13}$$

in which, $\frac{\partial \ddot{h}(t)}{\partial \alpha}$, $\frac{\partial \dot{h}(t)}{\partial \alpha}$ and $\frac{\partial h(t)}{\partial \alpha}$ are the sensitivity vectors of the acceleration, velocity and displacement impulse response functions with respect to system parameter α. $\frac{\partial [K]}{\partial \alpha}$ is the differentiation of the whole structure stiffness matrix to the system parameter change, and it can be derived with Equation (11). $h(t)$ and $\dot{h}(t)$ are solved by Equation (3) based on the finite element model of the structure. The sensitivity vectors $\frac{\partial \ddot{h}(t)}{\partial \alpha}$, $\frac{\partial \dot{h}(t)}{\partial \alpha}$ and $\frac{\partial h(t)}{\partial \alpha}$ can be obtained by solving Equation (13) with the time integration method, i.e. Newmark-β method mentioned in Section 2.1 for structural dynamic response calculation.

3.3 Damage identification algorithm

The objective function of the damage identification algorithm is to minimize the difference between the analytical impulse response function from finite element analysis and extracted impulse response function from measured responses

$$f_{obj}=\left\|\ddot{h}_a(t)-\ddot{h}_m(t)\right\|_2 \tag{14}$$

where $\ddot{h}_a(t)$ is analytical impulse response function from the finite element model analysis. $\ddot{h}_m(t)$ is the impulse response function extracted from the measured responses following Equation (10).

The first order sensitivity-based model updating method [14] without considering the second- and higher-order effects is adopted

$$\left[\frac{\partial \ddot{h}(t)}{\partial \alpha}\right]\{\Delta\alpha\}=\{\Delta\ddot{h}(t)\}=\{\ddot{h}_a(t)\}-\{\ddot{h}_m(t)\} \tag{15}$$

where $\Delta\alpha$ is the perturbation of system stiffness factors, $\left[\frac{\partial \ddot{h}(t)}{\partial \alpha}\right]$ is the sensitivity matrix of impulse response function with respect to system stiffness factors calculated from Equation (13). $\{\Delta\ddot{h}(t)\}$ is the difference between the analytical and extracted impulse response functions.

Equation (15) could be solved by using the pseudo-inverse of the sensitivity matrix as

$$\{\Delta\alpha\}=\left(\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^{-1}\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T\{\Delta\ddot{h}(t)\} \tag{16}$$

Since the sensitivity matrix $\left[\frac{\partial \ddot{h}(t)}{\partial \alpha}\right]$ is usually ill-conditioned, Tikhonov regularization [15] is used to stabilize the solution by defining a modified objective function which controls the errors between the accuracy and stability of the solution. The Tikhonov regularized solution is defined by minimizing the following objective function

$$J=\left\|\frac{\partial \ddot{h}(t)}{\partial \alpha}\Delta\alpha-\Delta\ddot{h}(t)\right\|^2+\lambda\|\Delta\alpha\|^2 \tag{17}$$

where λ is the optimal regularization parameter which balances the weight of the norm of the solution $\|\Delta\alpha\|$ and the minimization of the identification equation $\left\|\frac{\partial \ddot{h}(t)}{\partial \alpha}\Delta\alpha-\Delta\ddot{h}(t)\right\|$. The L-curve method [16] is employed to obtain this optimal regularization parameter λ. The solution of Equation (17) can be expressed as

$$\Delta\alpha=\left(\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T\frac{\partial \ddot{h}(t)}{\partial \alpha}+\lambda I\right)^{-1}\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T\Delta\ddot{h}(t) \tag{18}$$

where I is an identity matrix.

3.4 Damage identification procedure

An iterative damage identification procedure is used to update the initial finite element model and identify the structural damage. The first step is to extract the structural impulse response function from measured dynamic acceleration responses. The excitation force information is assumed available. The second step makes use of extracted impulse response functions from the structure to update the structural elemental stiffness factors iteratively with the sensitivity based model updating method. The difference between the analytical and extracted impulse response functions is minimized. Analytical impulse response function can be obtained from the finite element analysis. An initial intact finite element model is used as a baseline model for the identification. The iterative damage identification procedure is described as follows

Step 1: Acquire the dynamic acceleration responses at a limited number of measurement locations from the damaged structure.

Step 2: Extract the impulse response function with measured responses by Equation (10). It is noted that the ensemble-averaging technique can be employed to improve the accuracy of extracted impulse response functions.

Step 3: Compute the analytical impulse response function from Equation (3) with the structural finite element model. The difference between the analytical and extracted impulse response functions can be obtained.

Step 4: Calculate the sensitivity matrix of the impulse response function with respect to system stiffness parameters by Equation (13) with the numerical integration algorithms.

Step 5: Obtain the Tikhonov regularized solution of the perturbation vector of structural stiffness factors $\{\Delta\alpha\}$ from Equation (18).

Step 6: The finite element model is iteratively updated with $\alpha_{i+1} = \alpha_i + \Delta\alpha$ as the system parameters for the next iteration. Repeat Steps 3 to 5 until the following convergence criterion is satisfied.

$$\frac{\|\alpha_{i+1} - \alpha_i\|_2}{\|\alpha_i\|_2} \le Tolerance \tag{19}$$

where i denotes the ith iteration. The tolerance is taken as 1.0×10^{-4} in this study.

3.5 Optimal sensor placement for sensitivity based damage identification

The choice of sensor numbers and placed locations may affect the damage identification performance. It is interesting to investigate the optimal sensor placement for the sensitivity based damage identification. The Fisher information matrix [17], which is an indicator that can effectively reflect the contribution of each DOF to the vibrations of a structure, will be used in this study to find out the best locations for the impulse response function sensitivity based damage identification. The Fisher information matrix is widely used in the optimal sensor placement. In this paper, it is calculated based on the sensitivity of the impulse response function with respect to system parameters calculated from Equation (13)

$$F = \left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T \frac{\partial \ddot{h}(t)}{\partial \alpha} \tag{20}$$

Maximizing the Fisher information matrix will lead to a good damage identification if the measurement noise is assumed not correlated [17]. To account for the contribution from different DOFs to the Fisher information matrix, Kammer [18] defined an effective independence matrix which was formed as follows to estimate the contribution of each candidate DOF

$$E = \frac{\partial \ddot{h}(t)}{\partial \alpha}\left(\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T \frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^{-1}\left(\frac{\partial \ddot{h}(t)}{\partial \alpha}\right)^T \tag{21}$$

The terms on the diagonal elements of matrix E represent the contributions of the corresponding DOFs. The summation of the contribution from different time instants to the effective independence matrix will be calculated to estimate the importance of candidate sensor locations. The contribution at the ith DOF is expressed as

$$E_i = \sum_{j=(i-1)*nt+1}^{i*nt} diag(E_j) \tag{22}$$

The largest the value of E_i is, the more important the associated sensor location is. The above-mentioned optimal sensor selection method will be used in this study to define the best locations for placing the sensors with the purpose of impulse response function sensitivity based damage identification. It should be noted that in a previous study of selecting the optimal measurement locations [19], Fisher information matrix, which defines the locations that are most sensitive to structural damage, is used together with a noise sensitivity analysis. The locations that are most sensitive to structural damage and least sensitive to measurement noise are selected as the optimal vibration measurement locations. In the present study, however, the measurement noise is not considered in selecting the optimal sensor placement locations.

4 NUMERICAL STUDIES

Numerical studies on a simply-supported beam structure are conducted to demonstrate the accuracy and effectiveness of the proposed approach for structural damage identification. A 20 m long simply-supported Euler-Bernoulli beam structure is taken as an example. The Young's modulus and mass density are $3.3\times10^4 MPa$ and $2500 kg/m^3$, respectively. The moment of inertia and area of the cross-section of the beam structure are respectively 0.05 m^4 and 0.6 m^2. The finite element model of the beam structure consists of ten Euler-Bernoulli beam elements and eleven nodes, as shown in Figure 1. Each node has a translational degree-of-freedom (DOF) in the vertical direction and a rotational DOF. The entire structure has 22 DOFs in total. Rayleigh damping is assumed and the damping ratios for the first two modes are taken as $\xi = 0.012$. The first three natural frequencies of the intact structure are 4.08 Hz, 16.46 Hz and 36.81 Hz, respectively. Two scenarios with different types of excitation forces will be studied in this paper. The first scenario studies the damage identification problem of the tested beam structure under a multi-sine wave excitation force, and the second scenario considers an impact force as the excitation.

4.1 Scenario 1: Multi-sine wave excitation force

A multi-sine wave excitation force is applied on the transverse DOF of node 3, as shown in Figure 1. The sinusoidal excitation force is expressed as

$$F(t) = 8000(1 + 0.1\sin(4\pi t) + 0.05\sin(8\pi t)) \quad (23)$$

The sampling rate is set as 2000 Hz and the sampling duration is the first 0.5 s. The damage is simulated as a reduction in the stiffness, i.e. the reduction in the elastic modulus of a specific element. Dynamic analysis is performed with the damaged structure to obtain the responses, which will be used for the extraction of impulse response function and damage identification. White noise is added to the original calculated responses to simulate the noisy responses. A normally distributed random noise with zero mean and unit standard deviation is added and the noisy response is obtained as

$$\ddot{x}_n(t) = \ddot{x}_c(t) + E_p N_{oise} std(\ddot{x}_c(t)) \quad (24)$$

4.1.1 Extraction of impulse response function

10% stiffness reduction is introduced into the fifth element of the structure. Dynamic response analysis is conducted to obtain the "simulated" measured responses for the damage identification. The information of the applied excitation, i.e. the time-history and location, is assumed available in the extraction of impulse response functions. The transverse accelerations at Nodes 4, 7 and 8 are assumed available for the extraction of impulse response functions, which will be used for the subsequent damage identification in the next section. Figure 2 shows the measured accelerations in the vertical direction of Nodes 4, 7, and 8. The impulse response function at each measurement location can be extracted by solving Equation (10). The extracted impulse response functions from measured acceleration responses without noise effect are shown in Figure 3. The identified impulse response functions are very close to the true ones calculated from the finite element analysis by Equation (3), indicating the accuracy and effectiveness of the proposed procedure to extract the impulse response functions. The smeared noise in the measured acceleration responses will have an influence on the identification accuracy of the extracted impulse response functions. 5% noise effect is added to the original responses and these noisy responses will be used for the computation to investigate the noise effect on the extraction of impulse response functions. Ensemble-averaging technique could be employed to improve the identification accuracy of impulse response functions by performing repeated tests. Figure 4 shows the extracted impulse response functions with 5% noise effect and with 50 ensembles. It can be seen that the extracted impulse response functions match the analytical curves accurately in the initial period with high magnitudes. After 0.1s, the smeared noise significantly affects the accuracy of the extraction of impulse response functions. However, the identification results with 50 ensembles show a good agreement with the true impulse response functions. The relative error between the true and extracted impulse response functions is defined as

$$\text{Relative error } (\%) = \frac{\left\| \ddot{h}_{true}(t) - \ddot{h}_{extracted}(t) \right\|_2}{\left\| \ddot{h}_{true}(t) \right\|_2} \times 100\,(\%) \quad (25)$$

The relative errors between the true and extracted impulse response functions at Nodes 4, 7 and 8 in the first 0.1 second are listed in Table 1. The identification accuracy of impulse response functions from measured acceleration responses is significantly improved by employing the ensemble-averaging technique. These extracted impulse response functions from the measurements without and with noise effect, and with 50 ensembles will be used in the subsequent damage identification by using the iterative damage identification procedure described in Section 3.4.

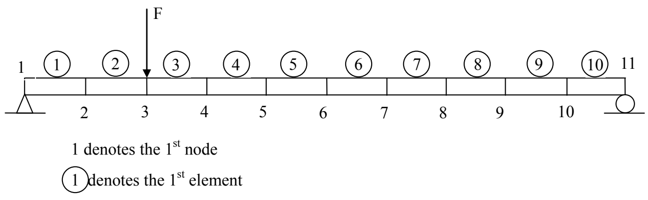

Figure 1. Finite element model of the simply-supported beam structure

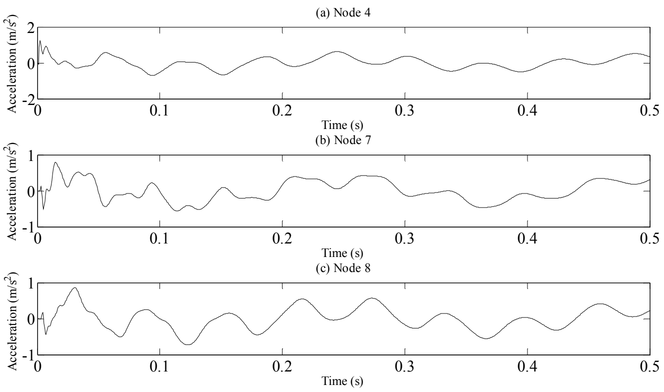

Figure 2. Measured acceleration responses from the damaged structure under a multi-sine force

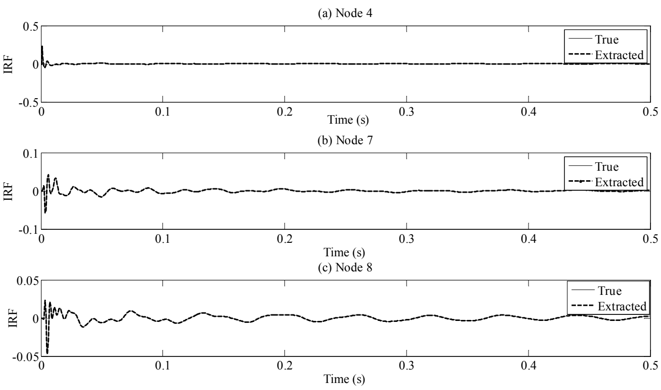

Figure 3. Extracted impulse response functions from measured acceleration responses without noises

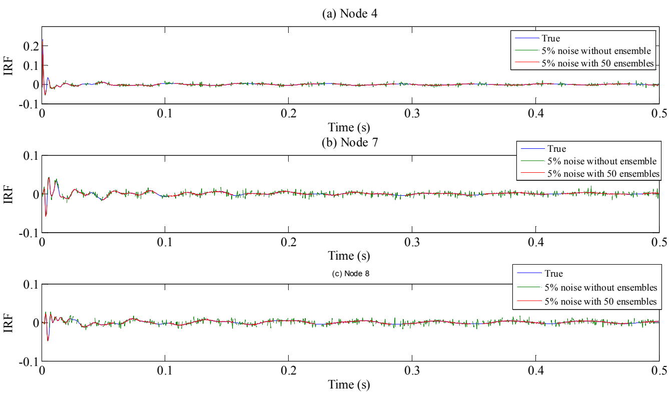

Figure 4. Extracted impulse response functions with noise effect and ensembles

Table 1. Relative errors in the first 0.1 s of the extracted impulse response functions

Relative error (%)	No noise	5% noise without ensemble	5% noise with 50 ensembles
Node 4	2.79×10^{-9}	27.11	3.80
Node 7	5.29×10^{-9}	45.86	6.34
Node 8	7.18×10^{-9}	63.44	8.75

4.1.2 Damage identification results

Figure 5 shows the damage identification results for the cases including no noise effect, 5% noise but without ensemble, and 5% noise with 50 ensembles. It can be seen from Figure 5 that using the measurements without noise effect can identify the simulated damage exactly, which demonstrates the correctness of the proposed damage identification approach. The identification results from measurements including 5% noise without ensemble show that the introduced damage can be detected effectively, however, several large false positives present, i.e. on the 4th and 10th elements. When the ensemble-averaging technique is employed, the damage identification results are improved with less false identifications. The required iterations and relative errors in the identification result are shown in Table 2. The identification using original responses with no noise effect gives an accurate identification of damage severity with the relative error of only 0.1%. For the identification with 5% noise but without ensembles, the identified damage is 10.92% and the relative error is 9.2%. A lesser number of iterations is required for convergence when employing the ensemble-averaging technique, while the relative error in the identified damage at the 5th element is improved to 5.4%.

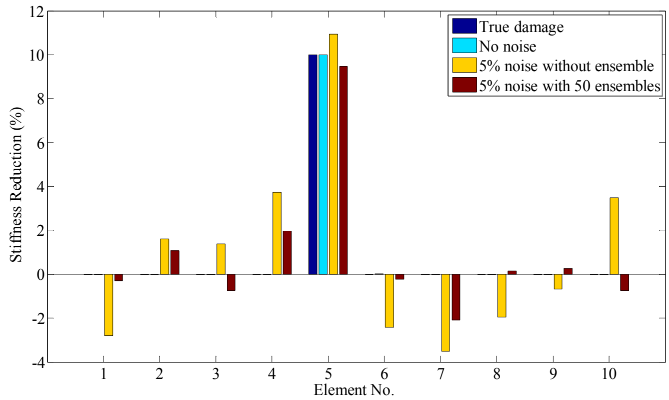

Figure 5. Damage identification results with noise effect and ensembles (Scenario 1)

Table 2. Identification iterations and errors

	Iterations	Identified damage at the 5th element	Relative error
No noise	8	9.99%	0.1%
5% noise without ensemble	10	10.92%	9.2%
5% noise with 50 ensembles	7	9.46%	5.4%

4.1.3 Sensitivity based optimal sensor placement for damage identification

The Fisher Information Matrix is calculated based on the sensitivity of impulse response function with respect to system stiffness parameter, which is obtained by Equation (13). Since only the responses at the translational DOFs could be measured, the transverse DOFs at all the eleven nodes are considered as candidates to possibly place the accelerometers. The contribution from each translational DOF is calculated with Equations (21) and (22) as shown in Figure 6. Three sensor locations are selected based on the amount of the contribution from each DOF to the Fisher Information Matrix shown in Figure 6. Therefore the transverse DOFs at node 2, 3 and 9 are chosen as the three optimal locations for the impulse response function based damage identification. Damage identification is conducted by using the measured responses on these locations from the damaged structure. The same procedure for the extraction of impulse response functions and subsequent damage identification is followed.

A multi-damage case will be considered herein to investigate the performance and robustness of the proposed damage identification approach with optimal sensor locations. 10% and 15% stiffness reductions are introduced into the 4^{th} and 7^{th} elements of the beam model. Measured responses without and with noise effect are used for the identification. Figure 7 shows the damage identification results with optimal sensor locations. The locations of the simulated damages are identified accurately, and the identified extents of local damages are close to the true values. With the use of optimal sensor locations and ensemble-averaging technique, the introduced damages can be identified effectively and accurately with nearly exact damage severity estimation and small false positives and false negatives.

4.2 Scenario 2: impact force

An impact force as shown in Figure 8 is considered as the excitation force in the second Scenario. The excitation location is the same as that in the first scenario shown in Figure 1. The optimal sensor locations can be obtained based on the effective independence matrix calculated with Equation (21). The contribution at each transverse DOF is computed from Equation (22) and is shown in Figure 9. Based on the observations with maximum contributions to the effective independence matrix, we also select three optimal sensor locations on Node 2, Node 3 and Node 5.

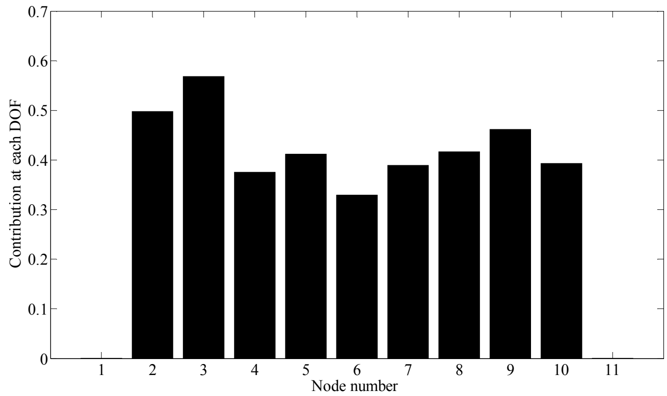

Figure 6. Contributions to Fisher Information Matrix at each DOF (Scenario 1)

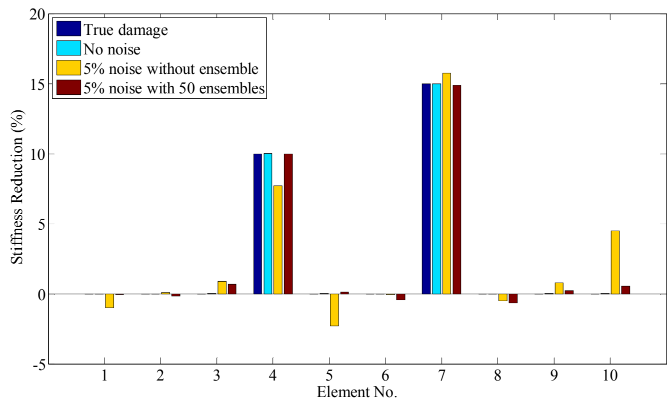

Figure 7. Damage identification results for the multi-damage case with optimal sensor locations (Scenario 1)

10% stiffness reduction is introduced into the fifth element. Measured acceleration responses from the damaged beam structure under the impact force are obtained and used for the extraction of impulse response functions. The sampling rate is 2000Hz. Similar to the cases in Section 4.1, 5% noise effect and 50 ensembles are considered. The extracted impulse response functions are shown in Figure 10. Only the response data in the first 0.1s are used for the extraction. It can be observed the noise effect has a significant influence on the accuracy of the extracted impulse response functions. However, using the ensemble-averaging technique can improve the extraction accuracy and get a much better estimation of impulse response functions. Figure 11 shows the damage identification results under impact force. It is observed that the identification with 50 ensembles from measured responses including 5% noise effect gives the correct damage location and closer damage extent estimation compared with the case without ensembles. Several large false identifications are observed on undamaged elements for the case with 5% noise without performing ensemble averaging estimations. Ensemble averaging always reduces noises in the signal, therefore leads to better damage identifications.

The above studies well demonstrate the effectiveness and performance of the proposed damage identification approach by using the extracted impulse response functions from measured accelerations. Good identification results are obtained with 5% noise effect included in the measured responses. It is proved that the ensemble-averaging technique can be performed to improve the damage identification. The advantages of the proposed damage identification approach using impulse response functions instead of measured dynamic responses directly lie in the merits that the repeated testes could be conducted to better estimate the impulse response functions through averaging, and an accurate estimation of impulse response functions will lead to good damage identification results. Further studies on the optimal sensor placement considering the noise effect in the measured responses can be conducted.

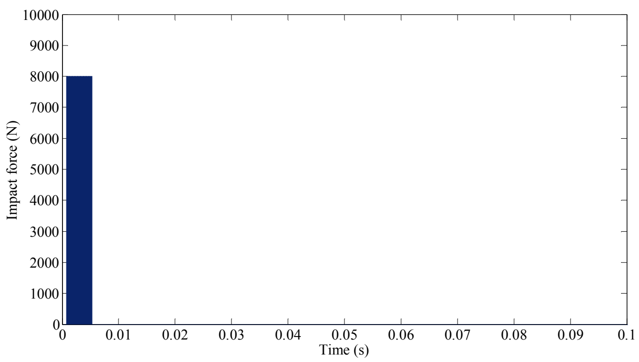

Figure 8. Impact force used in the Scenario 2

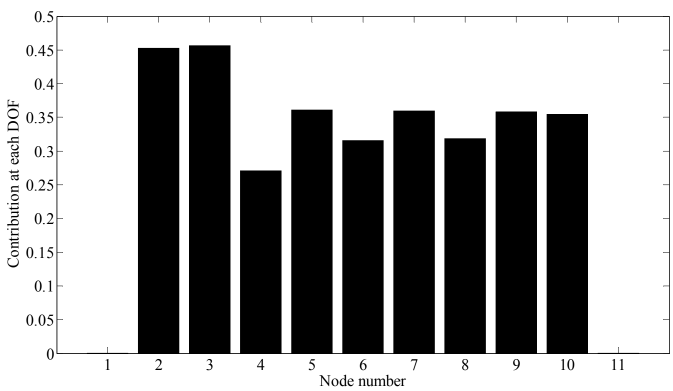

Figure 9. Contributions to Fisher Information Matrix at each DOF (Scenario 2)

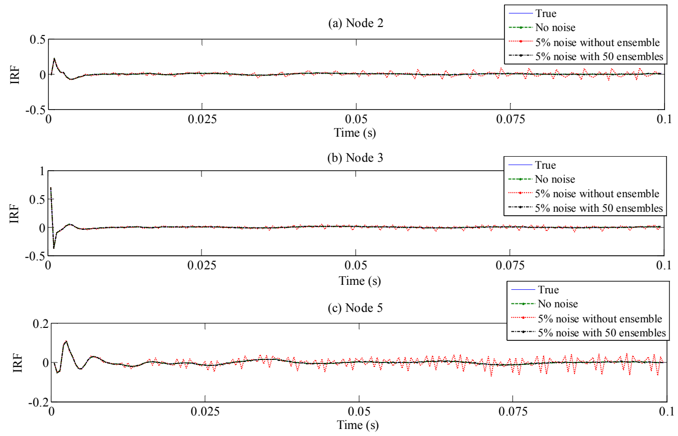

Figure 10. Extracted time domain impulse response functions (Scenario 2)

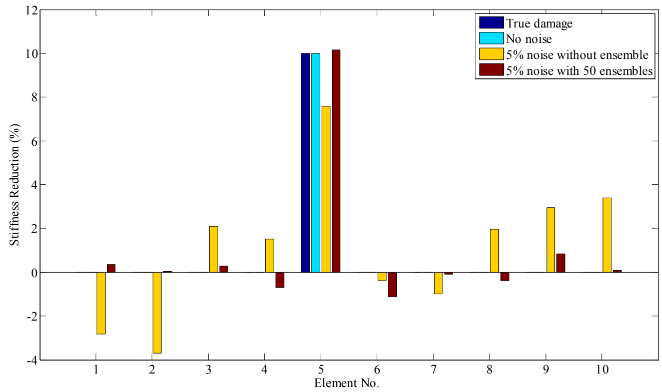

Figure 11. Damage identification results under impact force with optimal sensor locations (Scenario 2)

5 CONCLUSIONS

This paper presents a structural damage identification approach based on the time domain extracted impulse response functions and optimal sensor locations. The impulse response functions are extracted from the measured dynamic responses with the input available. The theoretical sensitivity of the impulse response function with respect to system stiffness parameters considering the damping model is derived. The first-order sensitivity based model updating technique is performed for the iterative model updating. The initial structural finite element model and acceleration measurements from the damaged structure are required. Local damage is identified as a reduction in the elemental stiffness factors. The impulse response function sensitivity based optimal sensor placement strategy is employed to investigate the best sensor locations for the proposed damage identification approach.

Numerical studies on an Euler-Bernoulli beam structure are conducted to validate the proposed approach for the extraction of time domain impulse response functions and the subsequent structural damage identification. Two scenarios with the beam structure under a multi-sine wave excitation force and an impact force respectively are investigated. 5% noise effect is considered in the measured acceleration responses. Optimal sensor locations are obtained based on the amount of the contribution at each DOF to Fisher Information Matrix. The simulated damage can be identified effectively in both scenarios. It is demonstrated that performing the ensemble-averaging calculations could improve the identification results.

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