eJSE International
Electronic Journal of Structural Engineering 14 - 2014

Design of Steel I-Beams: A Proposed Equation

E.Y. Sayed-Ahmed*
Construction and Architectural Engineering Dept., the American University in Cairo
on-leave from the Structural Engineering Dept., Faculty of Engineering, Ain Shams University, Egypt
*eysahmed@gmail.com / eysahmed@aucegypt.edu

H.A. Mustafa, A.A. Elserwi, and A.K. Dessouki
Structural Engineering Dept., Faculty of Engineering, Ain Shams University, Egypt

ABSTRACT: Design of laterally unsupported steel I-section beams according to Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD) techniques involves usage of multiple equations. According to most codes of practice, three distinct zones are established for the behaviour of laterally unsupported steel beams. Each of these zones is defined by a different design equation. In this paper, a single equation which defines the moment resistance is suggested for the three zones defining the behaviour of laterally unsupported steel beams. Results of the proposed equations are compared to those obtained using the design provisions of the Egyptian, Canadian and American codes of practice (ECP-LRFD 2008, CAN/CSA S16-09 2009 and AISC 2010, respectively). The proposed equation is also verified against results obtained from non-linear numerical analysis based on the finite element technique for steel I-beams.

1 INTRODUCTION

Local buckling and lateral-torsional buckling significantly affect the behaviour of steel I-section beams subject to flexure. The beam's flexure resistance is governed by a combination of local and lateral-torsional buckling resistances which lead to different failure modes. As such, design of steel I-section beams require the use of multiple equations that are controlled by local and lateral-torsional buckling behaviour as well as steel yielding. For these beams, the moment resistance depends on the cross-section compactness, the laterally unsupported length of the beam, the geometric properties of the cross-section and the yield strength of the steel. According to most codes of practice, three distinct zones are specified for behaviour of steel beams: elastic, elasto-plastic and fully plastic behaviour. Each of these zone has an equation for defining moment resistance of the beam.

Previous investigations to simplify the design procedures were performed by Sayed-Ahmed (2004) and. The possibility of adopting a simplified equation to calculate the moment resistance of steel I-section beams following CAN/CSA S16 and AISC provisions was investigated (Sayed-Ahmed and Loov 2005). Sayed-Ahmed (2004) also proposed an alternative simple design equation for the allowable bending stress of laterally unsupported steel I-section beams following the ECP-ASD (2001). However, both investigations were limited to built-up I-beams with a very small range of application.

Here, a single equation is proposed for calculating the moment resistance of steel I-section beams considering an extended range of application. The equation covers the design procedures according to the LRFD provision of the Egyptian, Canadian and American codes of practice (ECP-LRFD 2008, CAN/CSA S16-09 2009 and AISC 2010). These codes have been selected since they are the common codes of practice adopted in design of steel structures in the Middle East region. The proposed equation would cover all the three mentioned distinct zones defining the behaviour of steel beams. It also includes all the parameters considered by the codes of practice for beam's design. Results obtained via the proposed equations have been compared to those obtained by adopting the AISC (2010), CAN/CSA S16-09 (2009), and ECP-LRFD (2008). The equation is verified for both simply supported and cantilevers beams with different moment gradients that deem to be representative for most loading cases. Then, a nonlinear numerical model for steel I-beams is developed, verified and adopted in confirming the applicability of the proposed equation.

2 PROPOSED DESIGN EQUATION

2.1 *Equation Proposed by Sayed-Ahmed and Loov (2005)*

Sayed-Ahmed and Loov (2005) proposed a simplified equation to calculate the moment resistance of steel I-section beams and compared its results to those of CAN/CSA S16 and AISC provisions. The equation takes the following form:

$$M_n = M\left[1+\left[\frac{M}{M_{cr}}\right]^n\right]^{\frac{1}{n}} \qquad (1)$$
$$M_r = \varphi \cdot M_n$$

where M_n is the beam's nominal flexural strength, M_r is the beam's moment resistance, φ is the flexural resistance factor defined by codes of practice, and M_{cr} is the critical moment initiating lateral buckling.

For CAN/CSA S16-09, M is a moment which is taken equal to M_p for Class 1 and Class 2 sections or M_y for Class 3 sections:

$$M_y = S_x \cdot F_y$$
$$M_p = Z_x \cdot F_y \qquad (2)$$

where S_x and Z_x are the elastic and plastic section moduli about the major axis of inertia, respectively. F_y is the yield strength of the steel. The section compactness (Section's Class) is defined via the flange outstand-to-thickness and the web height-to-thickness ratios. For I-sections' beams

$$Class\ 1: \quad \frac{b_{fl}}{2t_{fl}} \le \frac{145}{\sqrt{F_y}} \ \& \ \frac{h_w}{t_w} \le \frac{1100}{\sqrt{F_y}}$$
$$Class\ 2: \quad \frac{b_{fl}}{2t_{fl}} \le \frac{170}{\sqrt{F_y}} \ \& \ \frac{h_w}{t_w} \le \frac{1700}{\sqrt{F_y}} \qquad (3)$$
$$Class\ 3: \quad \frac{b_{fl}}{2t_{fl}} \le \frac{200}{\sqrt{F_y}} \ \& \ \frac{h_w}{t_w} \le \frac{1900}{\sqrt{F_y}}$$

where F_y is the yield stress of the steel in MPa unit, b_{fl} and t_{fl} (Figure 1) are the width and thickness of the flange, respectively. h_w and t_w are the height and thickness of the web, respectively.

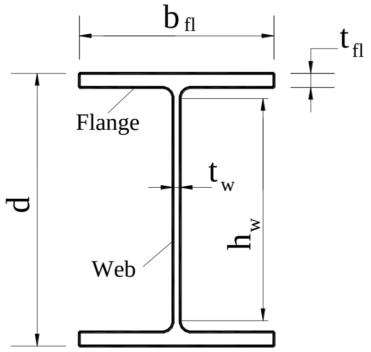

Figure 1. Typical notation for the cross-section of the steel I-beam adopted in the current investigation.

For AISC (2010) and ECP-LRFD (2008), M defined in Equation 1, is taken as

$$M = M_p - \left(M_p - M_y\right)\cdot\left(\frac{\lambda-\lambda_p}{\lambda_r-\lambda_p}\right) \le M_p \qquad (4)$$

where M_p and M_y are the plastic and yield moments, respectively and λ is the greater of the flange outstand-to-thickness ratio or the web height-to-thickness ratio. If both the flange and the web slenderness ratios are less than λ_p, the moment resistance should be based on M_p. It is worth mentioning that the AISC specifications consider the effect of the residual stresses on the yield moment via the following equation:

$$M_y = S_x \cdot \left(F_y - F_r\right) \qquad (5)$$

where F_r is the residual stress (69 MPa for hot-rolled sections and 114 MPa for built-up sections).

According to the AISC, the beam's cross-section compactness is defined by

$$Compact:$$
$$\frac{b_{fl}}{2t_{fl}} \le \lambda_p = 0.38\sqrt{\frac{E}{F_y}} \ \& \ \frac{h_w}{t_w} \le \lambda_p = 3.76\sqrt{\frac{E}{F_y}}$$
$$Non-compact: \qquad (6)$$
$$\frac{b_{fl}}{2t_{fl}} \le \lambda_r = 1.00\sqrt{\frac{E}{F_y}} \ \& \ \frac{h_w}{t_w} \le \lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

where E is the elastic modulus of the steel. The ECP-LRDF (2008) follows the same limits of Equation 6 in defining the section compactness.

The critical moment initiating lateral buckling is defined by

$$M_{cr} = C_b \cdot M_{ocr} = C_b \cdot \frac{\pi}{L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_y C_W} \qquad (7)$$

where, M_{cr} is the critical moment, L is the laterally unsupported length of the compression flange, E is the Young's modulus, I_y is the cross section moment of inertia about the weak axis, G is the shear modulus, J is the torsional constant and C_w is the warping constant. The ECP-LRFD (2008) introduces the following equation for the critical moment to

$$M_{cr} = S_x \cdot \sqrt{\left(\frac{1380 \cdot A_{fl}}{d \cdot L}\right)^2 + \left(\frac{20700}{\left(L/r_t\right)^2}\right)^2} \qquad (8)$$

where L is the beam's span, d is the overall depth of the cross-section of the I-beam, and A_{fl} is the area of the compression flange and 1/3 the compressive part of the web and r_t is the radius of gyration of this area (A_{fl}) about the section's minor axis of inertia.

CAN/CSA S16-09 defines the equivalent moment factor C_b by

$$C_b = 1.75 + 1.05 \cdot \left(\frac{M_A}{M_B} \right) + 0.3 \cdot \left(\frac{M_A}{M_B} \right)^2 \quad (9)$$

where M_A and M_B are the two end moments with M_A being the smaller one. The ratio M_A/M_B is positive for beams bent in double curvature and negative for beams bent in single curvature. CAN/CSA S16-09 adopts Equation 9 with an upper limit of 2.5 and uses C_b of 1.0 when bending moment between the end supports is greater than the end moments. On the other hand, AISC defines a general equation for C_b which is given by

$$C_b = \frac{12.5 \cdot M_{\max}}{3 \cdot M_1 + 4 \cdot M_2 + 3 \cdot M_3 + 2.5 \cdot M_{\max}} \quad (10)$$

where M_1, M_2, M_3 are the absolute values of the bending moments at the quarter, mid, and three quarter-points of the beam, respectively. $M_{max.}$ is the maximum moment acting on the beam.

Sayed-Ahmed and Loov (2005) investigation had a limited range of application: their study was verified against only three built-up I-beams. Furthermore, it only considered simply supported beams subjected to two equal and opposite moments. Thus, no consideration was given to various loading or end conditions and/or to the effect of moment gradient along the beam's length.

Mustafa (2011) further tested the applicability of the equation and recalibrated its parameter to CAN/CSA S16-09 (2009), AISC (2010) and ECP-LRFD (2008). The equation was further enhanced by including the effect of moment gradient.

2.2 *Application of the Proposed Equation to I-Beams Subjected to Constant Moment*

In order to adjust the exponent n of Equation 1 and recalibrate the equation, moment resistance of a wide range of hot-rolled I-section beams is considered in a comparative analysis (Mustafa 2011). The analysis is first based on calculating the nominal moments of simply supported beams composed of I-sections (IPE, HEB, SIB, built-up, mono-symmetric sections) and subjected to two equal and opposite moments (C_b = 1.0). The results of this analysis are compared to the moments calculated via the procedures adopted by AISC (2010), CAN/CSA S16-09 (2009) and ECP-LRDF (2008). Two common steel grades adopted in Egypt and Europe are considered in the analysis; these are S235 and S355 (ST37 and ST52) with yield strengths of 240 MPa and 360 MPa, respectively.

As such, the exponent n of Equation 1 is calibrated to 2.8 for the AISC provisions and 4.0 for both CAN/CSA S16-09 and ECP-LRFD provisions. For the majority of sections, these values of n approximately equalize the maximum and minimum differences between the proposed equation results and the codes' prediction for moment resistance of I-beams.

Samples of the analysis outcomes and the percentages of difference between nominal moment prediction of AISC (2010), CAN/CSA S16-09 (2009), and the ECP-LRFD (2008) and that of the proposed equation are shown in Figures 2, 3 and 4.

The figures include only beams with IPE sections as an indicative for the analysis. A wider comparative investigation for beams with other hot rolled and built-up sections is presented elsewhere (Mustafa 2011). However, the same behaviour was recorded for all the analysed cross sections.

Figures 2 to 4 indicate a good agreement of the proposed equation results with those obtained via the considered codes of practice provisions.

2.3 *Moment Gradient*

To this point, the proposed equation is calibrated against simply supported beams subjected to two equal and opposite moments (i.e. C_b = 1.0). A major challenge was to account for different loading (moment gradient) and boundary conditions.

In order to account for the said moment gradient, Mustafa (2011) related the exponent n of the proposed design equation to the equivalent moment factor C_b. Thus, the proposed equation is revised to

$$M_n = M \left[1 + \left[\frac{M}{M_{cr}} \right]^{n \cdot C_b} \right]^{\frac{1}{n \cdot C_b}} \quad (11)$$

$$M_r = \varphi \cdot M_n$$

where the exponent n in Equation 11 is 4.0 as adopted in the above mentioned investigation and C_b is the equivalent moment factor estimated using Equation 10. A wide range of hot-rolled (IPE, SIB and HEB) and built-up section beams have been considered in the verification analysis of Equation 11 with both the steel grades mentioned earlier. Cantilever beams subjected to uniformly distributed loads and tip concentrated loads were analysed. Using Equation 10, the equivalent moment factors C_b for the considered two loading cases were calculated to be 2.3 and 1.67, respectively (Mustafa 2011).

Nominal moment obtained using the proposed equation is compared to the moment predicted via the ECP-LRFD (2008) provisions. Samples of the investigation outcomes are shown in Figure 5 and 6. Once again, only beams with IPE and ST52 are shown here; the rest of the comparative study is presented elsewhere (Mustafa 2011). The same behaviour was recorded for all the analysed cross sections.

The investigation reveals that the deviation between the codes prediction and the proposed design equation is less than 8%. It is also evident from the analysis that this difference is consistently located at the elasto-plastic zone of the beams' behaviour.

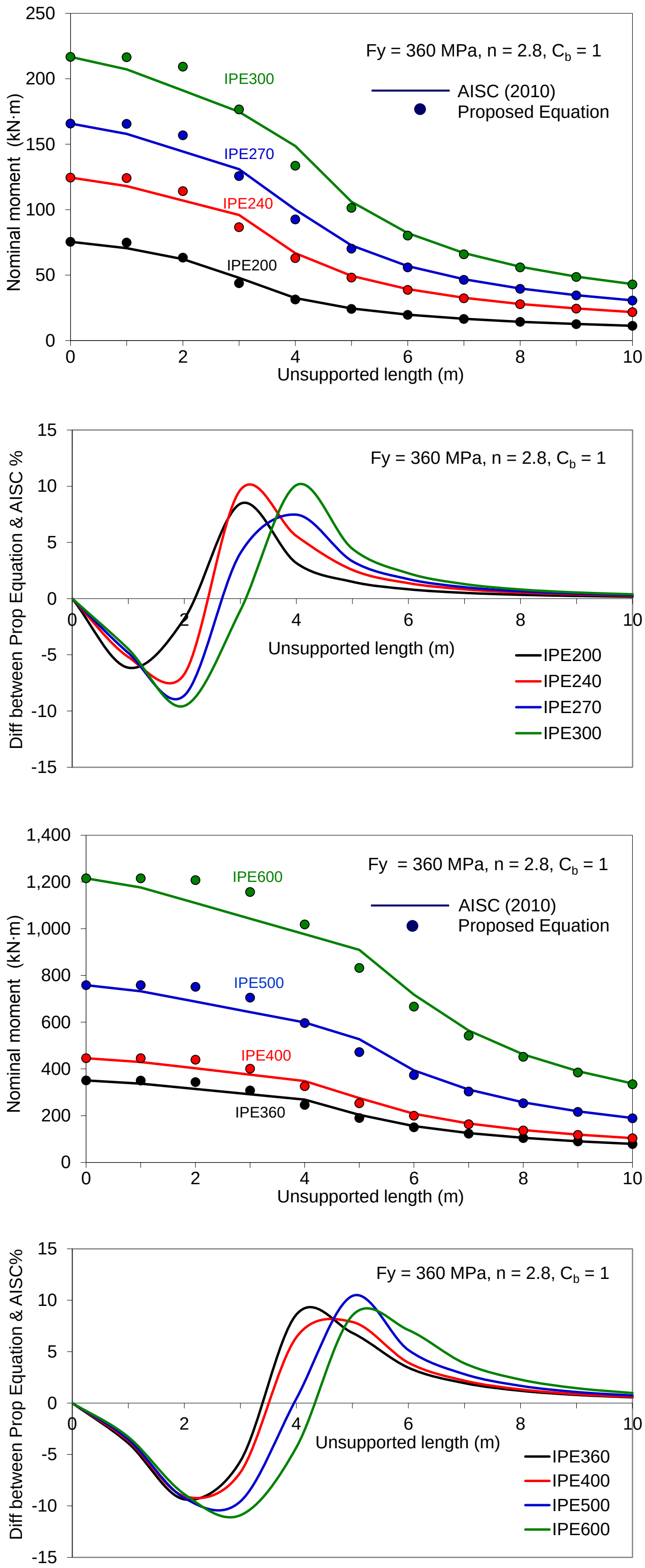

Figure 2. Nominal moment of simply supported I-beams subjected to constant moment according to AISC (2010) and the proposed equation.

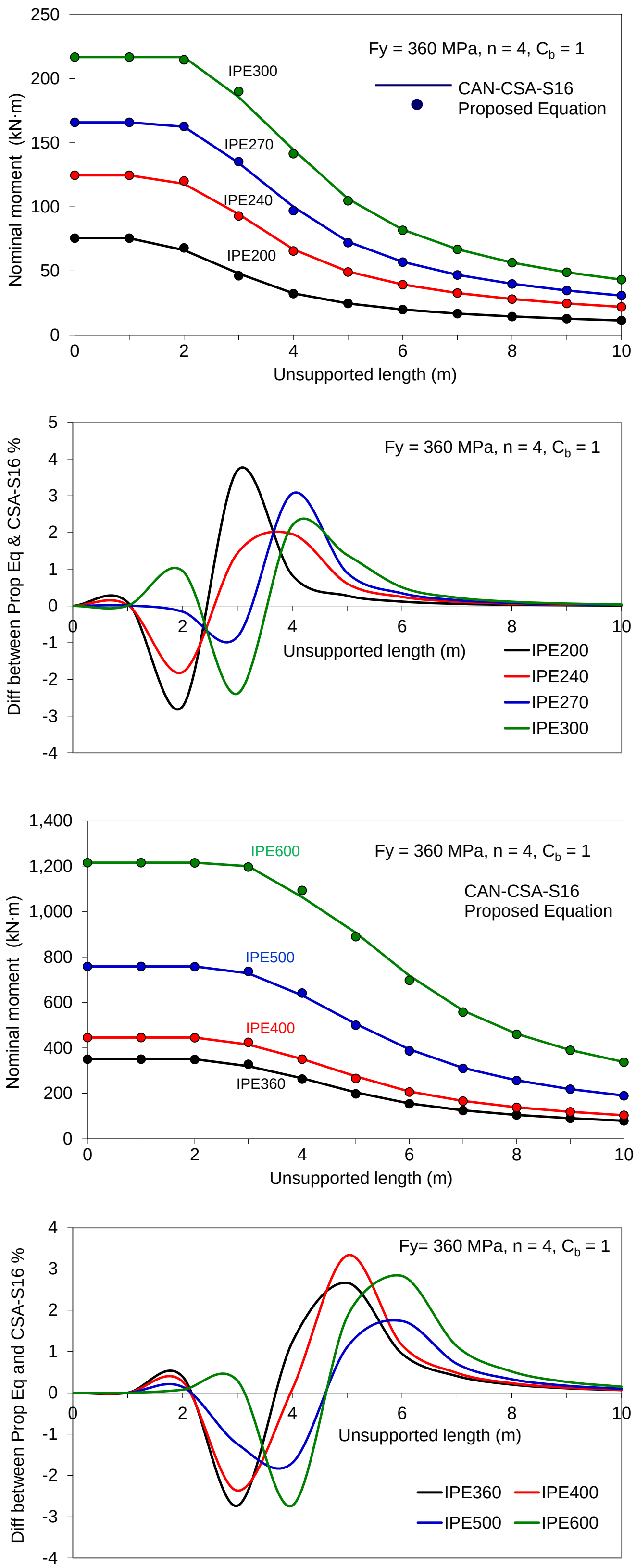

Figure 3. Nominal moment of simply supported I-beams subjected to constant moment according to CAN/CSA S16-09 (2009) and the proposed equation.

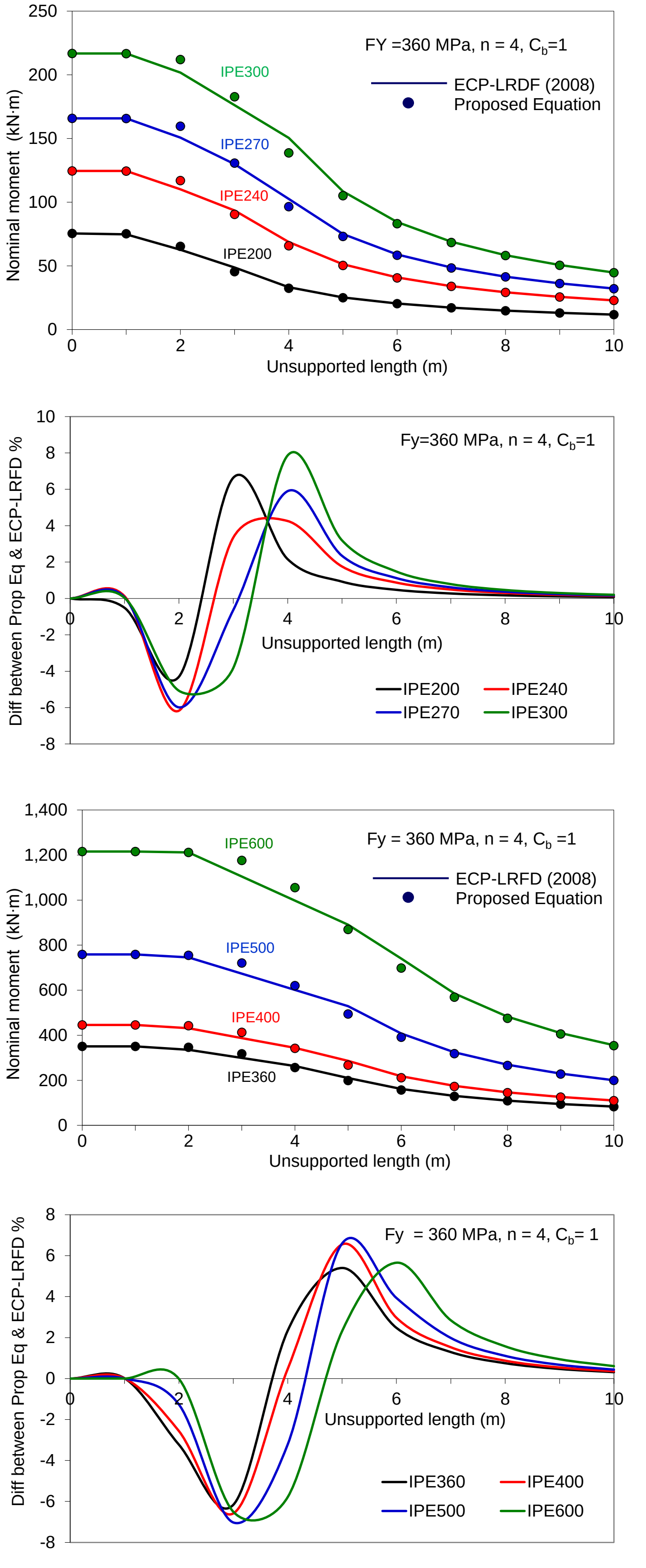

Figure 4. Nominal moment of simply supported I-beams subjected to constant moment according to ECP-LRFD (2008) and the proposed equation.

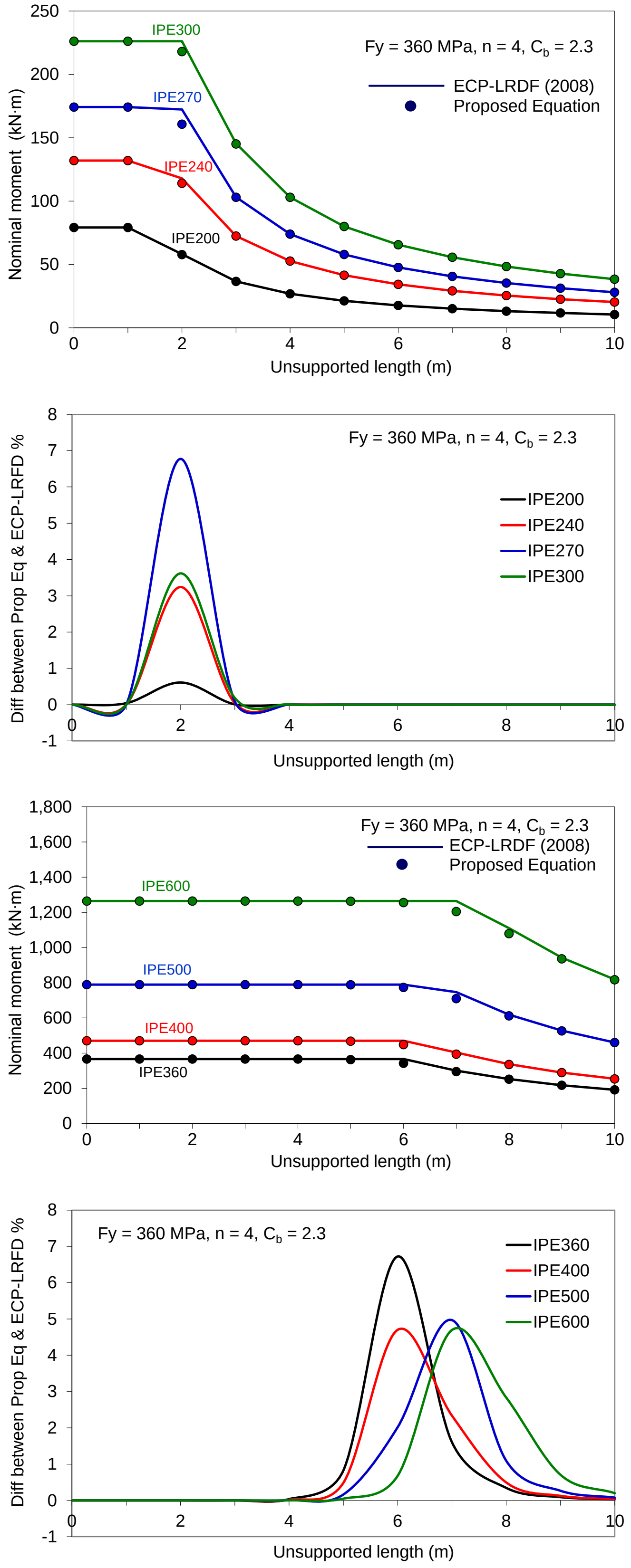

Figure 5. Nominal moment of cantilever I-beams subjected to a uniformly distributed load according to ECP-LRFD (2008) and the proposed equation.

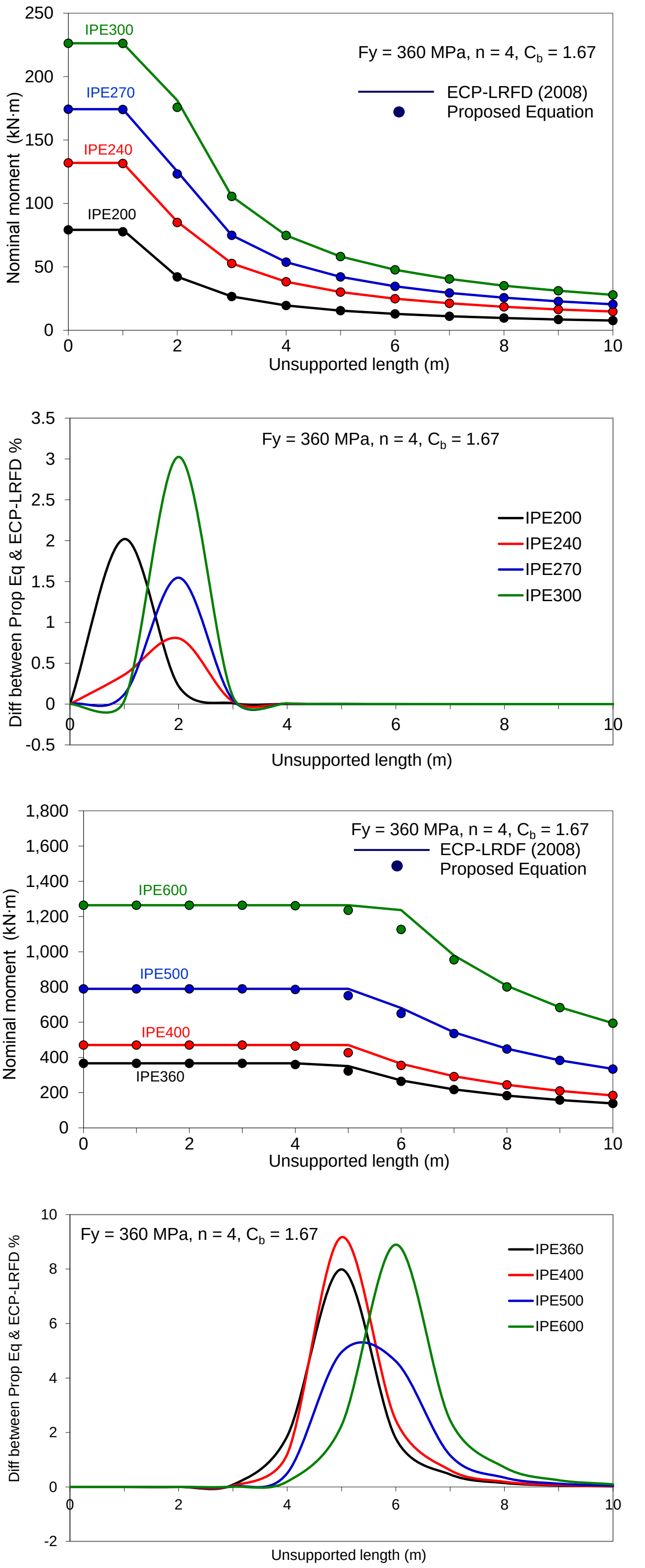

Figure 6. Nominal moment of cantilever I-beams subjected to a tip concentrated load according to ECP-LRFD (2008) and the proposed equation.

3 NUMERICAL ANALYSIS AND PROPOSED EQUATION VERIFICATION

3.1 *Description of the Numerical Model*

Nonlinear numerical analysis based on the finite element method is adopted to further verify the behaviour of the equation proposed for the design of I-beams. A 3-D finite element model has been built for the analysis of I-beams. The element adopted in the numerical analysis is a 4-node quadrilateral thick shell element with membrane and bending capabilities.

The element is assumed to be isotropic with a constant thickness. It has six degrees of freedom per node. Geometric and material nonlinearities have been included in element formulation and analysis.

Arc length solution technique with modified Newton-Raphson incremental procedures is adopted in the nonlinear analysis.

The beam's model is provided with two thick end plates where one node in the middle of each end plate is restrained. The middle node at one end plate is restrained from translation in the three directions in addition to rotation about the X-axis, while the middle node at the other end plate is restrained from movement in the Y- and Z-directions and rotation about the X-axis. Typical notations for cross-section components of the studied I-section steel beams are similar to those shown in Figure 1. On the other hand, a typical finite element mesh for one of the analysed beams is presented in Figure 7 along with the idealised stress strain curve (Salmon et al. 2009) for the steel adopted in the model.

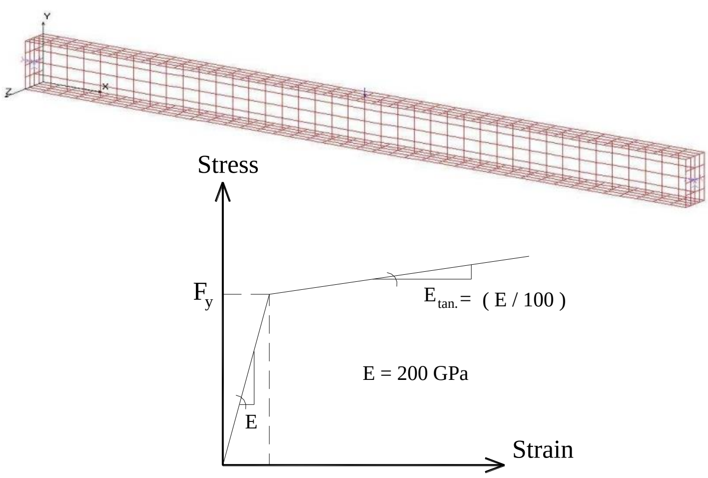

Figure 7. A typical finite element mesh of one of the analysed beams (above) and steel idealised stress-strain curve adopted in the finite element model (below).

To account for the effect of residual stress in the numerical model, a self-equilibrating stress similar to that described by Vila et al. (2004) is adopted in the numerical analysis. The residual stress distribution assumed over an I-section is shown in Figure 8.

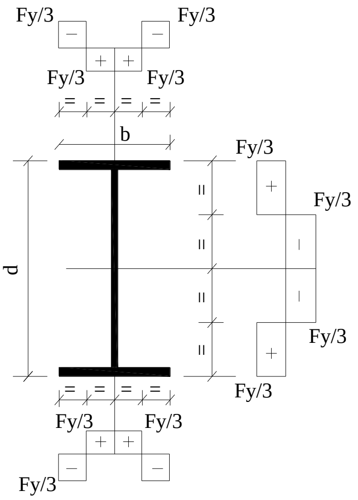

Figure 8. Assumed residual stress distribution over an I-section.

3.2 *Finite Element Model Verification*

As a verification process for the numerical model, the model is adopted to simulate previous experimental results for beams which were conducted by others (Lukey and Adams 1969, Sritawat and Nicholas 1975, Masahiro 1988, Kuhlmann 1989, and Kemp 1996). Details of the tested beams adopted in the verification process are listed Table 1. The results obtained numerically via the finite element model well agree with those experimentally determined (Table 1). Full details of the finite element model and its verification process are given elsewhere (Mustafa 2011).

3.3 *Numerical Verification of the Proposed Equation*

Results obtained for the nominal moment via the proposed equation are compared to the nominal moments obtained numerically using the finite element model. A wide range of simply supported and cantilevers I-section beams was considered in this process with the previously mentioned two types of steel grades (Mustafa 2011). Constant moments, tip cantilever concentrated loads and uniformly distributed loads were considered in the analysis. Sample of this comparative investigation is shown in Figure 9. Full details of the numerical investigation and its results are presented elsewhere (Mustafa 2011).

Samples of the percentages of difference between the results of the proposed equations, the data obtained by following the ECP-LRFD provisions and the outcomes of the numerical finite element model analysis are also shown in Figure 9.

It is evident from Figure 9 that moment capacities calculated by the proposed equation well match the moment capacities obtained by the finite element analysis in all the three zones of beams' behaviour (elastic, elasto-plastic and fully plastic). Furthermore, the proposed equation outcomes and the results of the finite element analysis are almost identical to those obtained by the ECP-LRFD in the elastic and fully plastic zones of beams' behaviour. The ECP-LRFD (2008) deviated by about 8% from the finite element model results in the elasto-plastic zone of beams' behaviour. The same order of difference is recorded between the results of the proposed equation an ECP-LRFD provisions. This behaviour was expected as the code of practice simply assumes a linear transition between the elastic and the plastic zones for the moment capacity in the elasto-plastic zone

Table 1. Comparison between experimental investigation results and the numerical model results.

No.*	Geometric Data (mm)				F_y (MPa)	L (m)	M_{FE}/M_{exp}
	D	b	t_{fl}	t_w			
1	360	160	15	8.1	250	3.2	0.98
2	200	190	15	9	250	1.26	0.90
3	240	120	13	8.3	250	1.62	1.03
4	320	180	17	10.6	250	1.63	0.91
5	240	200	20	13.5	250	2.01	1.12
6	200	124	12	8.6	250	3	0.94
7	260	150	12	7.72	250	6.1	1.02
8	260	150	12	7.72	250	3.05	1.05
9	260	150	12	7.72	250	3.66	1.06
10	260	150	12	7.72	250	2.44	1.02
11	248	125	4.5	3.2	330	1.5	0.93
12	248	125	4.5	3.2	330	2	1.06
13	248	125	4.5	3.2	330	2.85	1.05
14	300	150	4.5	3.2	330	1.65	0.91
15	300	150	4.7	3.4	330	1.8	1.02
16	300	150	4.7	3.4	330	2.4	0.93
17	300	150	4.7	3.4	330	3.35	1.05
18	200	150	4.7	3.4	330	1.8	0.92
19	200	150	4.7	3.4	330	2.4	1.10

* Series No. 1-2: Kemp 1996.
Series No. 3-4: Kuhlmann 1989.
Series No. 5-6: Lukey and Adams 1969.
Series No. 7-10: Sritawat and Nicholas 1975.
Series No. 11-19: Masahiro 1988.

4 REGRESSION ANALYSIS ON THE PROPOSED EQUATION RESULTS

Finally, a regression analysis has been performed on the results of the proposed equation to evaluate the correlation coefficient $R_{code/Eq.}$ between these results and the results of the ECP-LRFD (2008). Another correlation coefficient $R_{FEM./Eq}$ is also determined for the results of the proposed equation and those obtained from the numerical finite element analysis. These correlation coefficient are given by

$$R_{code/Eq.} = \sqrt{(S_t - S_r)/S_t}$$

$$\text{where} \quad S_t = \sum \left(M_n^{ECP} - \overline{M}_n^{ECP} \right)^2 \qquad (12)$$

$$S_r = \sum \left(M_n^{ECP} - M_n^{Eq.11} \right)^2$$

$$R_{FEM/Eq.} = \sqrt{(S_t - S_r)/S_t}$$

$$\text{where} \quad S_t = \sum \left(M_n^{FEM} - \overline{M}_n^{FEM} \right)^2 \qquad (13)$$

$$S_r = \sum \left(M_n^{FEM} - M_n^{Eq.11} \right)^2$$

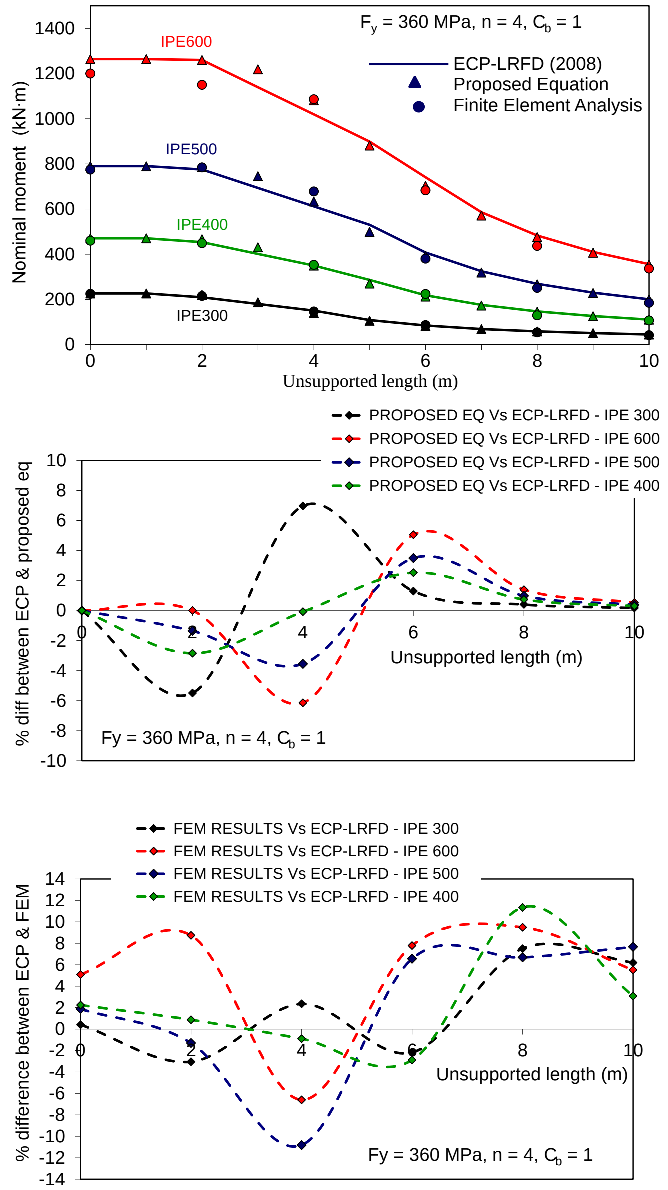

Figure 9. Samples of the nominal moments for I-sections beams obtained via the proposed equation, the ECP-LRFD (2008) provision and the numerical finite element model.

For $R_{code/Eq.}$, S_t is the total sum of the squares of the residuals between the nominal moments obtained using the ECP-LRFD (2008) provisions and their mean value while S_r is sum of the squares of the residuals between the nominal moments obtained using the code provisions and those obtained using the proposed equation. On the other hand, for $R_{FEM./Eq.}$, S_t is the total sum of the squares of the residuals between the nominal moments obtained using the finite element model and their mean value while S_r is sum of the squares of the residuals between the nominal moments obtained using the finite element model and those obtained using the proposed equation.

Full details of the regression analysis are presented by Mustafa 2011. The analysis reveals an excellent correlation coefficient between the data of the proposed equation and both the ECP-LRFD (2008) provisions or the numerical analysis outcomes. In both cases, the correlation coefficient was found to be greater than 0.98.

5 CONCLUSIONS

A simple equation is proposed for the design of steel I-beams. The performed investigation reveals that the outcomes obtained by the proposed equation well agree with the results of the ECP-LRFD (2008), AISC (2010) and CAN/CSA S16-09. The outcomes of the proposed equation also match the results of a nonlinear numerical analysis which is performed using the finite element technique. The acceptable difference in results between the proposed equation and the codes of practices ranges between 5% and 9% and it is localized in the elasto-plastic zone of beams' behaviour.

6 REFERENCES

AISC 2010. Specifications for Structural Steel Buildings. American Institute of Steel Construction. Chicago, USA.

CAN/CSA S16-09 2009. Canadian Standards Association 2009. Design of Steel Structures. Etobicoke, Ont., Canada.

ECP-ASD 2001. Egyptian Code of Practice for Steel Construction and Bridges – Allowable Stress Design. Ministry of Housing, Utilities and Urban development, Cairo, Egypt.

ECP-LRFD 2008. Egyptian Code of Practice for Steel Construction – Load and Resistance Factor design LRFD. Ministry of Housing, Utilities and Urban development, Cairo, Egypt.

Kemp, A. R., "Inelastic Local and Lateral Buckling in Design Codes", *Journal of Structural Engineering*, American Society of Civil Engineering, Vol. 122, No. 4, April 1996, pp. 374-382.

Kuhlmann, U., "Definition of Flange Slenderness Limits on the Basis of Rotation Capacity Values". *Journal of Construction Steel Research*, Elsevier, Vol. 14, No. 1, 1989, pp. 21-40.

Lukey, A. F. and Adams P. R., "Rotation Capacity of Wide Flanged Beams under Moment Gradient", *Journal of the Structural Division*, American Society of Civil Engineering, Vol. 95, No. ST6, 1969, pp. 1173-1188.

Masahiro, K., "Lateral-Torsional Buckling of Thin-Walled I-Beams", *Journal of Structural Engineering*, American Society of Civil Engineering, Vol. 114, No. 4, April 1988, pp. 841-855.

Mustafa, H. A. M., "Proposed Simple Equations for the Design of Steel I-Beams", PhD dissertation, Structural Engineering Department, Ain Shams University, Cairo, Egypt. 2011

Salmon, C.G., Johnson, J.E., and Malhas, F.A. Steel Structures: Design and Behavior. 5th ed., Prentice Hall, USA. 2009.

Sayed-Ahmed, E.Y., "Design of Laterally Unsupported Steel I-Beams According to the Egyptian Code of Practice: a Proposed Simple Equation", *Al-Azhar University Engineering Journal*, Al-Azhar University, Faculty of Engineering, Vol. 7, No. 5), Dec. 2004, pp. 1043 – 1063.

Sayed-Ahmed, E. Y., and Loov, R. E. 2005. Design of Laterally Unsupported Steel I-Beams According to the Egyptian Code of Practice: A Proposed Simple Equation. 1st Canadian Conf. on Effective Design of Struct., McMaster Univ., Hamilton, Ont., Canada, July 10 – 13, 2005. pp 137 – 146.

Sritawat, K. and Nicholas, T., "Inelastic Buckling of Simply Supported Steel I-Beams", *Journal of Structural Division*, American Society of Civil Engineering, Vol. 101, No. 7, July 1975, pp. 1333-1347.

Vila, R., Cazeli, R., and Simoes, D., "The Effect of Residual Stresses in the Lateral-Torsional Buckling of Steel I-Beams at Elevated Temperature", *Journal of Constructional Steel Research*, Elsevier, Vol. 3, No. 60, 2004, pp. 783-7913.