eJSE International

Special Issue: Electronic Journal of Structural Engineering 14(1) 2015

A Bayesian Damage Identification Technique Using Evolutionary Algorithms - a Comparative Study.

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ABSTRACT: In this paper, a one-stage model-based damage identification technique based on the response power spectral density of a structure is investigated. The technique uses a finite element updating method with a Bayesian probabilistic framework that considers the uncertainty caused by measurement noise and modelling errors. The efficacy of two different evolutionary algorithms – a genetic algorithm and a covariance matrix adaptation evolution strategy – is examined via numerical simulation of time-history response data for a beam structure. A range of different damage scenarios have been considered including: both single and multiple damage locations; varying damage severity; the introduction of noise and modelling errors and incompleteness in the number of captured modes and measurement response data. The results clearly show that both evolutionary algorithms implemented are effective and their overall performance, measured in terms of accuracy, is very similar. However, the covariance matrix strategy is found to be significantly superior in terms of its convergence rate and the number of function evaluations required to find the solution for both noisy and noise-free response data.

Keywords: Structural Health Monitoring, Power Spectral Density, Severity, Bayesian, Evolutionary Algorithm.

1 INTRODUCTION

Structural health monitoring (SHM), is the process of assembling general information describing the current condition of a structure, where the aim is to indicate the existence, location, and degree of damage, if damage occurs. Damage might occur in a structure after long-term deterioration under service loads such as fatigue and corrosion or due to extreme incidents such as earthquakes and impact loads. Structural health monitoring is categorized in three different general techniques: visual inspections, local experimental techniques and global methods. The local experimental techniques can be conducted using different kinds of embedded sensors, radio X-ray, laser scanning, radiographic, ultrasonic and thermal field methods etc. (Doebling et al. 1998). These techniques are generally very expensive, time consuming and ineffective for large and complex structural systems while visual inspections often miss critical problems in the structure under investigation.

Damage causes changes in structural physical properties, primarily: stiffness; mass and damping at damaged locations and consequently alters the dynamic response behavior of a structure. Therefore, monitoring of the changes in structural vibration response parameters is an important tool for the assessment of structural integrity and safety, on-time decision making regarding maintenance, rehabilitation and replacement requirements. The continuous/periodic monitoring of civil structures is essential to ensure their safety and acceptable performance during their life span and to prevent a catastrophe. Hence, estimation of the variations in dynamic response characteristics of the structure (such as its associated natural frequencies, mode shapes, modal damping, frequency response functions, etc.) provides useful information regarding the existence, location and severity of structural damage.

Structural vibration response monitoring methods can be classified into two categories: *model-based* and *non model-based* methods. Model-based methods locate and quantify damage by correlating an analytical model with test data from the damaged structure. These methods provide quantitative information describing the damage as well as damage existence. However, model-based methods are computationally intensive and require the updating of a finite element model. In contrast, non model-based methods assess damage by comparing the measurement data from the undamaged and damaged structure under consideration. These methods are computationally straightforward. However, generally they do not provide quantitative information about structural damage (Carden and Fanning 2004; Bayissa 2007; Fan and Qiao 2011).

In this paper, a model-based damage detection approach is introduced and investigated. When applying model-based methods for damage detection in civil structures, a significant amount of uncertainty caused by measurement noise in the response data and modelling error in the analytical model can arise. Therefore, there is a need for robust methods such as those offered by probabilistic frameworks rather than commonly used deterministic methods. Consequently, a Bayesian probabilistic framework representing the inverse problem is introduced. This framework is described in terms of updated probability density function, which can be used for statistical inference such as model characteristic predictions, sensitivity analysis, cost value predictions, reliability analysis, and structural health monitoring.

In the past, damage identification studies which were conducted based on power spectral density analysis of the structure have dealt mainly with the non-model based techniques which cannot be used for estimation of damage severity. In this study a one stage model-based damage identification technique is presented which can be used for detection, localization and estimation of the severity of damage using the mean square value of response power spectral density (MSV of response PSD) and a Bayesian probabilistic approach. The term one stage implies that the investigation for the existence, location and damage severity is performed via a one-stage process while in a two-stage process, the existence and location of damage are obtained in the first stage using a non-model based analysis and the severity is estimated in the second stage using a model based analysis.

The only assumption when computing MSVs is that the excitation and response are stationary random processes; the system is linear and time-invariant and the excitation is stationary ergodic white noise.

When updating the underlying finite element model, the discrepancy between the two data series described by an appropriate objective (or cost) function should be minimized. The effectiveness of two different evolutionary algorithms – a *genetic algorithm* and a *covariance matrix adaptation evolution strategy* – used as the global optimization procedure in this context, is subsequently evaluated.

Comprehensive numerical simulation experiments using the evolutionary algorithms and a finite element update procedure on a beam structure consisting of 10 elements are carried out. A number of scenarios are constructed by varying the locations and the severity of damage, introducing noise into the environment, and by incompleteness of the number of captured modes and measurement response data. The performance of the two algorithms is investigated and compared using statistical analysis.

The remainder of paper is organized as follows: In section 2, the literatures and problem formulation are discussed. The MSV of response PSD and the Bayesian probabilistic approach and its application in the area of damage identification are described in detail. In section 3, the evolutionary algorithms used in this study are depicted. In section 4, the method is presented, including a description of the beam structure and scenarios to be investigated. In section 5 numerical simulation results are given and analysed. Finally, in section 6, the results are discussed and conclusions drawn.

2 PROBLEM FORMULATION

In the following sub-sections, the formulations of the MSV of displacement response PSD for a beam structure and the Bayesian probabilistic approach are presented.

2.1 Mean Square Value of Response Power Spectral Density

Despite the extensive research conducted in the past on damage identification using alternative damage sensitive parameters and indicators, objective functions and optimization techniques, the search for damage sensitive parameters and effective algorithms is still in progress. Damage parameter indicators are usually determined based on the difference between healthy and damaged parameters of a structure under consideration. The sensitivity of damage indicators, which have been identified based on dynamic characteristic of the structure, varies for different types of damage. Carden and Fanning (2004) provided an overview of studies based on frequencies and mode shapes. However, neither frequency (Salawu 1997) nor modal response parameters are identified as being consistent in providing reliable information of a structure under investigation.

Damage identification techniques which are based on power spectral density (PSD) analysis can provide useful information regarding the existence and location of damage. Damage-sensitive vibration response parameters that utilize broadband frequency information (as opposed to resonance frequency based traditional counterparts (Bayissa et al. 2011) have strong physical relationships with structural dynamic properties. They can be employed in either non-model or model based damage identification studies. Additional features of this approach include: sensitivity to both local and global damage; low sensitivity to noise and modal truncation errors; identification of linear as well as nonlinear damage conditions. Moreover, the MSV can be computed either from experimental response data analysis or directly from an updated finite element model. Furthermore, it is a suitable damage parameter due to its computational simplicity and flexibility (determined from experimental modal analysis, time-domain and spectral-domain analysis) and flexibility in its applications (can be used for input-output as well as output-only damage identification problems) (Bayissa 2007). In the past, MSV of response PSD and its derivatives have been effectively used only for investigating the existence and location of damage (Bayissa and Haritos 2007).

For an elastic and isotropic simply supported beam, the forced vibration response at any point x from the end supports can be defined using superposition of the natural modes as follows (Newland 1984):

$$w(x,t)=\sum_{r=1}^{\infty}\frac{\phi_r(x)\phi_r(l)}{\left(\omega_r^2+2i\xi_r\omega_r\omega-\omega^2\right)}P_0e^{i\,\omega t} \tag{1}$$

where $w(x,t)$ is the lateral deflection at time t and distance x from its end, P_0 is the amplitude of the harmonic load at grid point l, ϕ_r is the mass-normalized mode shape and ω_r and ξ_r are the natural frequency and the damping ratio of the r^{th} mode.

The MSV which is described as the overall energy content of the signal and can be obtained from continuous or discrete signals in either the time-domain or spectral-domain, are given in equations (2) and (3), respectively as follows (Bayissa 2007):

$$MSV=\frac{1}{T}\int_0^T\left|y(t)\right|^2dt=\frac{1}{N}\sum_{n=0}^{N-1}\left|y[t_n]\right|^2 \tag{2}$$

$$MSV=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{yy}(\omega)d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2\,S_{pp}(\omega)\,d\omega=\sum_{n=0}^{N/2-1}\left|F[f_n]\right|^2 \tag{3}$$

where $y(t)$ is the time series signal, $y[t_n]$ is an N point sequence, $F[f_n]$ is the respective Fourier transform of the N point sequence data series of time length T, $S_{yy}(\omega)$ is power spectral density and $S_{pp}(\omega)$ is the excitation PSD. $|H(\omega)|$ is the frequency response function, ω is the excitation frequency (Bayissa 2007). In this study, the MSV parameter is determined using spectral-domain analysis (equation (3)) for damage identification in structures.

The effects of the cross-spectral terms on the responses that result from multiple input excitations are not considered since only single excitation input is used and finally the MSV of response (PSD) at a grid point k and harmonic excitation applied at a grid point l can be described as follows:

$$MSV_k=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\sum_{r=1}^{\infty}\frac{\left(\phi_r^k\right)^2\left(\phi_r^l\right)^2}{\left(\omega_r^2-\omega^2\right)^2+\left(2\xi_r\omega_r\omega\right)^2}\right)S_{pp}(l,\omega)\,d\omega \tag{4}$$

2.2 Bayesian Probabilistic Approach for Structural Damage Identification

Damage identification methods using the Bayesian probabilistic framework have been reported in the literature. Sohn and Law (2000) formulated a Bayesian probabilistic framework based on modal parameters. Using a large number of experimental data sets, they attempted to minimize the relative posterior probability function. Yuen and Katafygiotis (2001) proposed a Bayesian probabilistic technique using a time-domain approach in order to identify the modal characteristics of the structure in the condition of ambient data and have extended their study (2005) using noisy measurement response data without the knowledge of the input spectrum and presented that the updated probability distribution can be well approximated by a Gaussian distribution centered at the most probable values of the parameters. Furthermore, Guo and Li (2012) conducted a sensitivity analysis of frequency index, modal strain energy index and Bayesian theory, based on the frequency and modal strain energy, in a two-stage damage identification procedure and have presented that the result of using a Bayesian framework is more accurate than just using frequency or modal strain energy index based methods. Au and Zhang (2012) performed a similar study on a primary-secondary structure using a frequency-domain approach.

Other studies have also been reported in the literature of methods which can be used to quantify the uncertainties associated with modelling errors and process of constructing a mathematical model of a structure using Bayesian updating methods (Beck and Au 2002; Cheung and Beck 2009). Bayissa

(2007) has formulated the Bayesian posterior probability function considering MSVs response power spectral density but did not offer any solution for the inverse problem.

In order to employ the Bayesian probabilistic framework, the analytical model is parameterized in terms of structural stiffness K as an assembly of element stiffness matrices assuming that damage affects only the stiffness properties of the structure. The overall stiffness matrix $K(\beta)$ in terms of N_β number of elements is given as follows:

$$K(\beta) = \sum_{i=1}^{N_\beta} (1-\beta_i) K_i \tag{5}$$

where K_i is the stiffness matrix for the i^{th} element (or substructure) and β_i $(0 \le \beta_i \le 1)$ is a set of non-dimensional model parameters that represents the contribution of the i^{th} element stiffness to the global stiffness matrix. In the case that no stiffness loss has occurred, the value of β_i is 0 and in situations of damage for elements or substructures, it would be determined to be greater than 0. Therefore, the value of β_i is an indicator of the location as well as the amount of stiffness loss if any damage has taken place. The prediction error of the optimization function can be determined using the following equation:

$$PE_i = \left(\frac{\delta K_i^p - \delta K_i^a}{\delta K_i^a} \right) \times 100 \tag{6}$$

in which, δK_i^p is the predicted damage severity in percent; δK_i^a is the actual damage severity present in the structure and PE_i is the prediction error.

In order to implement Bayes' theorem, all the uncertain quantities were represented as probability distributions and then by creating the *posterior* conditional probabilities for the different variables of interest, inferences can then be determined. Accordingly, by multiplying the *prior* distributions and *likelihood* functions, the result of the statistical inverse problem is provided by the posterior probability distribution.

A joint posterior distribution for the set of model parameters conditioned on the observations can be obtained from Bayes' theorem (Gilks et al. 1996) which can be used as a damage indicator and is defined as follows (Sohn and Law 2000; Bayissa 2007):

$$p(\beta|D,M) = \frac{p(D,\beta|M)}{p(D|M)} = \frac{L(D|\beta,M)\,p(\beta|M)}{\int_B p(D|\beta,M)\,p(\beta|M)\,d\beta} \tag{7}$$

where D denotes vectors of measured data sets from the undamaged and damaged structural condition states, $\beta=[\beta_1,....,\ \beta_{N\beta}]^T$ indicates the non-dimensional model parameters included in the parameter space, $p(\beta|D,\mathrm{M})$ is the posterior density or the updated Probability Density Function (PDF) of the unknown parameters after observing the data; $p(D,\beta|M)$ is the joint probability distribution over all random quantities; $p(\beta|\mathrm{M})$ is the prior probability distribution function (PDF) of the initial model parameters β for a structural model class M, $L(D|\beta,\mathrm{M})$ is the likelihood density, also known as the conditional probability of observing the data D, $p(D|\mathrm{M})$ is the normalizing factor for the posterior PDF. In those situations in which the main sources of uncertainties are from modeling error and measurement noise, the measured response value $D(s)$ after considering measurement noise $\varepsilon_N(s)$, modeling error $\varepsilon_M(\beta)$ and computed response value $D(\beta)$, is defined as follows:

$$D(s) = D(\beta) + \varepsilon_M(\beta) + \varepsilon_N(s) \tag{8}$$

The Normal distribution can be used for defining a mathematical explanation for the numerical approximation error $\varepsilon_M(\beta)$ and measurement noise $\varepsilon_N(s)$, as follows (Bayissa 2007):

$$\varepsilon_M \sim \mathcal{N}\left(\bar{\mu}_\mu, \Sigma_\mu\right) \tag{9}$$

$$\varepsilon_N \sim \mathcal{N}\left(0, \Sigma_\mu\right) \tag{10}$$

$\overline{\mu_\mu}$ is the conditional expectation; $\sum_\mu$ is the positive definite covariance matrix of the approximation error that can be obtained using the Inverse-Wishart distribution as follows (Gelman et al. 2003):

$$\sum\nolimits_\mu \sim Inv-Wishart_{\nu_0}(\Lambda_0^{-1}) \tag{11}$$

The updated distribution after observing n samples of X can be stated as:

$$P(\mu,\Sigma) \sim NIW\left[\frac{\kappa_0\mu_0 + n\overline{\mathrm{X}}}{\kappa_0+n}, \kappa_0+n, \nu_0+n, \Lambda_0 + C + \frac{\kappa_0 n}{\kappa_0+n}\left(\overline{\mathrm{X}}-\mu_0\right)\left(\overline{\mathrm{X}}-\mu_0\right)^T\right] \tag{12}$$

where

$$\overline{X} = \frac{1}{n\sum_{i=1}^{n} X_i} \qquad C = \sum_{i=1}^{n}\left(X_i - \overline{X}\right)\left(X_i - \overline{X}\right)^T \tag{13}$$

The parameters ν_0 and Λ_0 indicate the degrees of freedom and the prior covariance matrix for the Inverse-Wishart distribution. Moreover, μ_0 and κ_0 are the prior mean and the number of prior measurements on the Σ, respectively.

For an independent and distributed zero mean Gaussian noise, the likelihood probability functions for the response measurements is the discrepancy between the theoretical parameters computed from the analytical model and those obtained from measured response data which can be defined as follows:

$$L(D|\beta,M) \propto \exp\left\{-\frac{1}{2}\left[D(s)-D(\beta)-\varepsilon_M\right]^T \times \sum\nolimits_{\mu}^{-1}\left[D(s)-D(\beta)-\varepsilon_M\right]\right\} \tag{14}$$

The conditional PDF of the response MSV parameters for a single data set can be expressed as follows:

$$L\left(\mu_n^0|\beta,M\right) = \prod_{\delta r=1}^{N_{\delta r}} L\left(\mu_{\delta r}^0|\beta,M\right) = \frac{1}{f_\mu(\sum)}\exp\left\{-\frac{1}{2}\sum_{\delta r=1}^{N_{\delta r}}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right]^T \times \sum\nolimits_{\mu}^{-1}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right]\right\} \tag{15}$$

in which, $L(\mu_n^0|\beta,M)$ is the conditional PDF for the MSV determined from the r^{th} frequency bandwidth, $\mu_{\delta r}^0(s)$ and $\mu_{\delta r}^0(\beta)$ indicate the vectors of the MSV determined from the measured and computed response data, respectively. s indicates the observed data set number, $s = 1, \ldots, N_s$. δr is the frequency bandwidth including the r^{th} mode, $\delta r = 1, \ldots, N_{\delta r}$. $\overline{\mu}_{\varepsilon_M}$ is the expected value of the modeling error; $f_\mu(.)$ is the normalizing factor for the conditional PDFs, given by $f_\mu(\sum) = [2\pi]^{N_{\delta r}/2}\left\|\sum_\mu\right\|^{1/2}$, Γ is a matrix that transforms the MSVs computed at full model degrees of freedom to the measurement grid points.

The prior PDF is assumed on the model parameters as white noise and the model parameters, β, can be described as uncorrelated Gaussian random variables of equal covariance centered around $\overline{\beta}$, $\beta \sim \mathcal{N}(\overline{\beta}, \sum_\beta)$. $\overline{\beta}$ is the best initial estimate of the model parameter distribution before any data is obtained and $\sum_\beta$ is the covariance of the prior PDF, which represents the initial level of uncertainty in the analytical model. Therefore, the prior PDF on the model parameters can be described using a multi-variate Normal distribution, as follows:

$$p(\beta|M) = \frac{1}{f_\beta(\sum_\beta)}\exp\left\{-\frac{1}{2}\sum_{i=1}^{N_\beta}\left[\beta_i-\overline{\beta}\right]^T \sum\nolimits_{\beta}^{-1}\left[\beta_i-\overline{\beta}\right]\right\} \tag{16}$$

where $p(\beta|M)$ is the prior PDF; $f_\beta(.)$ is the normalizing factor; $\sum_\beta$ expressed the level of confidence in the initial model parameters.

The joint posterior PDF of the model parameters can be computed by substituting the likelihood and prior PDFs given in Equation (15) and (16), respectively, into Equation (7), as follows:

$$p(\beta|D,M) = \overline{\chi}_1 \frac{1}{f_\mu(\sum)}\exp\left\{-\frac{1}{2}\sum_{\delta r=1}^{N_{\delta r}}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right]^T \times \sum\nolimits_{\mu}^{-1}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right]\right\} \tag{17}$$

in which, $\overline{\chi}_1$ is the normalizing factor for the posterior PDF of the model parameters. Finally, the posterior PDF can be described in the form:

$$p\left(\beta|D,M\right) = \tilde{f}\left(\sum\nolimits_\mu, \sum\nolimits_\beta\right)\exp\left[-\frac{1}{2}Q(\beta)\right] \tag{18}$$

where $Q(\beta)$ is the objective (or cost-function) and states the final objective of the problem. The objective function for Equation (17) is described as:

$$Q(\beta) = \frac{1}{2}\sum_{s=1}^{N_s}\sum_{\delta r=1}^{N_{\delta r}}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right]^T \sum\nolimits_{\mu}^{-1}\left[\mu_{\delta r}^0(s)-\Gamma\mu_{\delta r}^0(\beta)-\overline{\mu}_{\varepsilon_M}\right] + \frac{1}{2}\sum_{i=1}^{N_\beta}\left[\beta_i-\overline{\beta}\right]^T \sum\nolimits_{\beta}^{-1}\left[\beta_i-\overline{\beta}\right] \tag{19}$$

In order to determine the most probable values of the model parameters, some kind of optimization algorithm should be employed. By maximizing the posterior PDF, the maximum *a posteriori* estimate of the parameter of interest can be computed, as follows (Bayissa 2007):

$$\hat{\beta}_i = arg\ max\ p(\beta_i|D,M) \tag{20}$$

in which, $\hat{\beta}_i$ is an optimal model parameter that represents all the information required for assessment of structural damage.

3 OPTIMIZATION TECHNIQUES

In order to obtain the most probable values of the model parameters ($\hat{\beta}_i$), the objective function given in equation (19) should be minimized using an optimization algorithm. In order to deal with such a complex, high-dimensional, non-linear, non-differentiable and ill-condition function, some kind of global optimization algorithm is required. Two evolutionary algorithms are used in this study to meet this goal: a genetic algorithm (GA) and a co-variance matrix adaptation evolution strategy (CMA-ES). GAs have been widely used in the area of damage identification and CMA-ES is a relatively new but previously proven very effective in other domains. These two algorithms have been chosen in this comparison study to demonstrate the effectiveness and efficiency of the newly presented CMA-ES over GA to the SHM community. In the following sub-sections, the algorithms are described in detail. This is followed by a description of the parallel deployment of the algorithms.

3.1 Genetic Algorithms

Damage identification problems usually deal with non-smooth, noisy stochastic problems and non-differentiable multi-dimensional functions in which "local identifiable" optimization algorithms are not capable of locating the global maxima or minima. In the past, stochastic methods such as importance sampling and Markov chain Monte Carlo (MCMC) based methods have been widely used for different purposes such as: probability calculations, Bayesian computation, image analysis, optimization, solving partial differential equations, multidimensional integration in different areas of study (Gilks et al. 1996). However, these methods are not capable of solving the problem of global optimization.

In order to deal with complex high-dimensional, non-linear, non-differentiable and ill-condition problems, stochastic global optimization algorithms are typically be required. Genetic algorithms are a population based stochastic optimization technique inspired by natural evolution principles that can be used for discrete as well as continuous optimization problems (Goldberg 1989). GAs are global search techniques which deal with a large number of variables. Here, individuals in the population represent potential solutions – points in the search space – to the optimization problem. An iterative simulated evolutionary process creates new points in the search space by modifying selected points and continuously moving them toward more optimal regions (Reeves and Rowe 2002). The effectiveness of GAs in many problem domains may be attributed in part to the fact that gradient-based information is not a necessary requirement to guide the search; there is a non-dependency on the initial starting point; the algorithms are capable of handling a large number of parameters and constraints; it is possible to handle both discrete and continuous variables; the algorithms can be used to approximate the global minimum value of functions with several local minima; and there is a possibility to accept failed designs.

GAs have been widely used to detect the changes of frequencies or modal data of the structure under different damage conditions (Perera and Torres 2006; Meruane and Heylen 2011; Putha et al. 2012; Varmazyar 2013; Varmazyar et al. 2013). Other studies (Koh et al. 2010; Perera et al. 2010; Sandesh and Shankar 2010; Hsiao et al. 2012) have combined multiple algorithms, for example GAs and particle swarm optimization (Seyedpoor 2012), in order to detect damage in the structure. Other damage identification studies using heuristics such as Artificial Neural Networks have also been reported in the literature (Osornio-Rios et al. 2012).

Individuals in the GA population (the potential solutions) are represented by a *chromosome*. Each chromosome consists of a collection of *genes*. In a real-encoded GA, each gene represents a variable of the optimization problem. For instance, the chromosome $c_i = [c_{i1}, c_{i2}, \ldots, c_{im}]$ contains m genes where c_{i1} is the first gene (variable) of the chromosome c_i. Typically, the initial values of the genes within the chromosomes are randomly generated. The *fitness* or objective value of each chromosome c in the population set is evaluated. In order to create the next *generation*, the Darwinian principle of natural selection is invoked. Fitter individuals have a chance to reproduce. Here, "genetic material" is exchanged via artificial *crossover* and *mutation* operators between the selected parent individuals.

Crossover combines different parts of the selected parents' chromosomes. This process basically exchanges gene values between the parents, producing two new offspring. A *mutation* operator is usually employed to alter the values of a randomly selected gene. This process helps to maintain diversity and avoid the problems of premature convergence. Multiple iterations of this selection-evolutionary operator cycle will hopefully generate new, improved individuals (solutions to the problem).

Unfortunately, population-based approaches are inherently slow (or low rate of convergence) when are faced with very complicated and time-consuming objective functions. Despite the many advantages of meta-heuristics in complex search and optimization problems, the computational costs when using GAs can be expensive. However, using a suitable algorithm with the least number of function evaluations and parallel deployment of the algorithms provide a viable alternative to address such computational issues (Cantu Paz 2000). Clearly, there are many on-gong challenges when identifying the most appropriate meta-heuristics to use in damage identification problems.

In this problem domain, the actual genes c_{ij} encode values for the model parameters used in the Bayesian objective function. Thus, as the population evolves, new (and hopefully better) solutions (model parameters) will appear in the population. The effectiveness of the GA is typically correlated with the value of algorithm parameters: for example, the population size; the crossover and mutation rates; the chromosome encoding scheme, as well as the actually implementation of the evolutionary operators.

3.2 Covariance Matrix Adaptation Evolution Strategy

The second evolutionary algorithm examined in this study, is the Covariance Matrix Adaptation Evolution Strategy (CMA-ES), first introduced by Ostermeier et al. (1994), and subsequently revised and extended. CMA-ES has been identified as a "local identifiable" optimization technique (Hansen and Ostermeier 2001; Auger and Hansen 2005a) and is well suited to global optimization (Hansen and Kern 2004; Auger and Hansen 2005b; Hansen 2009).

A comparison study has been conducted to investigate the performance of 31 different search algorithms on 24 known test functions (Hansen et al. 2010) and it has been concluded that for noisy, multi-dimensional and difficult objective functions, variants of CMA-ES have demonstrated superior performance to other algorithms. In contrast to quasi-Newton optimization methods, which need approximate gradients of the functions, the CMA-ES approach does not even require their existence which makes it more practical in terms of having non-smooth, non-continuous and noisy problems.

The CMA-ES is a stochastic and non-elitist algorithm that adapts a covariance matrix of the distribution at each iteration based on successful steps and new individuals are generated using a multivariate Normal distribution (Ostermeier et al. 1994). As was the case for the GA, the genes encode values for the model parameters used in the Bayesian update. In CMA-ES there are typically a number of parameter values that can be tuned. However, it is possible to simply set the population size parameter and allow the algorithm to adapt and/or automatically use default values for other parameters. An iterative simulated evolutionary process creates new individuals in the search space by modifying selected points and continuously moving them toward more optimal regions. Multiple iterations of this selection-evolutionary operator cycle will generate new, improved individuals. These new individuals are sampled according to a multivariate Normal distribution in the $\mathbb{R}^n$. Individuals with better fitness value are selected and the new mean of the search distribution is a weighted average of selected points from the sample. A step-size is introduced to control the overall scale of the distribution (Ostermeier et al. 1994; Igel et al. 2007).

There are very few studies reported in the area of damage identification using the CMA-ES. Jafarkhani and Masri (2011) have conducted a research study on an experiment, based on the natural frequencies and mode shapes of the structure and the CMA-ES technique implemented demonstrated a promising performance in this area.

3.3 Parallel Processing of Evolutionary Algorithms

Unfortunately, population-based evolutionary algorithms such as GAs, which need a large number of fitness evaluations are inherently slow when they are faced with very complicated and time consuming objective functions. Taking advantage of parallel deployment rather than running in sequential mode, has partly solved this problem. There are two types of parallelization: first, using multiple populations of genetic algorithm known as coarse-grain parallelism, which can be applied on one or multiple processors and second, applying one population of GAs on a number of processors known as micro-grain parallelism (Punch 1998).

Adeli and Kumar (1995) presented coarse-grained effective parallelization strategies on the GA-based structural optimization. They employed both a penalty-function and an augmented Lagrangian technique and concluded that the high scalability of the developed coarse-grained method provided a cost-effective alternative for structural optimization on a cluster of workstations. Some improvement was also seen in running time of the program in Meruane and Heylen's study (2011); however, there are some limitations in employing a number of processors which are equal to the number of populations. While increasing the number of populations, this would inversely increase the run-time as the program needs more function evaluations to reach the optima.

On the other hand, different investigations have presented conflicting results regarding the speed-up of the run-time. Punch (1998) suggested that these conflicting reports might be due to the nature of the problems. However in the second method, which only parallelises the objective function, using multiple processors simultaneously can speed up the computation time. Single population parallel deployment is a relatively straightforward task to be implemented on the GA and the CMA-ES. The use of a computing cluster and/or a computer with multiple cores will typically significantly decrease the time taken to find an optimal solution but will not affect the accuracy of the results (Cantu Paz 2000).

4 NUMERICAL SIMULATION STUDY

In this paper, a one-stage model-based Bayesian probabilistic damage identification approach is implemented.This approach is based on the MSV of response PSD of the structure, which can be used to detect, localize and estimate the severity of damage regions. An evolutionary algorithm is employed to minimize the objective function, which represents the minimum discrepancy between the two data series described when updating the finite element model.

To evaluate the efficacy of the proposed approach, a series of numerical simulation experiments are conducted using a simply supported beam structure with 10 elements. The use of a simple rather than a complex structure will enable the fundamental study of the effects of damage to vibrational characteristics without having to deal with the complicated dynamic interactions associated with complex structures. Moreover, since most of the existing methods are demonstrated on beam-like structures, this provides an opportunity to compare the proposed method to all existing damage identification techniques. On the other hand, beam-like structures are one of the most popular structural systems that are employed in various types of structures such as bridges, buildings, masts and etc.

The material properties of the beam are mass density of 2500 kg/m^3, Young's modulus of 30GPa with dimension of 10m x 0.3m x 0.4m. Damage is simulated by reducing the Young's modulus and the level of severity induced is directly related to the percentage reduction in the Young's modulus. Two different damage conditions are analyzed: damage introduced into a single location; and damage introduced in multiple locations. The locations of damage for both the single and multiple damage condition are illustrated in Figure 1. The severity of damage at given locations is also considered as Table 1. A MATLAB toolbox known as CALFEM (Austrell et al. 2004), has been used to develop the parameterised FEM (Finite Element Modelling) of the beam. Localized damage was simulated by reduction in the Young's modulus of: 5%, 10%, 15% and 20%, and a broadband impact hammer excitation simulated using an impulsive load with a single integration time step, has been implemented. A constant modal damping ratio of 0.01 was applied and the first 10 flexural modes determined for computation of the response MSVs at each nodal point. Three different scenarios are examined in this study:

(i) An accurate numerical model and noise-free response data (full set of measurement grid points and complete set of modes);
(ii) An approximate numerical model and noisy response data (full set of measurement grid points but incomplete set of modes);
(iii) An approximate numerical model and incomplete noisy response data (incomplete sets of both measurement grid points and modes).

4.1 Scenario (i): An Accurate Numerical Model and Noise-Free Response Data

In this scenario, a noise-free measurement response data set and an error free numerical model using the first 10 flexural modes, a single frequency bandwidth, $N_{\delta r}=1$ with a sampling rate of 2500 Hz, are applied for damage identification. $\sum_{\mu}$ is the positive definite covariance matrix of the approximation error which was obtained from the Inverse Wishart distribution of the resampled model in this study. The value of the transformation matrix for all degrees of freedom was considered 1, $\Gamma = [1,....,1]^T$. The number of model degrees of freedom where MSVs were computed, was the same as the simulated measurements points. Finally, the MSVs of PSDs response computed from the numerical experimental data were used along with the MSVs obtained from the FE model to optimize the objective function.

4.2 Scenario (ii): An Approximate Numerical Model and Noisy Response Data

In this scenario, a limited number of modal measurements compared to scenario (i) in the presence of a noisy measurement response and modelling errors are considered. In order to simulate measurement uncertainty caused by the vibration noise during data acquisition, the time domain response histories obtained at each node were polluted with varying levels of spatially Gaussian random noise 5% and 10%. Ensemble averaging using 10 samples was conducted on the frequency domain data sets. In the case of the approximate numerical model, a constant level of random noise, up to 1% was introduced to the analytical data. An ensemble averaging of 10 samples was used over their transform in the spectral domain.

This level of the error obtained was kept the same during the optimization.

The number of model degrees of freedom where MSVs were computed, was the same as the simulated measurements points. A single frequency bandwidth, $N_{\delta r}=1$, that included only the first 5 flexural modes for both numerical experimental and analytical models was used. Consequently, the MSVs in both sets of numerical experimental data and FE model were determined. Finally, an optimization algorithm was used to maximize a *posteriori* (objective function), which would inversely detect the location and estimate the severity of damage that would have been induced for the different damage conditions. In this study each *gene* represents a β value and each *chromosome* indicates an individual from the evolutionary algorithm population. Furthermore, a typical response PSD with 10% noise level implemented for computation of MSVs and subsequent damage identification is presented in Figure 2.

4.3 Scenario (iii): An Approximate Numerical Model and Incomplete Noisy Response Data

For this scenario, the robustness of the proposed method is investigated in a condition of modal incompleteness and a limited amount of measurement data in the presence of noisy response data and modelling error. In order to simulate these complex conditions, firstly, the MSVs are computed at only half of the beam nodes (or 5 measurement grid points) and secondly, half of the flexural modes (5 modes) considered in scenario (i) are taken into account in order to calculate MSVs. The simulated measurement data set was polluted with varying levels of spatially Gaussian random noise (5% and 10%). In the case of the approximate analytical model, a constant level of random noise, up to 1% was introduced to the numerical data. Finally, an optimization algorithm was applied to inversely detect, locate and estimate the severity of damage and the results are discussed in section 5.6. Figure 1(c)–(d) illustrate the finite element model nodes, the grid points at which MSVs were computed, the location of the induced structural damage and the substructure elements of the coarse measurement grid points of the beam. Therefore, this scenario consisted of not only measurement noise and modelling errors but incompleteness in the number of captured modes and measurement data.

5 RESULTS

5.1 *Genetic Algorithms Parameter Setting*

A study with different numbers of population varying from 10 to 150 has been conducted using real-coded GA. The population was initialized randomly with gene values in the range of 0 to 1 representing the damage parameters. The performance across a number of different population sizes (varying from 10 to 150) was investigated. Figure 3 plots the most interesting results. After all, a population size of 120 individuals (chromosomes) was selected. A number of empirical trials were then used to determine the best GA parameter settings in the problem domain based on the given population size.

Figure 4 plots the results when different combinations of the GA operators were employed and the objective function values averaged over running 20 trials. The best combination of the GA operators (determined by the best value of the objective function) were then selected and applied for different situations of the structure: Selection – the tournament method was chosen for the type of parents' selection for creating the next generation; Crossover – two-point crossover, with a crossover fraction of 0.8; Mutation – adaptive feasible mutation was used for the mutation function and in order to scale the raw fitness scores to values in the range, the proportional fitness scaling function was employed.

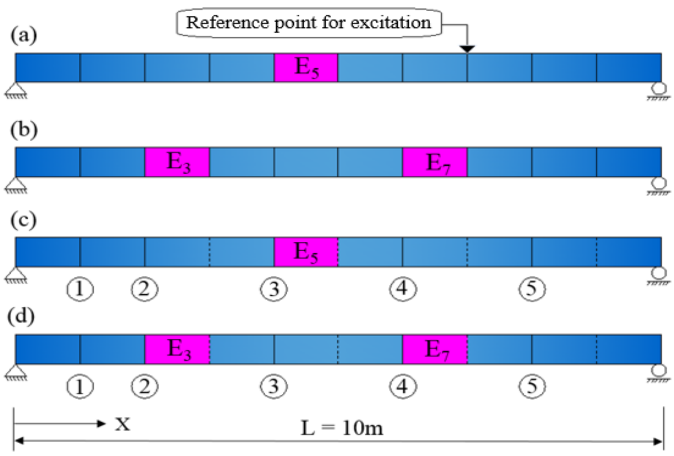


Figure 1 FE model of the beam with damage locations and simulated measurement grid points indicated: (a) single damage condition (at model element 5); (b) multiple damage condition (at model elements 3 and 7); (c) single damage condition with indicated grid points considered; (d) multiple damage condition (at model elements 3 and 7) with indicated grid points considered.

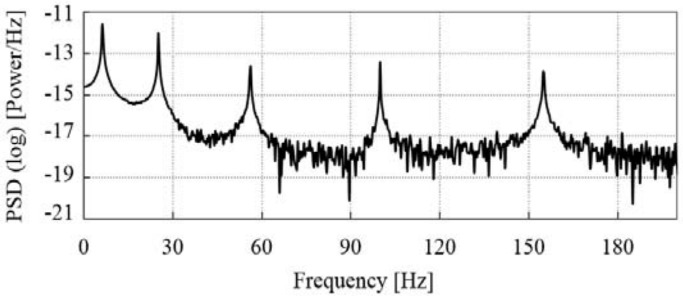


Figure 2 A typical response PSD plot with incomplete number of modes and 10% noise level ensemble averaged over 10 samples

Table 1
Simulated structural damage conditions applied to the beam

Structural damage conditions (%)		
Single damage location	Multiple damage locations	
E_5	E_3	E_7
5	5	5
10	10	10
15	15	15
20	20	20
-	5	15

5.2 *Covariance Matrix Adaptation Evolution Strategy Parameter Setting*

A study with different numbers of population varying from 10 to 150 has been conducted and Figure 5 plots the most important ones of CMA-ES and the objective function values averaged over running 20 trials. The population was initialized randomly with values in the range of 0 to 1 representing the damage parameters. A number of empirical trials were then used to determine the best CMA-ES population number in the problem domain. The best population number of the CMA-ES algorithm determined by the best value of the objective function also considering the least number of function evaluations, was then selected and applied for different situations of the structure. In this study, a CMA-ES with a population size of 60 individuals was used.

5.3 *Parallel Implementation of GA and CMA-ES*

In order to investigate the speed-up effects when multiple processors are used, a range of simulation experiments were carried out using a varying number of processors for the GA and CMA-ES. Figure 6 plots the speed-up α (the percentage of the running time) when the number of processors was varied from 1 to 12. The results of both algorithms are in total agreement and seen to be inversely proportional to the number of processors, and as the number of processors increases, there is a significant improvement in running time. This emphasizes the fact that the speed-up effects depend on the type of the objective function and are not affected by the optimization algorithms implemented. In the sections that follow, the results are presented using the algorithms with 12 processors. Note: a prefix "P" is identified with parallel implementation of the evolutionary algorithms in the remainder of the paper.

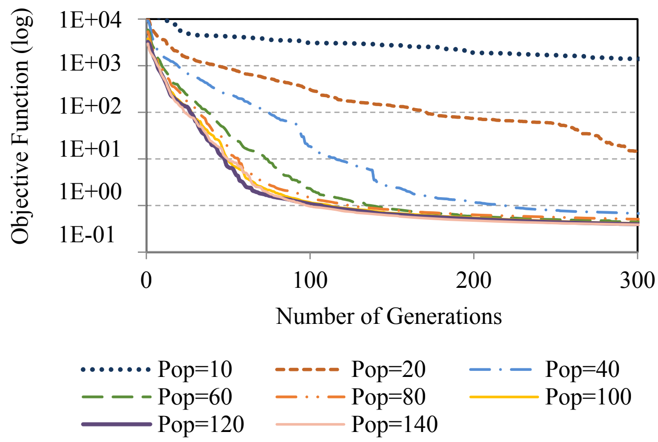
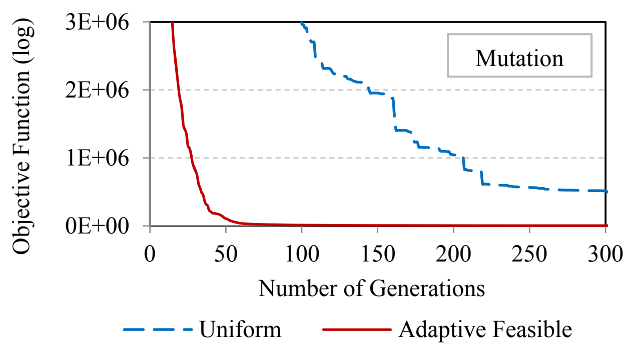
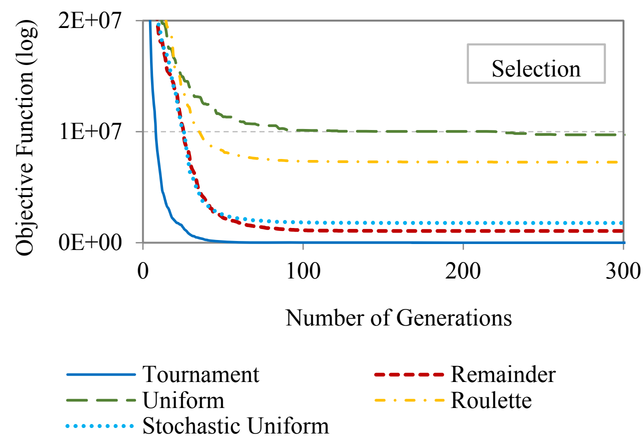


Figure 3 Convergence curve in case of different population number averaged over 20 runs for the GA.





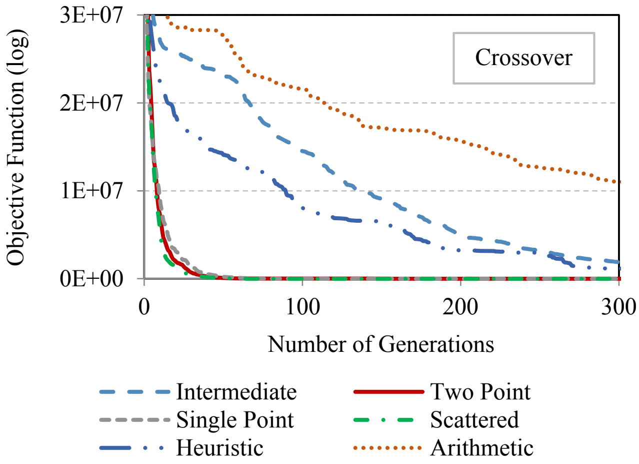


Figure 4 Convergence curve in case of: top - mutation operators, middle - selection processes and bottom- crossover operators.

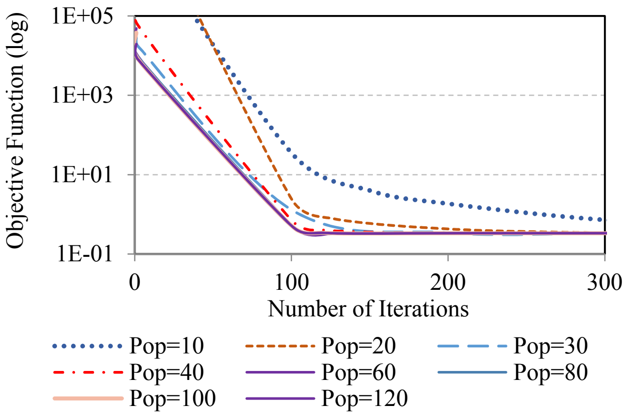


Figure 5 Convergence curve in case of different population number averaged over 20 runs for the CMA-ES.

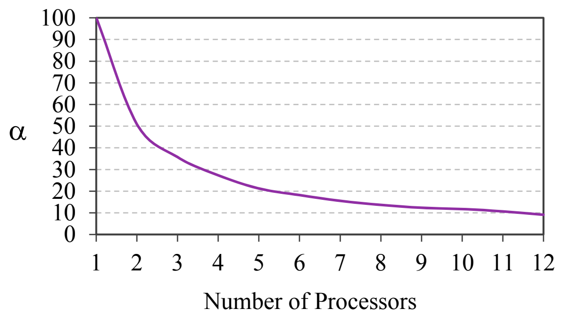


Figure 6 Time improvement using different number of processors

5.4 *Scenario (i) An Accurate Numerical Model and Noise-Free Response Data*

Figure 7 plots results of the damage identification process for both algorithms for single, multiple and irregular locations of damage. The figures clearly demonstrate that in the condition of noise free response data and an accurate numerical model, the proposed method is able to perfectly localize and determine the damage severity introduced into the beam, in the case of single, multiple and irregular damage locations. Although, the results of damage identification given are for 5% to 20% damage severity level, the method is capable of detecting damage even as low as 1% damage severity.

5.5 *Scenario (ii) An Approximate Numerical Model and Noisy Response Data*

The results presented in Figures 8 and 9 and Tables 2 to 5 are obtained using noisy response data and incomplete modal data for both PGA and P-CMA-ES algorithms. The results obtained for both algorithms are very similar and clearly show that the proposed methods are capable of locating and estimating the stiffness loss in the presence of noise ranging from 5 to 10% and by considering only 50% of the original number of flexural modes for single and multiple locations of damage. For a single location of damage, the maximum error observed is about 14% corresponding to the 15% level of damage and 10% noise condition given in Tables 2 and 3.

In the case of multiple locations of damage, the proposed approach could accurately predict the location as well as the severity of damage in the condition of various levels of noise, 5 and 10% for all damage levels (regular). The maximum predicted error observed is in Tables 4 and 5 for 10% damage and 10% noise level and is about -28% for the multiple regular damage condition for both algorithms. However, this error level observed may not necessarily reflect the performance of the proposed technique as the noise level is quite high and the damage level is quite small. By increasing the level of damage, the estimated error reduced to only -7% for 20% damage level for both algorithms.

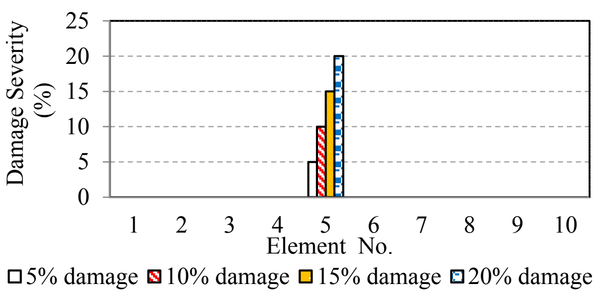
Furthermore, the damage identification results for cases where irregular levels of damage are applied, are presented in Figures 8 and 9. The results show that the proposed method is found to accurately predict the location of irregular damage with the noise level ranging from 5 to 10%. The severity of damage was predicted with the maximum estimated error of -16% for 5% noise and -29% for 10% noise for the small damage level and the maximum of -4% for 15% damage level and 10% noise for both PGA and P-CMA-ES algorithms given in Tables 4 and 5. It also can be clearly seen that the P-CMA-ES is able to estimate the damage with a slightly better approximation than the PGA for multiple damage locations (Tables 4 and 5).

5.6 *Scenario (iii) An Approximate Numerical Model and Incomplete Noisy Response Data*

In this section, damage identification results of a more practical damage scenario are presented in Figures 10 to 13 and Tables 2 and 3 using a PGA and a P-CMA-ES. The results clearly show that using both algorithms, the method is capable of both locating and estimating the stiffness loss for a single location of damage in the presence of significant noise levels ranging from 5 to 10% and by considering only 50% of the original number of both grid points and vibration modes. In the case of 5% noise level, the method could perfectly indicate the location of damage which was in the middle of the beam. However, in the case of 10% noise, a false identified location is seen in element 10 for all damage extents. The prediction error for estimated severity is decreased by increasing the level of damage. Therefore, the maximum error observed using a PGA is about -15% and -19% for 5% and 10% noise and about -21% and -16% using the P-CMA-ES corresponding to the 5% level of damage. However, this error level observed may not necessarily reflect the general performance of the proposed technique as the noise level is quite high and the damage level is quite small.

In the case of multiple locations of damage and for high levels of noise; 5 and 10%, and a low level of damage, the proposed method is found to have limitations in predicting the location as well as the extent of damage for both regular and irregular multiple damage conditions. However, for 5 and 10% noise, both algorithms failed to predict the exact locations and severity of damage (Figures 10 to 13).

Comparing the results of the PGA in this scenario and considering the number of populations in both algorithms, it is revealed that the results are slightly more accurate using the P-CMA-ES and the algorithm can overcome the problem of local optima better to progress the process of optimization and to return a more accurate solution than otherwise.



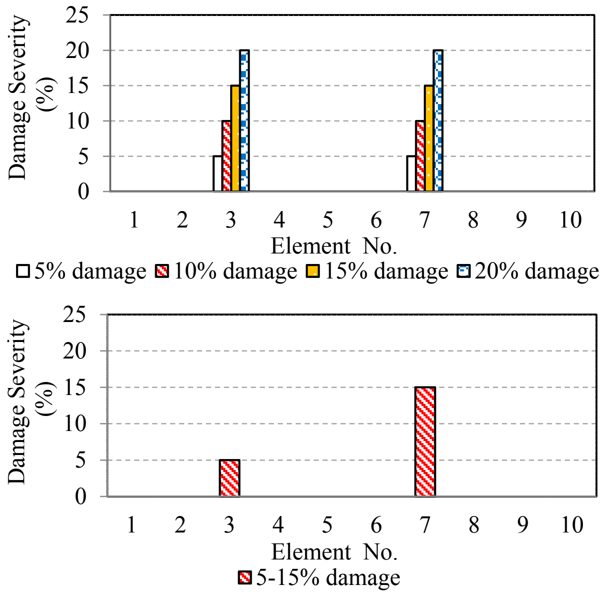


Figure 7 Damage identification using the Bayesian approach of MSV response PSD in the case of a PGA or a P-CMA-ES: top-single, middle-multiple, bottom- multiple irregular in a noise free environment (Scenario (i)).

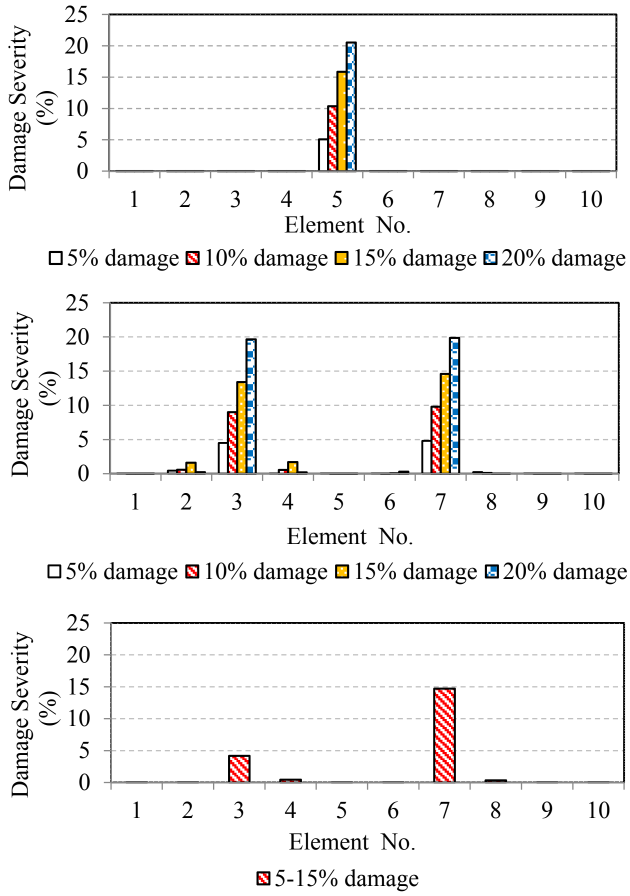


Figure 8 Damage identification using the Bayesian approach of MSV response PSD, a PGA or a P-CMA-ES: top-single, middle-multiple, bottom- multiple irregular with 5% noise for response data (Scenario (ii)).

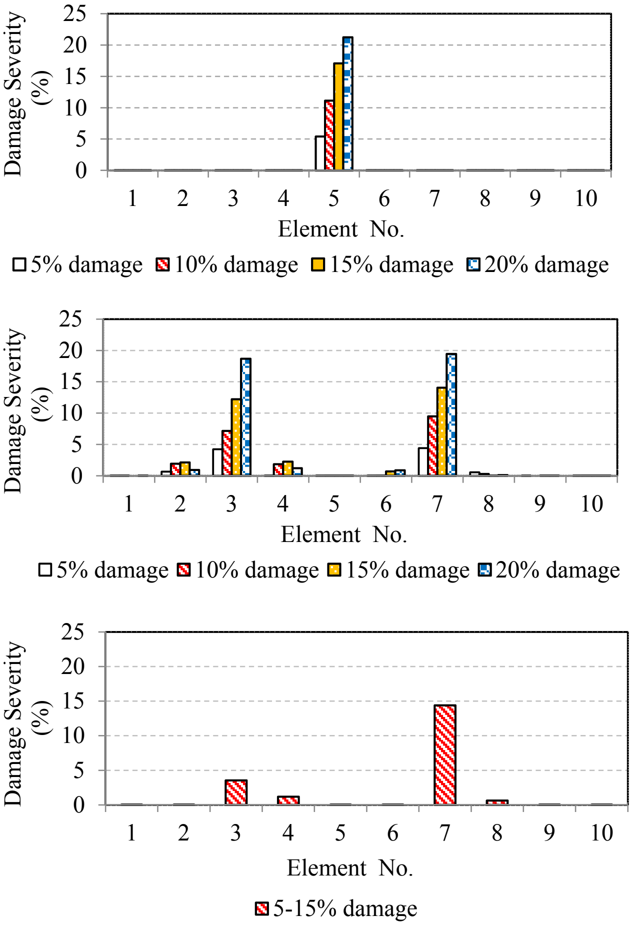
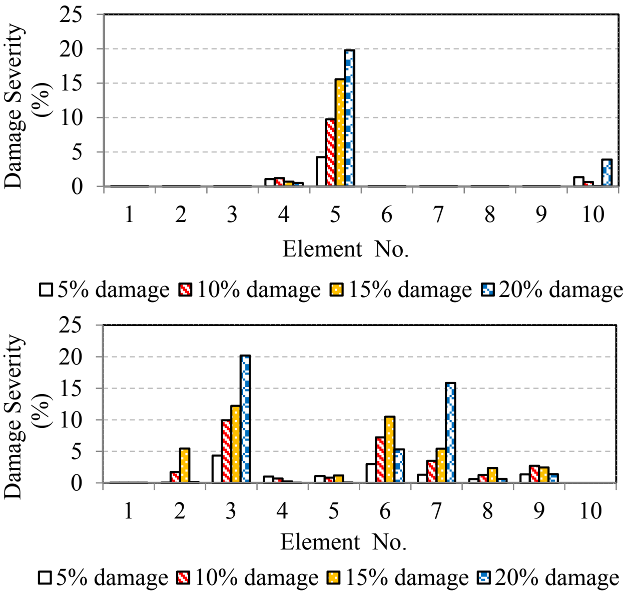


Figure 9 Damage identification using the Bayesian approach of MSV response PSD, a PGA or a P-CMA-ES: top-single, middle-multiple, bottom- multiple irregular with 10% noise for response data (Scenario (ii)).



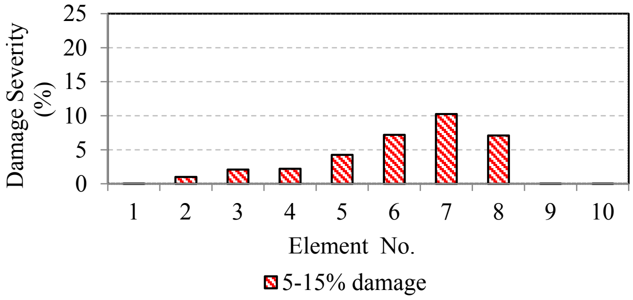


Figure 10 Damage identification using the Bayesian approach of MSV response PSD and a PGA: top-single, middle-multiple, bottom- multiple irregular with 5% noise for response data (Scenario (iii)).

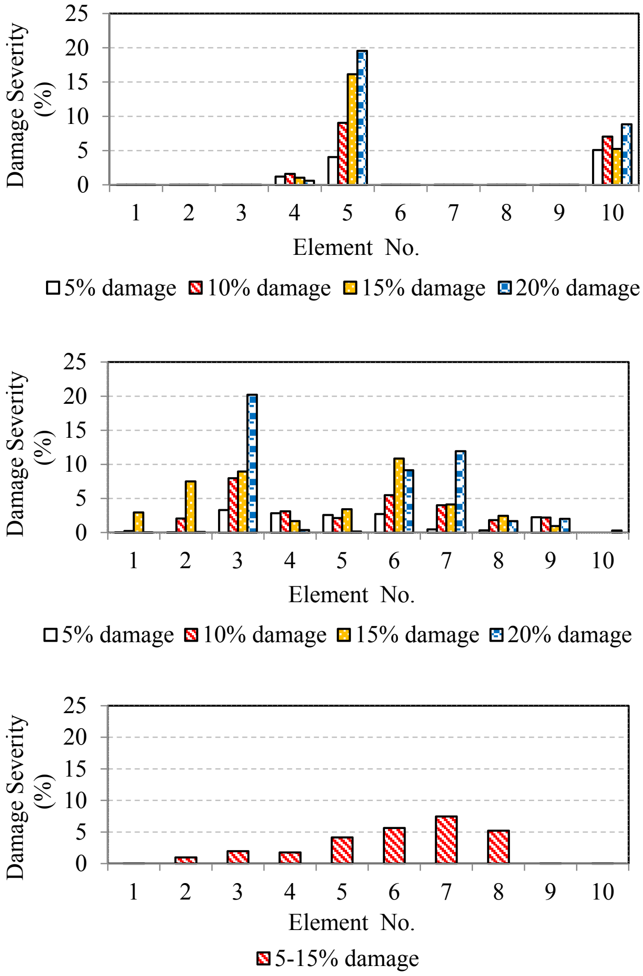


Figure 11 Damage identification using the Bayesian approach of MSV response PSD and a PGA: top-single, middle-multiple, bottom- multiple irregular with 10% noise for response data (Scenario (iii)).

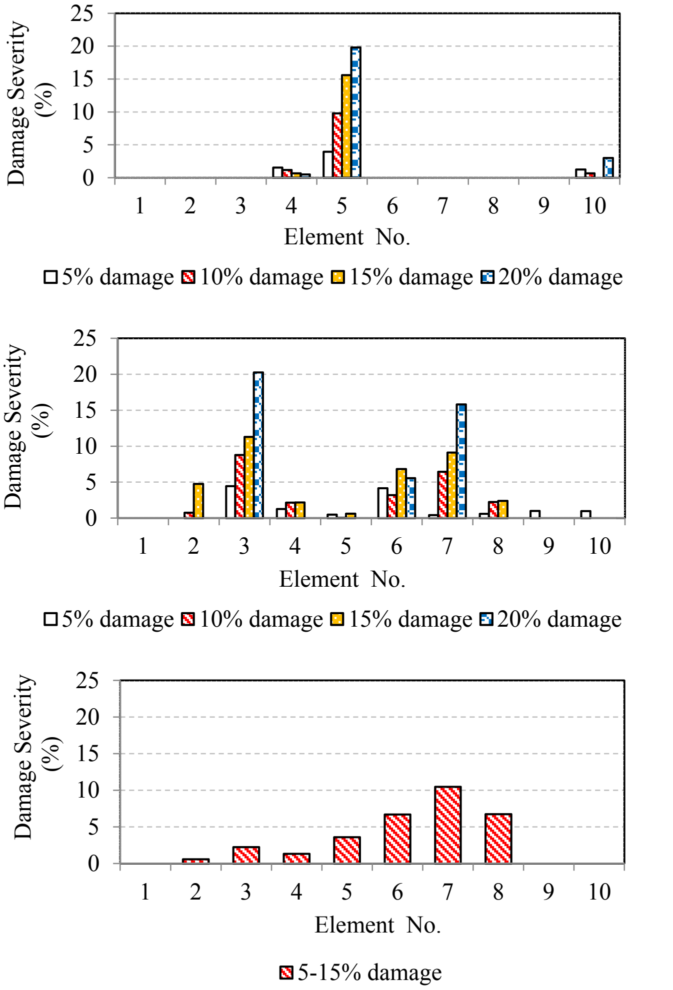
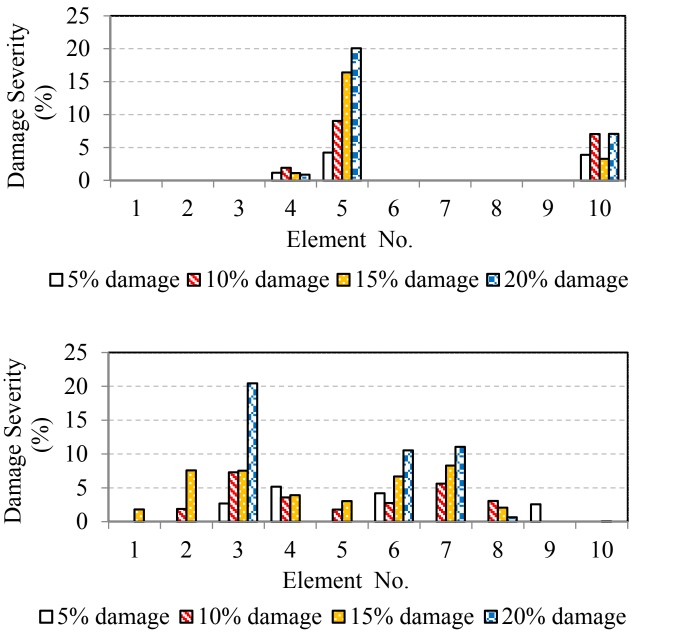


Figure 12 Damage identification using the Bayesian approach of MSV response PSD and a P-CMA-ES: top-single, middle-multiple, bottom- multiple irregular with 5% noise for response data (Scenario (iii)).



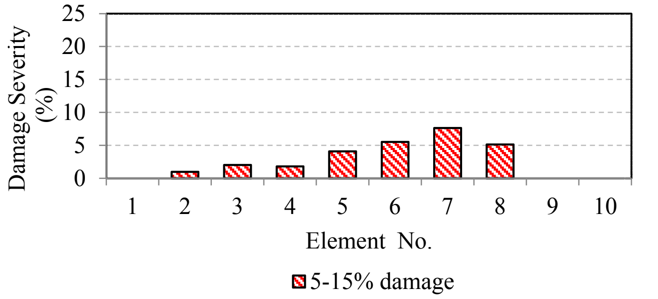


Figure 13 Damage identification using the Bayesian approach of MSV response PSD and a P-CMA-ES: top-single, middle-multiple, bottom- multiple irregular with 10% noise for response data (Scenario (iii)).

5.7 *Comparative study*

In this section, a series of comparative studies between the GA and CMA-ES are carried out to identify the more effective solution technique considering the accuracy of the results, the number of fitness function evaluations and using different statistical tests.

In the preceding sections, the performance of the PGA and P-CMA-ES applied to the Bayesian objective function are investigated and the results demonstrated that both algorithms were capable of detecting the location and estimating the degree of damage reasonably accurately as a damage identification process. However, the CMA-ES algorithm deals better with the condition of multiple locations of damage and incomplete noisy response data. Moreover, the CMA-ES exhibits superior performance, overcoming the problem of local optima, returning an improved (lower) value for the objective function.

Despite the significant performance capabilities of the evolutionary algorithms in this domain, they were found to be computationally intensive and required a large number of fitness evaluations. An algorithm that produces a (near) optimal result in the least number of function evaluations should be preferred over the other algorithm. Therefore, in the next stage of analysis, the number of fitness evaluations required to reach the optima is examined closely. Figures 14 and 15 plot the results averaged over running 5 trials. The population size is fixed at 120 individuals for the GA and 60 individuals for the CMA-ES.

Table 2
Single damage (element 5) identification results of averaged over 5 runs by the PGA

Case	Noise	Degree of damage (%)	Predicted damage (%)	Generation No.	Predicted Error (%)
Scenario (ii)	5%	5	5.07	1493	+1.46
		10	10.36	1865	+3.64
		15	15.85	1743	+5.64
		20	20.52	4033	+2.59
Scenario (ii)	10%	5	5.41	2498	+8.26
		10	11.11	1943	+11.14
		15	17.08	1010	+13.86
		20	21.22	3358	+6.12
Scenario (iii)	5%	5	4.25	1496	-15.05
		10	9.76	2209	-2.44
		15	15.56	1821	+3.72
		20	19.78	1218	-1.09
Scenario (iii)	10%	5	4.05	1428	-19.01
		10	9.03	1362	-9.65
		15	16.13	11559	+7.54
		20	19.56	1000	-2.21

Table 3
Single damage (element 5) identification results of averaged over 5 runs by the P-CMA-ES

Case	Noise	Degree of damage (%)	Predicted damage (%)	Iteration No.	Predicted Error (%)
Scenario (ii)	5%	5	5.10	205	+2.10
		10	10.39	170	+3.90
		15	15.87	184	+5.82
		20	20.54	191	+2.70
Scenario (ii)	10%	5	5.44	186	+8.85
		10	11.15	184	+11.45
		15	17.10	214	+14.03
		20	21.24	212	+6.21
Scenario (iii)	5%	5	3.96	205	-20.84
		10	9.77	232	-2.29
		15	15.59	312	+3.95
		20	19.80	226	-1.02
Scenario (iii)	10%	5	4.22	221	-15.66
		10	9.04	206	-9.55
		15	16.41	243	+9.41
		20	20.06	198	+0.32

Table 4
Multiple damage (elements 3, 7) identification results of averaged over 5 runs by the PGA

Case	Noise	Degree of damage at elements 3, 7 (%)		Predicted damage at elements 3, 7 (%)		Generation No.	Prediction Error (%)	
Scenario (ii)	5%	5	5	4.49	4.81	1736	-10.19	-3.82
		10	10	9.01	9.80	2685	-9.95	-1.98
		15	15	13.40	14.59	3229	-10.67	-2.70
		20	20	19.64	19.85	2933	-1.82	-0.74
		5	15	4.19	14.71	2535	-16.19	-1.97
Scenario (ii)	10%	5	5	4.24	4.42	1598	-15.14	-11.57
		10	10	7.15	9.48	2618	-28.48	-5.17
		15	15	12.21	14.06	2960	-18.60	-6.27
		20	20	18.67	19.43	3119	-6.67	-2.84
		5	15	3.55	14.40	1512	-29.02	-4.03

Table 5
Multiple damage (elements 3, 7) identification results of averaged over 5 runs by the P-CMA-ES

Case	Noise	Degree of damage at elements 3, 7 (%)		Predicted damage at elements 3, 7 (%)		Iteration No.	Prediction Error (%)	
Scenario (ii)	5%	5	5	4.51	4.80	185	-9.80	-4.00
		10	10	9.02	9.80	194	-9.80	-2.00
		15	15	13.40	14.61	196	-10.67	-2.60
		20	20	19.68	19.86	195	-1.60	-0.70
		5	15	4.21	14.72	164	-15.80	-1.87
Scenario (ii)	10%	5	5	4.26	4.41	172	-14.80	-11.80
		10	10	7.19	9.49	146	-28.10	-5.10
		15	15	12.20	14.07	149	-18.67	-6.20
		20	20	18.66	19.56	132	-6.70	-2.20
		5	15	3.56	14.41	145	-28.80	-3.93

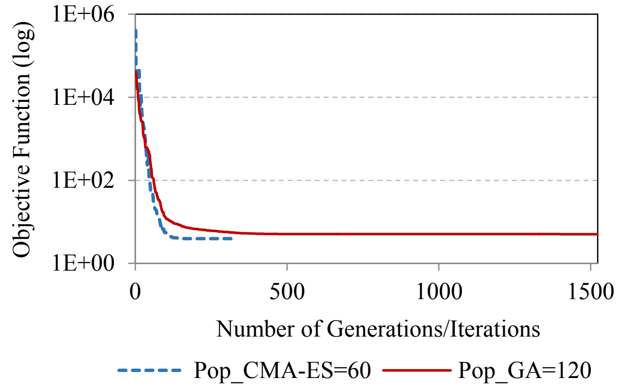


Figure 14 Convergence curve for the CMA-ES with 60 number of populations and the GA with 120 number of populations.

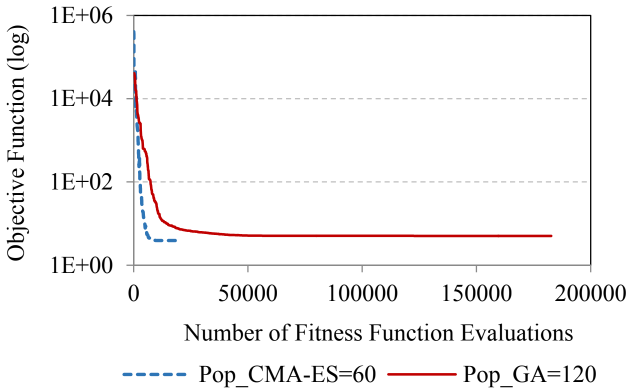


Figure 15 Convergence curve for the CMA-ES and GA in terms of number of function evaluations

It can be clearly seen from the graphs that both algorithms improve (decrease the objective function value) with increasing number of generations/iterations but the CMA-ES compared to the GA reaches the optima in a significantly fewer number of generations/iterations and lower number of function evaluations.

The first statistical analysis between the two algorithms is conducted using a t-test. The t-test is a statistical test with a null (or H-naught) hypothesis. H and P are two outputs of the t-test in which the H-value either accepts the hypothesis with the value of 0 or rejects it with the value of 1. The P-value indicates the significance of the difference; the smaller the P-value, the more significant the difference between the two data series (Milton and Arnold 1995).

A number of statistical tests were then carried out to determine if there was a significant difference between the algorithms examined. The null hypothesis tested was that there was no significant difference between the values of objective function by each algorithm. Consequently, a t-test in Microsoft Excel was conducted. For an equal number of data sets of 50 for each series and unequal variance, the null hypothesis is rejected which means there is a significant difference between the two data series (p-value = 9.96E-17). This indicates that the means of both data series are significantly different and these two series are not to be treated the same. The mean value of the GA is much larger than the CMA-ES considering the variances of the two data samples. Since the goal is to minimize the objective function, the CMA-ES is then selected as the one with the lowest objective function values.

The second statistical test has been conducted to compare the performance of the two algorithms (Milton and Arnold 1995). In this comparison study between the GA and CMA-ES, the best values of objective functions from the GA trials are labeled as Y_i and the CMA-ES trails are considered $Y_{i'}$ where i = 1 to n, and n is the number of trials equal to 30. The performance difference of the two algorithms is specified as $Z_i = Y_i - Y_{i'}$. The associated mean and quasi-variance are computed based on the following equations respectively:

$$Z(n) = \frac{\sum_i Z_i}{n} \tag{21}$$

$$S^2(n) = \frac{\sum_i [Z_i - Z(n)]^2}{n-1} \tag{22}$$

The lower and upper limits of the mean value of Z can be obtained as follows:

$$Z(n) \pm t_{n-1,1-\alpha/2}\sqrt{\frac{S^2(n)}{n}} \tag{23}$$

The number of populations for both algorithms is analogous to the number of iterations in CMA-ES. The number of 50 trials has been conducted and the best 30 trials are selected to compare the performance. The objective function values are compared using Equations 21 to 23 and are reflected in Table 6. From Table 6, it can be seen that the fitness function improves with more function evaluations and finally at a generation/iteration number of 300, the best (minimum) objective function value can then be obtained.

In the case that the upper and lower limits of Z_i are positive, the GA fitness function is greater than CMA-ES for a confidence interval of 95% ($\alpha = 0.1$), indicating that the CMA-ES is performing better for minimization of the problem. As in no cases, is the upper and lower limits of Z_i negative, it can be concluded that CMA-ES outperforms the GA in all cases even with increasing number of generations/iterations.

6 DISCUSSION AND CONCLUDING REMARKS

In this paper, a one-stage model-based damage identification technique using the MSV of response PSD and a Bayesian probabilistic approach has been presented. An evolutionary algorithm is then employed to update the finite element model. A GA and a CMA-ES were implemented on a Bayesian probabilistic objective function to solve an inverse problem which takes into account modelling and measurement uncertainties and the subsequent investigation of the existence, location, and estimation of structural damage severity.

A 10 element beam structure was investigated to evaluate the performance of the proposed approach. A range of different damage scenarios was considered: both single location and multiple damage locations; varying damage severity; the introduction of noise and modelling errors and incompleteness in the number of captured modes and measurement response data. The results obtained clearly show that the proposed approach is able to accurately detect the severity, as well as the location of damage, through a one-stage model-based damage identification process using both P-CMA-ES and PGA algorithms. However, in some cases when multiple damage locations are considered, the accuracy of location and severity of damage found is affected by high levels of noise in the condition of incomplete response data. Problems such as multi-dimensionality, ill-conditioning and non-linearity were overcome via employment of the one-stage damage identification approach, which highlights the outstanding features of the proposed damage detection technique. Furthermore, the performance of the covariance matrix adaptation evolution strategy has been compared with the genetic algorithm in terms of number of function evaluations and using different statistical tests and the CMA-ES demonstrated a superior performance in the area of damage identification that needs a lower number of generated function evaluations to obtain optima.

The results also indicate that the MSV of response PSD is sensitive to structural damage existence, location and damage severity. Moreover, the parallel deployment of the CMA-ES and GA has been investigated and the results demonstrate significant improvement in speeding up the optimization process.

In this study, the effectiveness of the proposed method was limited to an investigation on a beam structure. In future work, it would be beneficial to conduct further studies on plate structures. It will also be useful to implement real experimental data from a structure to further demonstrate the effectiveness of the proposed method.

Table 6
Comparison of GA and CMA-ES

Populations	Generations/ Iterations	$Z(n)$	(n)	$S^2(n)$	LL	UL	*Conclusion*
60	50	58.18	30	3350.58	+76.13	+40.22	CMA-ES is better
60	150	5.09	30	3.12	+5.64	+4.54	CMA-ES is better
60	200	3.13	30	1.10	+3.46	+2.81	CMA-ES is better
60	300	1.40	30	0.34	+1.59	+1.22	CMA-ES is better

7 REFERENCES

Adeli, H. and Kumar, S., "Distributed Genetic Algorithm for Structural Optimization,". *Journal of Aerospace Engineering*, 8(3), 1995, 156-163.

Au, S.-K. and Zhang, F.-L., "Ambient Modal Identification of A Primary–Secondary Structure By Fast Bayesian FFT Method," *Mechanical Systems and Signal Processing,* 28, 2012, 280-296.

Auger, A. and Hansen, N., "Performance Evaluation of An Advanced Local Search Evolutionary Algorithm," Paper presented at the Evolutionary Computation, 2005a, The 2005 IEEE Congress on.

Auger, A. and Hansen, N., "A Restart CMA Evolution Strategy With Increasing Population Size," Paper presented at the Evolutionary Computation, 2005b, The 2005 IEEE Congress.

Austrell, P. E. , Dahlblom, O. and Lindemann, J., "CALFEM: A Finite Element Toolbox," Version 3.4: *Structural Mechanics*, 2004, LTH.

Bayissa, W. and Haritos, N., "Structural Damage Identification in Plates Using Spectral Strain Energy Analysis," *Journal of Sound and Vibration*, 307(1), 2007, 226-249.

Bayissa, W. L., "Damage Identification and Condition Assessment of Civil Engineering Structures Through Response Measurement," PhD thesis, The University of Melbourne, Department of Civil and Environmental Engineering, 2007.

Bayissa, W. L. , Haritos, N. and Sofi, M., "Comparative Study of Broadband Damage Localization Methods Applied to Test Data," *Ocean engineering*, 38(2-3), 2011, 329.

Beck, J. L. and Au, S.-K., "Bayesian Updating of Structural Models and Reliability Using Markov Chain Monte Carlo Simulation," *Journal of Engineering Mechanics*, 128(4), 2002, 380-391.

Cantu Paz, E., "Efficient and Accurate Parallel Genetic Algorithms," Kluwer Academic Publishers, 2000.

Carden, E. P. and Fanning, P., "Vibration Based Condition Monitoring," a review. *Structural health monitoring*, 3(4), 2004, 355.

Cheung, S. H. and Beck, J. L., "Bayesian Model Updating Using Hybrid Monte Carlo Simulation With Application to Structural Dynamic Models With Many Uncertain Parameters," *Journal of Engineering Mechanics*, 135(4), 2009, 243-255.

Doebling, S. W. , Farrar, C. R. and Prime, M. B., "A Summary Review of Vibration-Based Damage Identification Methods," *Shock and vibration digest*, 30(2), 1998, 91-105.

Fan, W. and Qiao, P., "Vibration-Based Damage Identification Methods: a Review and Comparative Study," *Structural health monitoring*, 10(1), 2011, 83.

Gelman, A. , Carlin, J. , Stern, H. and Rubin, D., "Bayesian Data Analysis," Second Edition (Chapman & Hall/CRC Texts in Statistical Science): Chapman and Hall/CRC, 2003.

Gilks, W. R. , Richardson, S. and Spiegelhalter, D. J., "Markov Chain Monte Carlo in Practice," Chapman & Hall, 1996.

Goldberg, D., "Genetic Algorithms in Search, Optimization and Machine Learning," Addison-Wesley Longman Publishing Co., Inc, 1989.

Guo, H. Y. and Li, Z. L., "Structural Damage Identification Based on Bayesian Theory and Improved Immune Genetic Algorithm," *Expert Systems with Applications*, 39(7), 2012, 6426-6434. doi: 10.1016/j.eswa.2011.12.023.

Hansen, N., "Benchmarking a BI-Population CMA-ES on the BBOB-2009 Function Testbed," Paper presented at the Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers, 2009.

Hansen, N. , Auger, A. , Ros, R. , Finck, S. and Pošík, P., "Comparing Results of 31 Algorithms From the Black-Box Optimization Benchmarking," BBOB-2009. Paper presented at the Proceedings of the 12th annual conference companion on Genetic and evolutionary computation, 2010.

Hansen, N. and Kern, S., "Evaluating the CMA Evolution Strategy on Multimodal Test Functions," Paper presented at the Parallel Problem Solving from Nature-PPSN VIII, 2004.

Hansen, N. and Ostermeier, A., "Completely Derandomized Self-Adaptation in Evolution Strategies," *Evolutionary computation*, 9(2), 2001, 159-195.

Hsiao, F. Y. , Wang, S. H. , Wang, W. C. , Wen, C. P. and Yu, W. D., "Neuro-Fuzzy Cost Estimation Model Enhanced By Fast Messy Genetic Algorithms for Semiconductor Hookup Construction," *Computer-Aided Civil and Infrastructure Engineering*, 27(10), 2012, 764-781.

Igel, C. , Hansen, N. and Roth, S., "Covariance Matrix Adaptation for Multi-Objective Optimization,". *Evolutionary computation*, 15(1), 2007, 1-28.

Jafarkhani, R. and Masri, S. F., "Finite Element Model Updating Using Evolutionary Strategy for Damage Detection," *Computer-Aided Civil and Infrastructure Engineering*, 26(3), 2011, 207-224.

Koh, B. H. , Choi, J. H. and Jeong, M. J., "Damage Detection through Genetic and Swarm-Based Optimization Algorithms," International Conference on Engineering, Science, Construction and Operations in Challenging Environments, 4, 2010, 2330.

Meruane, V. and Heylen, W., "An Hybrid Real Genetic Algorithm to Detect Structural Damage Using Modal Properties," *Mechanical systems and signal processing*, 25(5), 2011, 1559.

Milton, J. and Arnold, J. C., "Introduction to Probability and Statistics: Principles and Applications for Engineering and the Computing Sciences," McGraw-Hill (New York), 1995.

Newland, D. E. "An Introduction to Random Vibrations and Spectral Analysis," Longman, 1984.

Osornio-Rios, R. A. , Amezquita-Sanchez, J. P. , Romero-Troncoso, R. J. and Garcia-Perez, A., "MUSIC-ANN Analysis for Locating Structural Damages in a Truss-Type Structure by Means of Vibrations," *Computer-Aided Civil and Infrastructure Engineering*, 27(9), 2012, 687-698.

Ostermeier, A. , Gawelczyk, A. and Hansen, N., "A Derandomized Approach to Self-Adaptation of Evolution Strategies," *Evolutionary Computation*, 2(4), 1994, 369-380. doi: 10.1162/evco.1994.2.4.369

Perera, R. , Fang, S.-E. and Ruiz, A., "Application of Particle Swarm Optimization and Genetic Algorithms to Multiobjective Damage Identification Inverse Problems With Modelling Errors," *Meccanica*, 45(5), 2010, 723.

Perera, R. and Torres, R., "Structural Damage Detection via Modal Data with Genetic Algorithms," *Journal of Structural Engineering*, 132(9), 2006, 1491-1501. doi: 10.1061/(asce)0733-9445(2006)132:9(1491)

Punch, W. F., "How Effective Are Multiple Populations in Genetic Programming," *Genetic Programming*, 98, 1998, 308-313.

Putha, R. , Quadrifoglio, L. and Zechman, E., "Comparing Ant Colony Optimization and Genetic Algorithm Approaches for Solving Traffic Signal Coordination Under Oversaturation Conditions," *Computer-Aided Civil and Infrastructure Engineering*, 27(1), 2012, 14-28.

Reeves, C. R. and Rowe, J. E.. "Genetic Algorithms: Principles and Perspectives : a Guide to GA Theory," Kluwer Academic Publishers, 2002.

Salawu, O. S., "Detection of Structural Damage Through Changes In Frequency: a Review," *Engineering structures*, 19(9), 1997, 718.

Sandesh, S. and Shankar, K., "Application of a Hybrid of Particle Swarm and Genetic Algorithm for Structural Damage Detection," *Inverse Problems in Science & Engineering*, 18(7), 2010, 997-1021. doi: 10.1080/17415977.2010.500381

Seyedpoor, S. M., "A Two Stage Method for Structural Damage Detection Using a Modal Strain Energy Based Index and Particle Swarm Optimization," *International Journal of Non-Linear Mechanics*, 47(1), 2012, 1-8. doi: 10.1016/j.ijnonlinmec.2011.07.011

Sohn, H. and Law, K. H., "Bayesian Probabilistic Damage Detection of a Reinforced-Concrete Bridge Column," *Earthquake Engineering & Structural Dynamics, 29*(8), 2000, 1131-1152. doi: 10.1002/1096-9845(200008)29:8<1131::aid-eqe959>3.0.co;2-j

Varmazyar, M., "Response Measurement Approaches towards Damage Identification Using Evolutionary Algorithms," PhD Thesis, The University of Melbourne, Department of Infrastructure Engineering, 2013, http://hdl.handle.net/11343/40977

Varmazyar, M. , Haritos, N. , Kirley, M. and Peterson, T., "A one Stage Damage Detection Technique Using Spectral Density Analysis and Parallel Genetic Algorithms," *Key Engineering Materials, 558*, 2013, 1-11. doi: 10.4028/www.scientific.net/KEM.558.1

Yuen, K.-V. and Katafygiotis, L. S., "Bayesian Time–Domain Approach for Modal Updating Using Ambient Data," *Probabilistic Engineering Mechanics, 16*(3), 2001, 219-231.

Yuen, K. V. and Katafygiotis, L. S., "Model Updating Using Noisy Response Measurements Without Knowledge of the Input Spectrum," *Earthquake engineering & structural dynamics,* 34(2), 2005, 167-187.