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Column Analogy in Multi-Cell Structures with Fixed Columns

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ABSTRACT: A procedure is presented for the analysis of multi-cell structures such as multi-span and multi-storey frames by use of the method of column analogy, normally applicable to analysis of single-span and closed frames. The multi-cell structure is first divided into a number of cells each of which can be readily analyzed by column analogy for the applied external loads. A number of simultaneous equations are then written to restore continuity of the isolated cells. Certain moment-coefficients are computed also by column analogy, and used in writing the continuity equations. Solution of these equations yields moment corrections which are superimposed to the moments of the isolated cells to arrive at the final bending moments of the multi-cell structure.

The procedure is presented for the analysis of frames with fixed columns, and is applied for the analysis of three examples in order to illustrate the details of the solution. Results obtained are in excellent agreement with values calculated using the stiffness method.

KEYWORDS: indeterminate structures, multi-cell structures, moment coefficients, superposition.

1 INTRODUCTION

The method of column analogy is a powerful tool for the analysis of statically indeterminate structures in the form of single-cells such as single-span and closed frames (Cross, 1930 and 1945). It is advantageous in taking care of sidesway without any special consideration like, for instance, in the analysis by moment distribution. Also, in the analysis of multi-cell structures with few cells by the method of virtual work, column analogy provides a simplification of the solution by choosing a closed cell as the main system. Nevertheless, column analogy is not as of yet applicable independent of other methods in the analysis of multi-span or multi-storey frames of the type shown in Figure 1.

The objective of this investigation is to extend the use of the forgotten column analogy (Sözen, 2002) to the analysis of such multi-cell structures. The presented procedure makes use of column analogy and the principle of superposition.

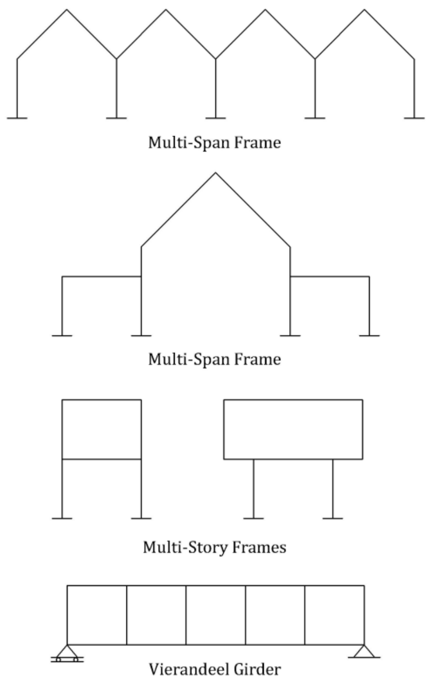


Figure 1. Multi-Cell Structures

2 METHOD OF ANALYSIS

Briefly, to begin the solution, the multi-cell structure is divided into a number of isolated single-cell structures. This step is accomplished by slicing longitudinally every member in common between two adjacent cells into two halves. Each cell can then be readily analyzed for external loads by column analogy as usual. In doing so, each cell deforms independently, and the two halves of the sliced members would not fit in together. This situation calls for a correction in order to restore continuity.

For this reason, intercellular correction statical moment diagrams are introduced as unknowns. For each sliced member, two correction moment diagrams are needed. Continuity moment coefficients are computed in each cell corresponding to statical moment diagrams of unit values. In order to restore continuity of the sliced members, two moment conditions are written for every member in common, one at each end, with the help of the continuity coefficients, giving as many equations as the number of unknown correction moment diagrams. These unknowns can then be evaluated by solving the continuity equations simultaneously.

Finally, the bending moment diagram of the multi-cell structure is determined by use of the principle of superposition of moments of isolated cells and correction moments.

3 PROCEDURE

For a detailed description of the suggested method of analysis, consider the three-span frame of Figure 2(a) with moments of inertia of the columns: I in exterior columns and 2I in interior columns. This is a simplifying but not a necessary assumption.

3.1 *Division of Multi-Span Frame; Case "O"*

In Figure 2(b) the multi-span frame is divided into three isolated spans with their inertia and loads. Every interior column is sliced into two columns each with inertia I.

The bending moment diagrams for the isolated frames can be readily determined by column analogy. This step is called Case "0" in which the moment at any section i is denoted by M_{i0}, except at sections top (or bottom) of sliced half-columns which are designated by k and k' in the half-columns lying on the left and right sides, respectively. The moments at these column sections are denoted by M_{k0} and M'_{k0}, respectively.

3.2 *Correction Moment Diagrams*

Each isolated frame will now deform independently under the action of its external loads. In order to restore continuity of the multi-span frame, it is necessary to add correction moment diagrams. These diagrams are obtained by assuming statical moment diagrams in the form of pairs of equal linear diagrams of negative (or positive) signs in the sliced columns of the isolated frames as shown in Figure 2(c). These moment diagrams involve n unknown moments X_n equal in number to twice the number of sliced interior columns. In the frame of Figure 2(a) there are four unknown moments X_1 to X_4 as sketched in Figure 2(c).

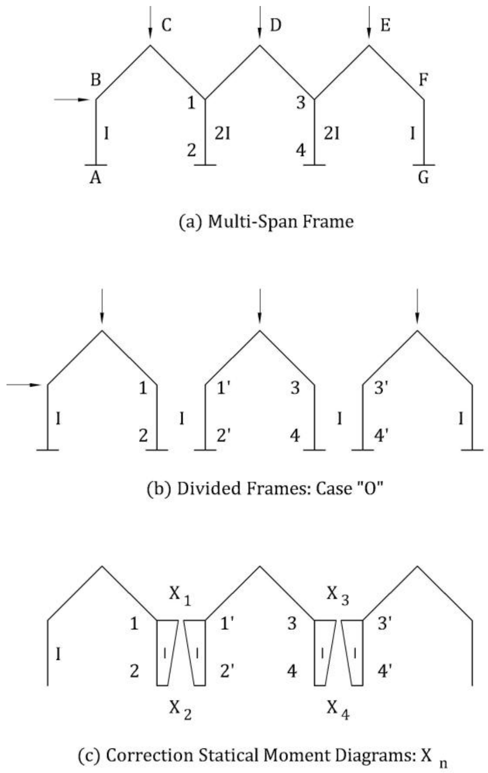


Figure 2. Division of Frame and Correction Moments

3.3 *Continuity Restoration*

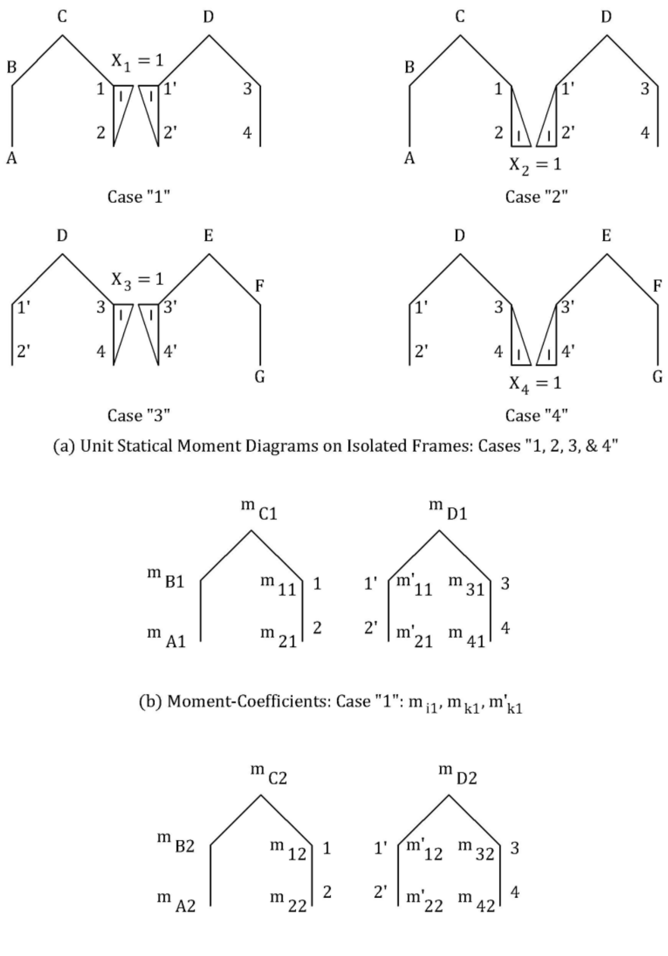
In the multi-span frame of Figure 2(a), the two halves of each of the sliced columns 1-2 and 3-4 must undergo identical deformations in order that they fit together forming the original columns. Thin situation will be satisfied only when the bending moments in the two halves of every column are identical. This condition of continuity may be briefly stated as follows:

$$M_k = -M'_k, or\ M_k + M'_k = 0, or\ M^*_k = 0 \qquad (1)$$

at the top and bottom sections of the column. M_k and M'_k denote the final bending moments at sections k and k' of the left and right sliced half-columns, respectively. M^*_k is the algebraic sum of M_k and M'_k.

3.4 *Moment-Coefficients; Cases "n": m_{in}, m_{kn} and m'_{kn}*

In order to satisfy the continuity condition (1), a set of moment-coefficients need to be determined first for every case of loading $X_n = 1$ as shown in Figure 3(a). These coefficients are in fact the moments induced in the isolated frames by the shown statical moment diagrams, and are determined by column analogy. They are denoted by m_{in}, m_{kn}, and m'_{kn} at sections i, k, and k', respectively. Consider, for example, the two isolated frames adjacent to column 1-2, Figure 3(a). Two cases of loading are analyzed. In Case "1", a statical linear moment diagram as shown is applied to both left and right frames with a unit value for X_1. Both frames are analyzed by column analogy to determine the moment-coefficients m_{i1}, m_{k1} and m'_{k1} in frame and column sections, Figure 3(b). In Case "2", Figure 3(a), a linear statical moment diagram as shown is applied to both left and right frames with a unit value for X_2. Again both frames are analyzed to determine the moment-coefficients m_{i2}, m_{k2} and m'_{k2} in frame and column sections, Figure 3(c). Other cases of loading, i.e. Cases "3" and "4" are treated similarly, but in view of identity of all isolated frames, it is noted that Case "3" is a repetition of Case "1", while Case "4" is a repetition of Case "2", so that $m_{k1} = m_{k3}$, $m'_{k1} = m'_{k3}$ and $m_{k2} = m_{k4}$, $m'_{k2} = m'_{k4}$.



(a) Unit Statical Moment Diagrams on Isolated Frames: Cases "1, 2, 3, & 4"

(b) Moment-Coefficients: Case "1": m_{i1}, m_{k1}, m'_{k1}

(c) Moment-Coefficients: Case "2": m_{i2}, m_{k2}, m'_{k2}

Figure 3. Moment-Coefficients; Cases "n"

3.5 *Continuity Moment-Coefficients; Cases "n": m^*_{kn}*

Having obtained the moment-coefficients m_{kn} and m'_{kn} for all cases of loading $X_n = 1$, they are added to get the so called continuity moment-coefficients, denoted by m^*_{kn} at top and bottom sections k of the columns in Case "n", hence $m^*_{kn} = m_{kn} + m'_{kn}$. In Figure 4 these coefficients are scored for the mentioned n (four) cases of loading. For illustration, in Case "1" the continuity coefficient m^*_{11} at the top section of column 1-2 is the sum of the coefficients m_{11} and m'_{11} of Figure 3(b), while the continuity coefficient at the top of column 3-4 is $m^*_{31} = m_{31}$. These coefficients are very useful in writing the continuity equations (1).

3.6 *Equations of Continuity*

In general, the final moment at a section k and k' of a sliced half-column can be determined by superposition of : (1) moments M_{k0} or M'_{k0} due to external loads; Case "0", and (2) moments due to the n unknown correction moments X_n. In a sliced half-column lying on the left side

$$M_k = M_{kO} + \sum_n m_{kn} \cdot X_n \tag{2}$$

and in a sliced half-column lying on the right side

$$M'_k = M'_{kO} + \sum_n m'_{kn} \cdot X_n \tag{3}$$

From which the continuity equation (1) can be written in the form

$$(M_k + M'_k) = (M_{kO} + M'_{kO}) + \sum_n (m_{kn} + m'_{kn}) \cdot X_n = 0 \tag{4}$$

or briefly

$$M^*_k = M^*_{kO} + \sum_n m^*_{kn} \cdot X_n = 0 \tag{5}$$

in which $M^*_k = M_k + M'_k$, and $M^*_{k0} = M_{k0} + M'_{k0}$. Equation (5) is written at all top and bottom sections of the sliced columns giving as many equations as the number of unknown correction moments X_n. In the illustrated three-span frame of Figure 2(a), there are four continuity equations (5) written at sections 1, 2, 3, and 4 which may be written with the help of the continuity moment-coefficients of Figure 4. in the form

$$\begin{bmatrix} M_{10}^* \\ M_{20}^* \\ M_{30}^* \\ M_{40}^* \end{bmatrix} + \begin{bmatrix} m_{11}^* & m_{12}^* & m_{13} & m_{14} \\ m_{21}^* & m_{22}^* & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33}^* & m_{34}^* \\ m_{41} & m_{42} & m_{43}^* & m_{44}^* \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = 0 \qquad (6)$$

or, briefly in matrix form

$$[M_{kO}^*] + [m_{kn}^*]\,[X_n] = 0 \qquad (7)$$

Solving these equations simultaneously the values of the correction moment X_n are found, and the final bending moments in the multi-span frame can then be readily computed by superposition.

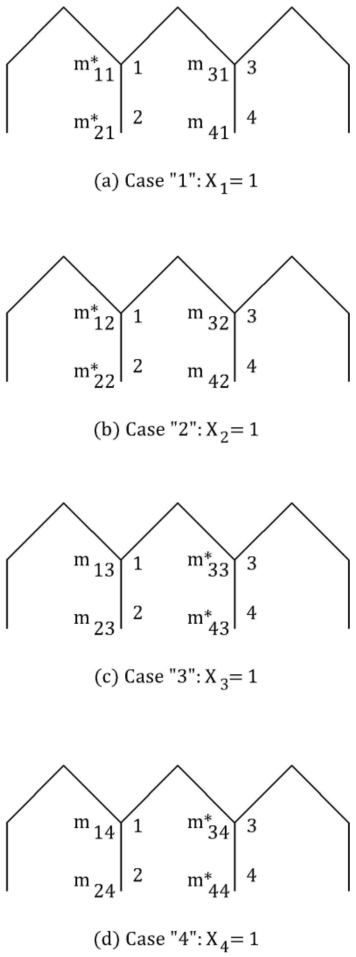


Figure 4. Continuity Moment-Coefficients; m^*_{kn}

3.7 *Bending Moment in Multi-Span Frame*

In general, the moment at any section i of the multi-span frame is obtained by superposition as the sum of : (1) moment due to external loads, denoted by M_{i0} in sections not in columns, and $\bar{M}_{k0} = M_{k0} - M'_{k0}$ in column sections (with the same sign as of M_{k0}), and (2) moment due to correction moment diagrams. At section i not in columns, the correction moment is $\Sigma\, m_{in} \cdot X_n$, while in column sections k the correction moment is $\Sigma\,(m_{kn} - m'_{kn}) \cdot X_n = \Sigma\, \bar{m}_{kn} \cdot X_n$, in which $\bar{m}_{kn} = m_{kn} - m'_{kn}$ is the moment-coefficient in case n for computing correction moment at top (or bottom) sections k of a full column of the multi-span frame. The moment-coefficients m_{in} and $\bar{m}_{kn}$ are shown in Figure 5. for the n cases $X_n = 1$. By superposition the final bending moment in the multi-span frame is then, at sections not in columns

$$M_i = M_{iO} + \sum m_{in} \cdot X_n \qquad (8)$$

and

$$\bar{M}_k = \bar{M}_{kO} + \sum \bar{m}_{kn} \cdot X_n \qquad (9)$$

in column sections.

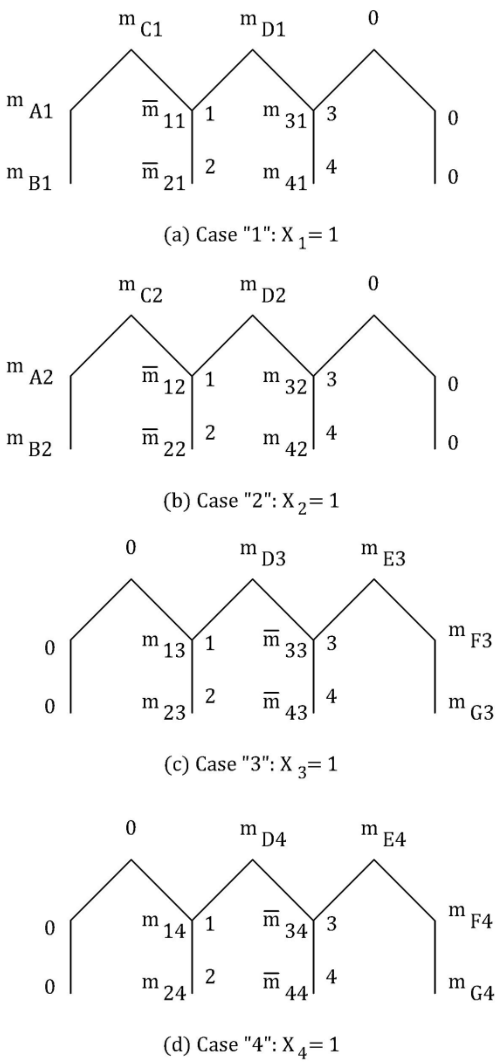


Figure 5. Correction Moment-Coefficients: m_{in} and m□$_{kn}$

4 RESULTS

For illustration of the proposed method of analysis, the following examples are solved.

4.1 *Example 1, Vierandeel Girder*

The dimensions, inertia and loads are given in Figure 6(a). The girder is divided into five isolated frames, but in view of symmetry only the three isolated frames: a, b and c of Figure 6(b) are considered. The Loads and equilibrants of every isolated frame are given in the figure. In Figure 7(a) the properties of the analogous column section are given. Figures 7(b), (c) and (d) show the statical moments of frames a, b and c due to the applied loads; Case "0". The straining actions and the indeterminate moments are given besides the drawing of every frame.

The computed moments M_{i0}, M_{k0} and M'_{k0} at the various sections are scored in column (3) of Table 1. In Figure 6(c) the correction statical moment diagrams X_1 to X_4 are shown. Figures 6(d), (e), (f), and (g) show the unit statical moment diagrams of Cases "1, 2, 3, and 4". Symmetry of diagrams on frame c is noticed in Cases "3 and 4". The straining actions and indeterminate moments are given in Figures 7(e) and (f) for Case "1". The resulting moment-coefficients are registered in columns (4) and (5) of Table 1. The corresponding coefficients of Cases "2, 3 and 4" are deduced from Case "1" in columns (6) to (11) of the table. In columns (12) to (16) of Table 2 are given the values of M^*_{k0} and the continuity moment-

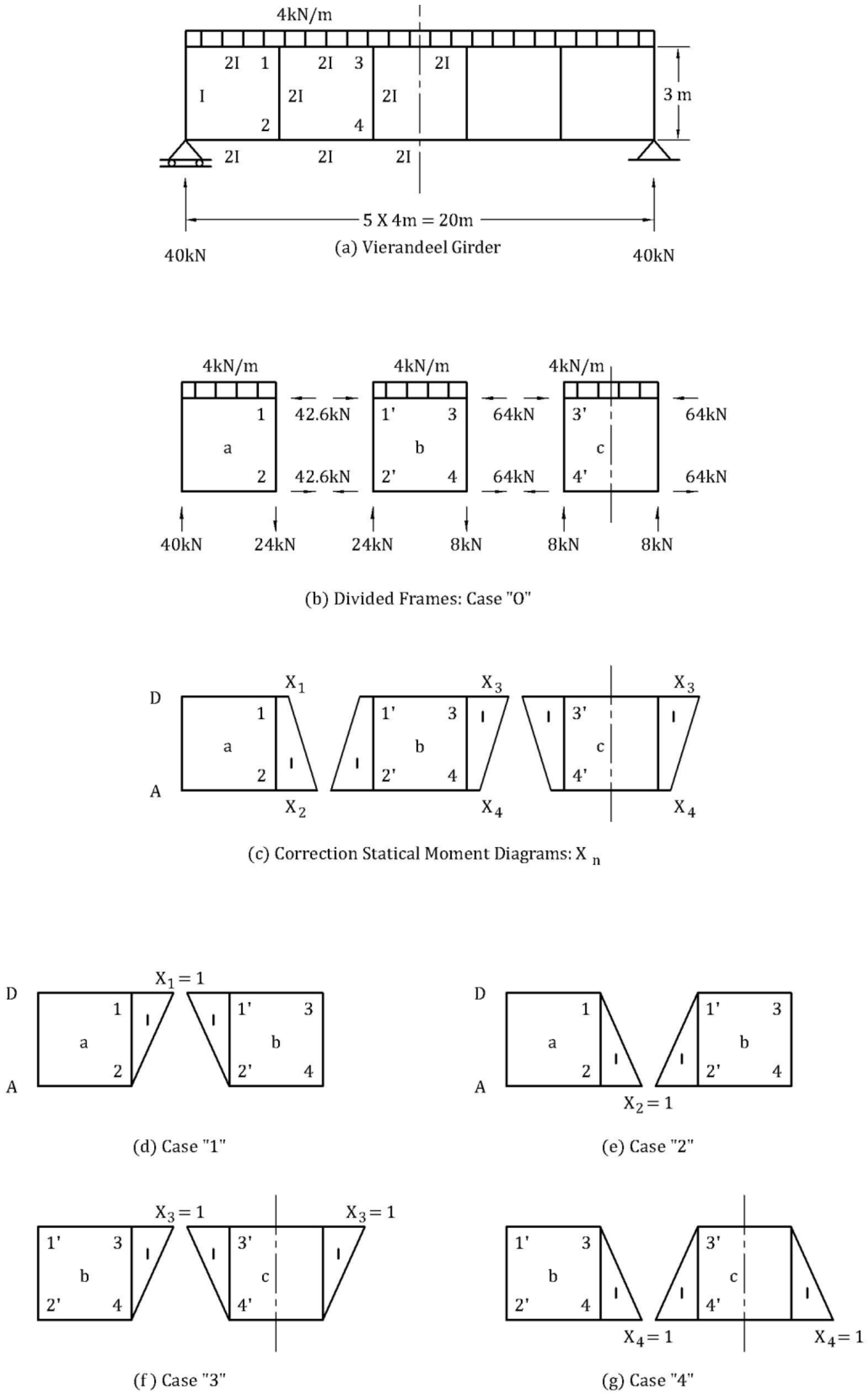


Figure 6. Division of Girder and Correction Moments

coefficients m^*_{kn}. Now the equations of continuity are easily written with the help of Table 2 in the form of Eqs. (6), giving four equations at column sections k = 1, 2, 3, and 4. Solution of these equations gives: X_1 = 5.2075859, X_2 = -11.571428, X_3 = 6.643143 and X_4 = -11.813766. Finally, Eq. (8) is applied to determine the final bending moments M_i with the help of Table 1, and Eq. (9) yields the final column moments M_k with the aid of columns (17) to (21) of Table 3. The results are written in columns (22) and (23) of the table. The bending moment diagram of the Vierandeel girder is drawn in Figure 8 on the tension side.

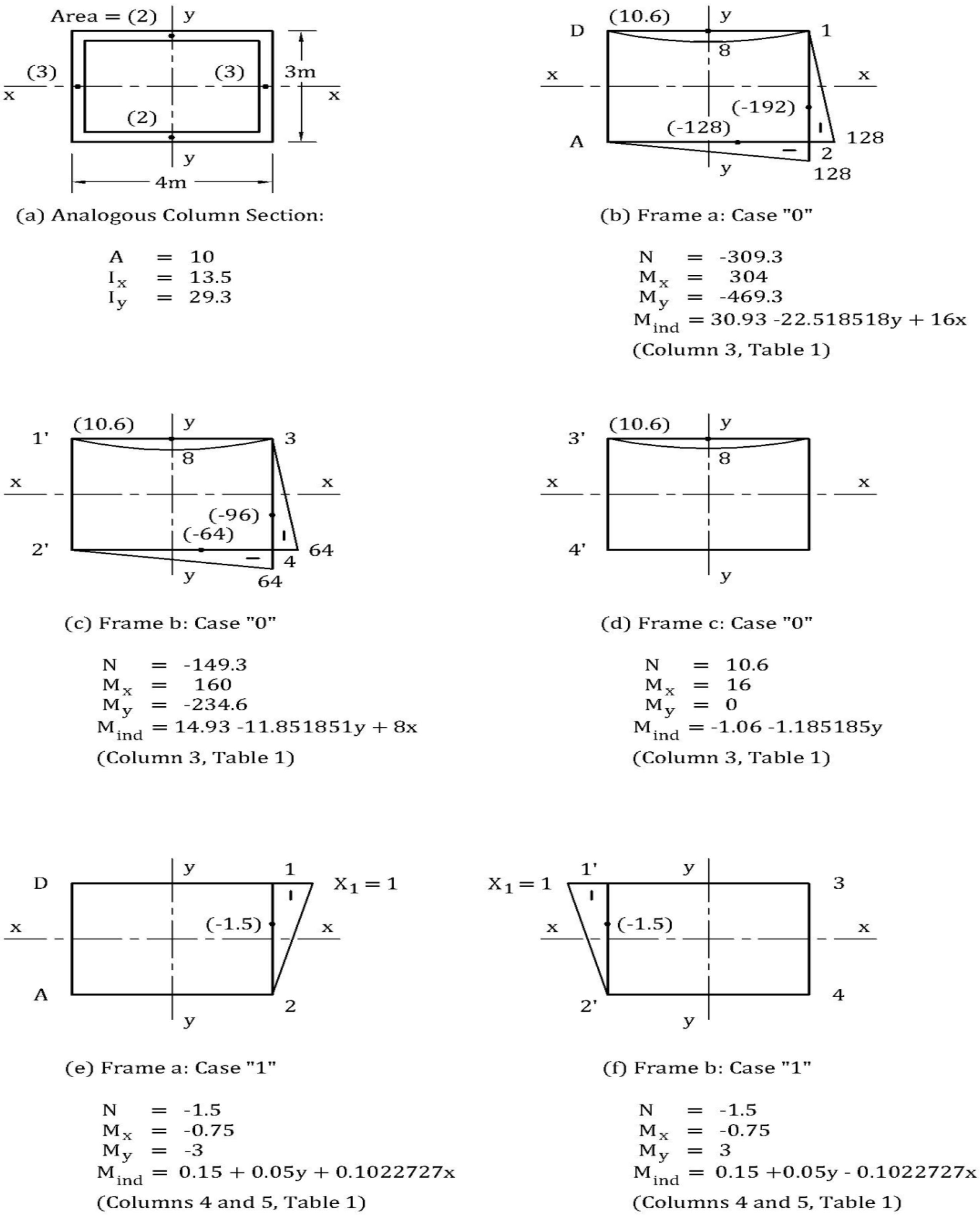


Figure 7. details of Solved Example 1.

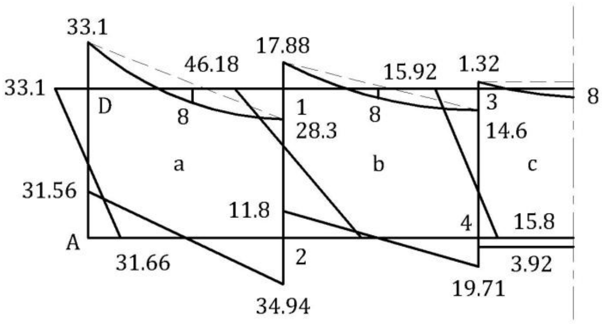


Figure 8. Bending Moment Diagram (kN.m), Example 1.

Table 1. Case "0", and Moment-Coefficients; Cases "1, 2, 3, and 4"

Frame	Section i, k, k'	Case "0"	Case "1"		Case "2"		Case "3"		Case "4"	
		M_{10}, M_{k0}, M'_{k0}	m_{i1}	$m_{k1}\ m'_{k1}$	m_{i2}	$m_{k2}\ m'_{k2}$	m_{i3}	$m_{k3}\ m'_{k3}$	m_{i4}	$m_{k4}\ m'_{k4}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
a	D	-34.84	0.0287878		-0.1378786					
	A	32.71	-0.1378786		0.0287878					
	1	29.15	0.4378786	-0.5621214	0.2712122	0.2712122				
	2	-31.28	0.2712122	0.2712122	0.4378786	-0.5621214				
b	1'	-18.84	0.4378786	-0.5621214	0.2712122	0.2712122	0.0287878	0.0287878	-0.1378786	-0.1378786
	2'	16.71	0.2712122	0.2712122	0.4378786	-0.5621214	-0.1378786	-0.1378786	0.0287878	0.0287878
	3	13.15	0.0287878	0.0287878	-0.1378786	-0.1378786	0.4378786	-0.5621214	0.2712122	0.2712122
	4	-15.28	-0.1378786	-0.1378786	0.0287878	0.0287878	0.2712122	0.2712122	0.4378786	-0.5621214
c	3'	-2.84					0.4666666	-0.5333333	0.1333333	0.1333333
	4'	0.71					0.1333333	0.1333333	0.4666666	-0.5333333

Table 2. Continuity Moment-Coefficients; m^*_{kn}

Frame	Section k	M^*_{k0}	m^*_{k1}	m^*_{k2}	m^*_{k3}	m^*_{k4}
(1)	(2)	(12)	(13)	(14)	(15)	(16)
a	1	10.31	-1.1242428	0.5424244	0.0287878	-0.1378786
	2	-14.57	0.5424244	-1.1242428	-0.1378786	0.0287878
b	3	10.31	0.0287878	-0.1378786	-1.0954547	0.4045455
	4	-14.57	-0.1378786	0.0287878	0.4045455	-1.0954547

Table 3. Correction Moment-Coefficients and Final Bending Moment

Frame (1)	Section i, k, k' (2)	$\overline{M}_{k0}$ (17)	$\overline{m}_{k1}$ (18)	$\overline{m}_{k2}$ (19)	$\overline{m}_{k3}$ (20)	$\overline{m}_{k4}$ (21)	M_1 (22)	$\overline{M}_k$ (23)
a	D						-33.099078	
	A						31.659981	
	1	48	0	0	-0.0287878	0.1378786	28.297533	46.179894
	2	-48	0	0	0.1378786	-0.0287878	-34.943408	-46.743396
b	1'						-17.88236	
	2'						11.800552	
	3	16	0.0287878	-0.1378786	-0.0287878	0.1378786	14.605774	15.92526
	4	-16	-0.1378786	0.0287878	0.1378786	-0.0287878	-19.711311	-15.79509
c	3'						-1.3194799	
	4'						-3.9162268	

4.2 *Example 2, Vierandeel Girder*

The same girder of example 1 is here subjected to an anti-symmetrical loading as shown in Figure 9(a). The two horizontal 10kN loads at D and B can be replaced by four 5kN loads at D, C, A, and B without affecting the bending moments in the girder. The girder is divided into five isolated frames as shown in Figure 9(b), but in view of antisymmetry only three isolated frames a, b and c are considered. The statical moments for the three frames in Case "0" are identical, Figure 9(d). The straining actions and the indeterminate moments are given in the figure, and the bending moment diagram is drawn in Figure 9(e). It is noticed that the values of M^*_{k0} in all column sections 1, 2, 3, and 4 are zero. Consequently, the condition of continuity, Eq. (5), is satisfied and no correction moments are needed. The bending moment diagram of the Vierandeel girder is drawn in Figure 9(f) on the tension side.

4.3 *Example 3,Multi-Span Frame*

In Figure 10(a) is sketched a three-span frame, its dimensions, moments of inertia and loads. This frame is similar to that of Figure 2(a) for which the solution was previously described. The frame is divided into three identical frames a, b and c of Figure 10(b). The properties of the analogous column section are given in Figure 10(c). In Figures 10(d) and (e) are sketches of the statical moments in a typical isolated frame due to vertical and horizontal loads, respectively. The straining actions and the equations of the indeterminate moments are given alongside each loading. The computed moments M_{i0}, M_{k0} and M'_{k0} at the various sections are registered in column (3) of Table 4. In this frame four unknown correction statical moments X_1 to X_4 as shown in Figure 2(c) are needed for continuity restoration. In Figures 10(f) and (g) are sketched the unit statical moment diagrams of Cases "1 and 2", respectively. The straining actions and equations of indeterminate moments are given in the figures. The resulting moment-coefficients are scored in columns (4) to (7) of Table 4. The corresponding coefficients of Cases "3 and 4" are given in columns (8) to (11). In columns (12) to (16) of Table 5 are found the values of M^*_{k0} and the continuity moment-coefficient m^*_{kn} as obtained from Table 4. Four continuity equations similar to Eqs. (6) are now written with the help of Table 5 at column sections k = 1, 2, 3, and 4. Solution of these equations gives: X_1 = -43.565642, X_2 = 68.7477, X_3 = -44.235842 and X_4 = 61.029173. Eq. (8) is then used to find the final bending moment M_i with the help of Table 4. Eq. (9) gives the final bending moments $\overline{M}_k$ with the aid of columns (17) to (21) of Table 6. The results are registered in columns (22) and (23) of the table. The bending moment diagram is drawn in Figure 10(h) on the tension side.

5 VALIDATION OF RESULTS

The final values of the member end moments, shown in Figures 8, 9 (f) and 10 (h) for the three solved examples, and calculated in Tables (3) and (6) for the first and third example are compared with stiffness method results obtained using GT STRUDL. GT STRUDL is a commercial structural design and analysis software program developed by the Computer Aided Structural Engineering Center at GA Tech. The comparisons are shown in Tables 7, 8, and 9 (a) and (b). The absolute percentage difference in the values of the moments is also provided. Results show excellent agreement between the presented procedure and the moments calculated using the stiffness method.

6 CONCLUSION

The forgotten method of column analogy used to analyze statically indeterminate single span and closed frames is extended, using the principle of superposition, to the analysis of multi-cell and multi-span frames with column fixed to the ground.

7 ACKNOWLEDGMENT

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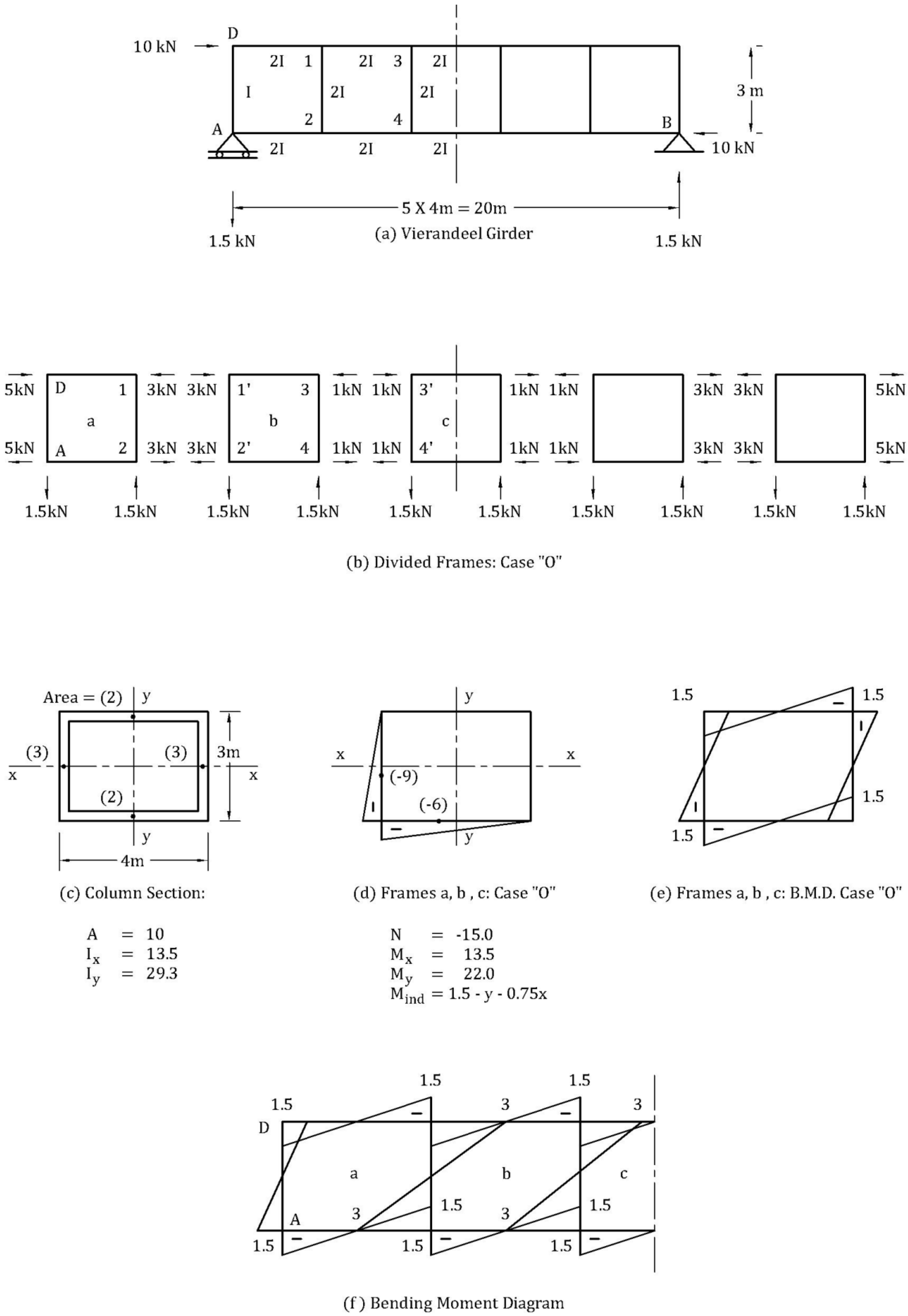

Figure 9. Detail of Solved Example 2.

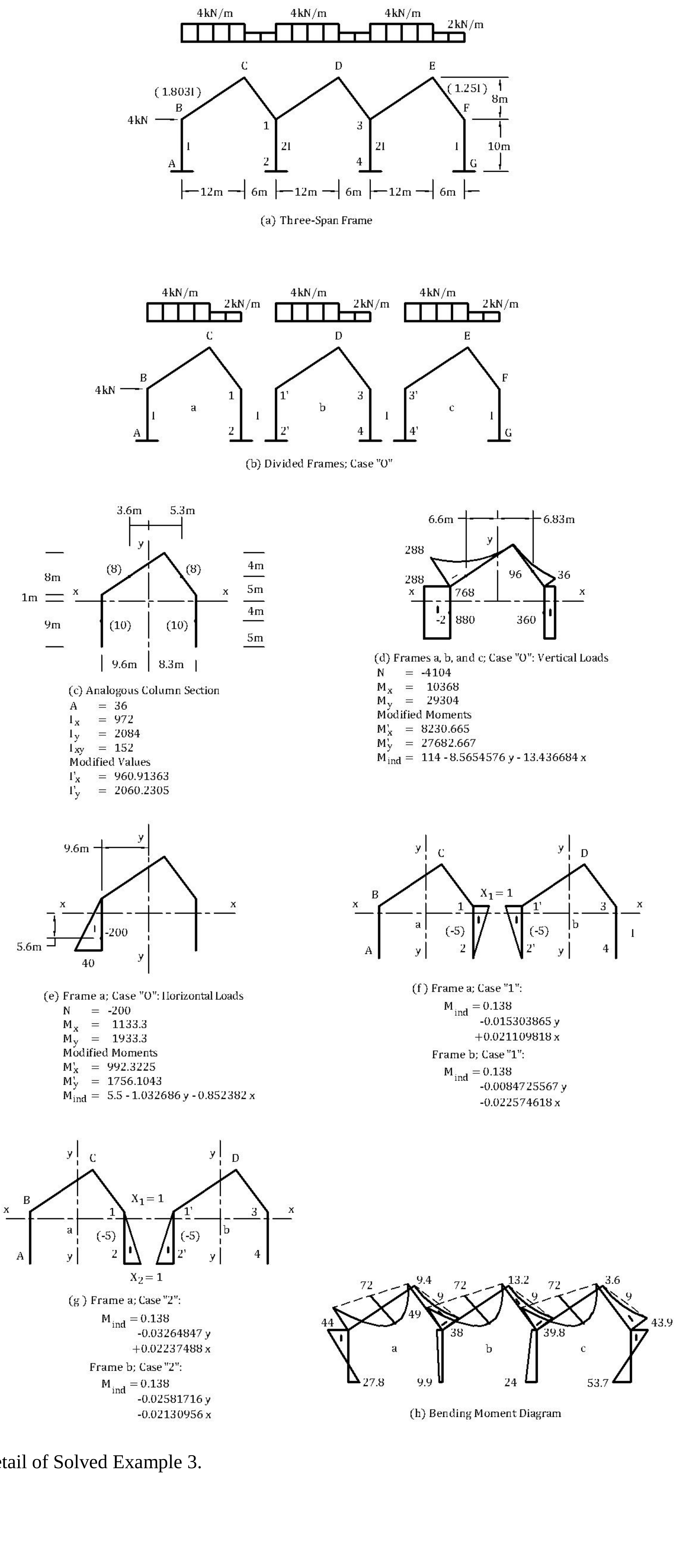

Figure 10. Detail of Solved Example 3.

Table 4. Case "0", and Moment-Coefficients; Cases "1, 2, 3, and 4"

Frame	Section i, k, k'	Case "0"	Case "1"		Case "2"		Case "3"		Case "4"	
		M_{10} , M_{k0} , M'_{k0}	m_{i1}	m_{k1} m'_{k1}	m_{i2}	m_{k2} m'_{k2}	m_{13}	m_{k3} m'_{k3}	m_{14}	m_{k4} m'_{k4}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
a	B	-39.91948	-0.0804764		-0.1100503					
	A	16.055471	0.0725617		0.2164374					
	C	-0.168882	0.0504098		-0.1027397					
	1	-45.118133	0.2994998	-0.7005002	0.2926977	0.2926977				
	2	50.863296		0.4525388		-0.3808173				
b	1'	-52.67751	0.3486376	-0.6513624	0.3190641	0.3190641	-0.0804754	-0.0804764	-0.1100503	-0.1100503
	2'	32.97705		0.4333629		-0.4227655		0.0725617		0.2164347
	D	5.558628	0.0099614		-0.1431872		0.0504098		-0.1027397	
	3	-42.53782	-0.0577053	-0.0577053	-0.0645074	-0.0645074	0.2994998	-0.7005002	0.2926977	0.2926977
	4	43.11675		0.0270197		0.1936636		0.4525388		-0.3808173
c	3'	-52.67751					0.3486376	-0.6513624	0.3190641	0.3190641
	4'	32.97705						0.4333629		-0.4227655
	E	5.558628					0.0099614		-0.1431872	
	F	-42.53782					-0.0577053		-0.0645074	
	G	43.11675					0.0270197		0.1936636	

Table 5. Continuity Moment-Coefficients; m^*_{kn}

Frame	Section k	M^*_{k0}	m^*_{k1}	m^*_{k2}	m^*_{k3}	m^*_{k4}
(1)	(2)	(12)	(13)	(14)	(15)	(16)
a	1	-97.795643	-1.3518626	0.6117618	-0.0804764	-0.1100503
	2	83.840346	0.8859017	-0.8035828	0.0725617	0.2164347
b	3	-95.21533	-0.0577053	-0.0645074	-1.3518626	0.6117618
	4	76.0938	0.0270197	0.1936636	0.8859017	-0.8035828

Table 6. Correction Moment-Coefficients and Final Bending Moment

Frame (1)	Section i, k, k' (2)	$\overline{M}_{k0}$ (17)	$\overline{m}_{k1}$ (18)	$\overline{m}_{k2}$ (19)	$\overline{m}_{k3}$ (20)	$\overline{m}_{k4}$ (21)	M_1 (22)	$\overline{M}_k$ (23)
a	B						-43.974647	
	A						27.784661	
	C						-9.428135	
	1	7.559377	-0.0491378	-0.0263664	0.0804764	0.1100503	-38.043741	11.043803
	2	17.886246	0.0191759	0.0419482	-0.0725617	-0.2164347		9.935647
b	1'						-49.087544	
	2'							
	D						-13.219251	
	3	10.13969	-0.0577053	-0.0645074	-0.0491378	-0.0263664	-39.844114	8.783445
	4	10.1397	0.0270197	0.1936636	0.0191759	0.0419482		23.988299
c	3'						-48.627569	
	4'							
	E						-3.620686	
	F						-43.922011	
	G						53.74064	

Table 7. Member End Moments (kN.m) – Example 1

	A	D	1-D	1-2	2-A	2-1	3-1	3-4	4-2	4-3
Column Analogy	31.66	-33.10	28.30	46.18	-34.94	-46.74	14.61	15.93	-19.71	-15.80
GT STRUDL	31.66	-33.10	28.30	46.17	-34.95	-46.74	14.61	15.92	-19.72	-15.79
% Difference	0	0	0	0.02	0.03	0	0	0.06	0.05	0.05

Table 8. Member End Moments (kN.m) – Example 2

	A	D	1-D	1-2	2-A	2-1	3-1	3-4	4-2	4-3
Column Analogy	-1.500	1.500	-1.500	-3.000	1.500	+3.000	-1.500	-3.000	1.500	3.000
GT STRUDL	-1.501	1.501	-1.499	-3.000	1.499	3.000	-1.500	-3.000	1.500	3.000
% Difference	0.07	0.07	0.07	0	0.07	0	0	0	0	0

Table 9 (a). Member End Moments (kN.m) - Example 3

	A	B	C	1-C	1-2	2	D
Column Analogy	27.78	-43.97	-9.428	-38.04	11.04	9.936	-13.22
GT STRUDL	27.79	-43.99	-9.452	-38.09	11.05	9.940	-13.24
% Difference	0.04	0.05	0.25	0.13	0.09	0.04	0.15

Table 9 (b). Member End Moments (kN.m) - Example 3

	3-D	3-4	4	E	F	G
Column Analogy	-39.84	8.783	23.99	-3.621	-43.92	53.74
GT STRUDL	-39.85	8.790	24.00	-3.646	-43.93	53.75
% Difference	0.03	0.08	0.04	0.69	0.02	0.02