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Statical Analysis of Pile Groups Containing Batter Piles

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ABSTRACT: The traditional statical method uses two approaches for the analysis of a loaded pile group containing batter piles. The first approach uses batter piles to resist a partial lateral load; the residual lateral load is then equally distributed between each pile in the pile group; the distributed residual lateral load in each pile then results in the shear force in that pile. The second approach adjusts the slopes of the batter piles in order to resist the complete lateral load and as a result, there are no shear forces in the piles. Three examples are presented in this paper to demonstrate the use of the two approaches. The results of these three examples are then compared to the "exact" results obtained by using either computer software or a hand calculation. The study presented in this paper has concluded the following: (1) The first approach may result in significant errors; therefore, its accuracy is questionable. (2) The results obtained from the second approach are quite close to the "exact" results; therefore, the second approach is quite an accurate approximate approach for the analyses of pile groups containing batter piles.

1 TRADITIONAL STATICAL METHOD

The traditional statical method (Teng 1962; Poulos and Davis 1990) for the analysis of a pile group containing batter piles is described step by step, using the example shown in Fig. 1, as follows:

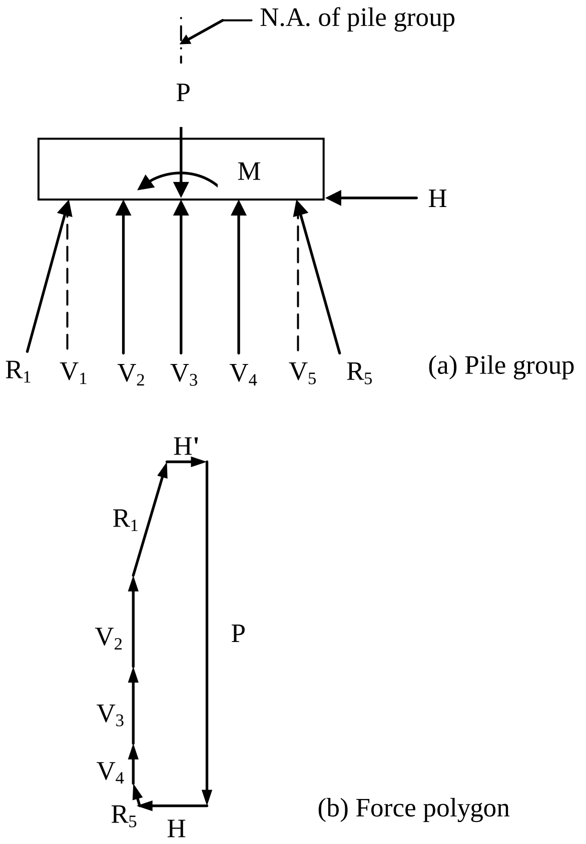

Figure 1. Traditional statical method for the analysis of a pile group containing batter piles

1. Assume the pile cap is rigid and the piles are elastic.
2. Treating the pile group as if all piles were vertical results in

$$V_i = \frac{P}{n} \pm \frac{M}{\Sigma(x^2)} x \tag{1}$$

where V_i = the axial force in a vertical pile or the vertical component of the axial force in a batter pile, P = a vertical load, n = the total number of piles, M = an overturning moment, and x = the distance of a pile to the neutral axis of a pile group.
3. Determine the slopes of the batter piles. The axial force R_i in each batter pile may thus be determined.
4. Compute the net horizontal component of the axial forces in batter piles.
5. Subtract the net horizontal component of the axial forces in batter piles from the horizontal load H to obtain the residual horizontal force H' (referring to the force polygon presented in Fig. 1(b)).
6. The traditional statical method uses two approaches to resolve the residual horizontal force H'. Approach I: the residual horizontal force H' is equally distributed between each pile in the pile group. Approach II: the residual horizontal force H' is eliminated by amending the slopes of the batter piles.

2 APPLICATION OF THE TRADITIONAL STATICAL METHOD

The following three examples illustrate the application of the traditional statical method. Example 1 illustrates Approach I. Examples 2 and 3 illustrate Approach II.

Example 1: Analysis of the offshore pile group shown in Fig. 2 using the traditional statical method Approach I. The pile group is acted upon by a vertical force P = 444.8 kN (100 kips), a moment M = 325.4 kN·m (240 ft-kips), and a horizontal force H = 35.6 kN (8 kips). The vertical equivalent free-standing lengths of the piles are assumed to be 11 m (36 ft). Batter piles for Piles 1 and 5 are used.

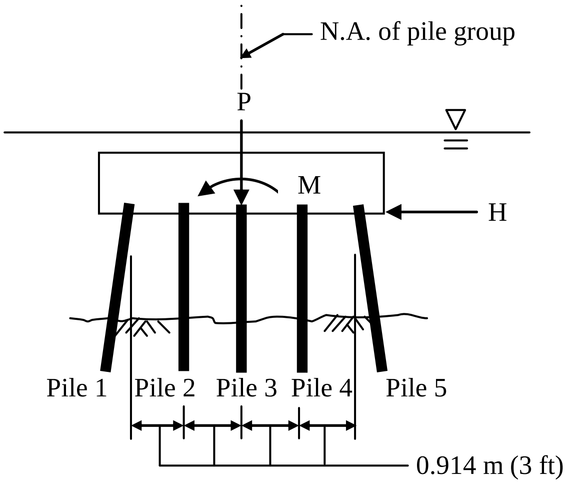

Figure 2. Pile group illustrating Examples 1 and 2

Using Eq. (1), one has

$$\sum(x^2) = (1.828)^2 + (0.914)^2 + (0.914)^2 + (1.828)^2 = 8.354\ \text{m}^2\text{, and}$$

$$V_i = \frac{444.8\text{kN}}{5} \pm \frac{325.4\text{kN}\cdot\text{m}}{8.354\text{m}^2}(x)$$

From which, the vertical component of the axial force in Pile 1, V_1 = 160.1 kN, the axial force in Pile 2, V_2 = 124.5 kN, the axial force in Pile 3, V_3 = 89.0 kN, the axial force in Pile 4, V_4 = 53.4 kN, and the vertical component of the axial force in Pile 5, V_5 = 17.8 kN.

Using a 1:8 slope for the batter piles (Piles 1 and 5) results in

The axial force in Pile 1,

$$R_1 = (160.1\text{ kN})\left(\frac{\sqrt{(8)^2 + (1)^2}}{8}\right) = 161.3\text{ kN, and}$$

The axial force in Pile 5,

$$R_5 = (17.8\text{kN})\left(\frac{\sqrt{(8)^2 + (1)^2}}{8}\right) = 17.9\text{ kN.}$$

The net horizontal component of the axial forces in batter Piles 1 and 5 with a slope of 1:8 thus can be computed as

$$(V_1 - V_5)(1/8) = (160.1\text{ kN} - 17.8\text{ kN})(1/8) = 17.8\text{ kN}$$

The residual horizontal force H' can then be computed as

$$H' = 35.6\text{ kN} - 17.8\text{ kN} = 17.8\text{ kN}$$

Thus, the residual horizontal force, which is distributed to each pile = (17.8 kN)/(5) = 3.6 kN. Therefore, the shear force in each pile is 3.6 kN.

Example 2: Using the traditional statical method Approach II by amending the slopes of the batter piles in Example 1 so that the residual horizontal force can be eliminated.

The slope of the batter piles is amended from 1 horizontal : 8 vertical to 1 horizontal : 4 vertical. Thus, the residual horizontal force can be eliminated as shown below:

$$H' = 35.6\text{ kN} - (160.1\text{ kN} - 17.8\text{ kN})(1/4) = 0$$

Referring to Example 1, the final pile axial forces are then computed as

$$R_1 = (160.1\text{ kN})\left(\frac{\sqrt{(4)^2 + (1)^2}}{4}\right) = 165.0\text{ kN,}$$

V_2 = 124.5 kN,

V_3 = 89.0 kN,

V_4 = 53.4 kN, and

$$R_5 = (17.8\text{kN})\left(\frac{\sqrt{(8)^2+(1)^2}}{8}\right) = 18.3 \text{ kN}$$

Note that since H' = 0, there is no residual horizontal force to be distributed to each pile. Therefore, there is no shear force in each pile.

Example 3: Analysis of the offshore pile group shown in Fig. 3 using the traditional statical method Approach II. The pile group is acted upon by a vertical force P = 444.8 kN (100 kips), a moment M = 325.4 kN·m (240 ft-kips), and a horizontal force H = 26.7 kN (6 kips). The vertical equivalent free-standing lengths of the piles are assumed to be 11 m (36 ft). Batter piles for Piles 1, 2, 4, and 5 are used.

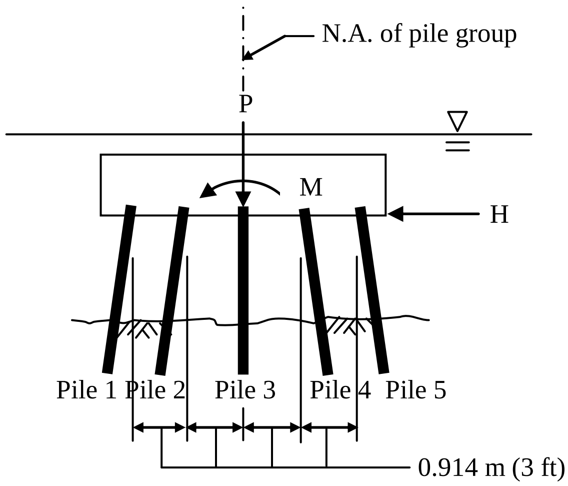

Figure 3. Pile group illustrating Example 3

Referring to Example 1, the vertical component of the axial force in Pile 1, V_1 = 160.1 kN, the vertical component of the axial force in Pile 2, V_2 = 124.5 kN, the axial force in Pile 3, V_3 = 89.0 kN, the vertical component of the axial force in Pile 4, V_4 = 53.4 kN, and the vertical component of the axial force in Pile 5, V_5 = 17.8 kN.

Using a 1:8 slope for the batter piles (Piles 1, 2, 4, and 5) results in

The axial force in Pile 1,

$$R_1 = (160.1 \text{ kN})\left(\frac{\sqrt{(8)^2+(1)^2}}{8}\right) = 161.3 \text{ kN},$$

The axial force in Pile 2,

$$R_2 = (124.5 \text{ kN})\left(\frac{\sqrt{(8)^2+(1)^2}}{8}\right) = 125.5 \text{ kN},$$

The axial force in Pile 4,

$$R_4 = (53.4 \text{ kN})\left(\frac{\sqrt{(8)^2+(1)^2}}{8}\right) = 53.8 \text{ kN, and}$$

The axial force in Pile 5,

$$R_5 = (17.8\text{kN})\left(\frac{\sqrt{(8)^2+(1)^2}}{8}\right) = 17.9 \text{ kN.}$$

Also, the residual horizontal force can be computed as shown below:

H' = 26.7 kN – (160.1 kN + 124.5 kN – 53.4 kN – 17.8 kN) (1/8) = 0

Since H' = 0, there is no residual horizontal force to be distributed to each pile. Therefore, there is no shear force in each pile.

3 EVALUATION OF THE ACCURACY OF THE TRADITIONAL STATICAL METHOD

The computer software SAP2000 (SAP2000 Educational 1997) is used to evaluate the accuracy of the results obtained by using the traditional statical method presented in Examples 1, 2, and 3. Two computer models for two cases (one is assumed to have a pinned condition at the top and bottom of the equivalent free-standing pile length, the other is assumed to have a fixed condition at the top and bottom of the equivalent free-standing pile length) have been considered for each example, as shown in Figs. 4, 5, and 6. The pile caps are assumed to be rigid. Also, HP12×74 steel piles (AISC 2005) are assumed to be used for the pile groups. The vertical equivalent free-standing length of the piles is h = 11 m for each model. The spacing between each pile is s = 0.914 m.

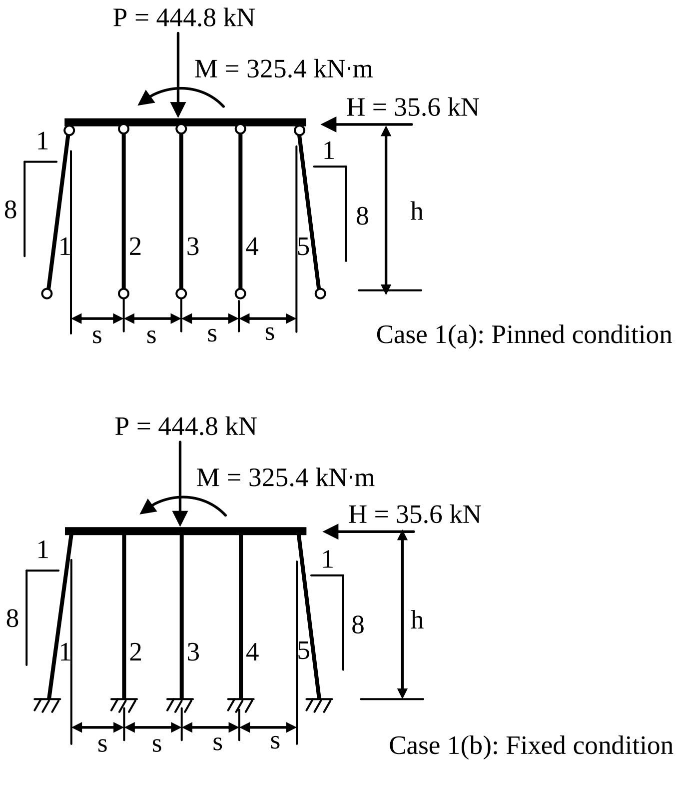

Figure 4. Computer models for the pile group illustrated in Example 1

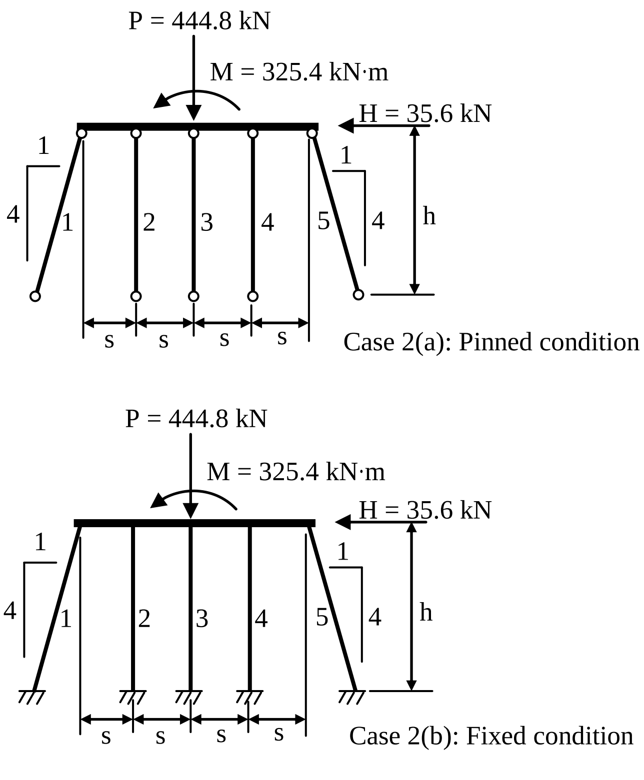

Figure 5. Computer models for the pile group illustrated in Example 2

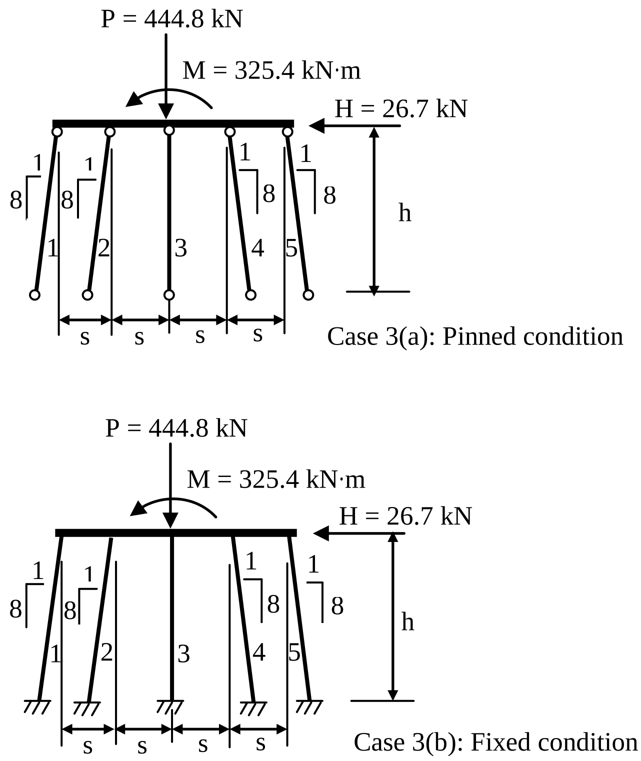

Figure 6. Computer models for the pile group illustrated in Example 3

Pile axial and shear forces obtained by using the traditional statical method Approach I presented in Example 1 (in which the residual horizontal force H' is not eliminated) and by using SAP2000 are tabulated in Table 1.

Table 1. Pile axial and shear forces obtained by using the traditional statical method Approach I presented in Example 1 and by using SAP2000

Pile	Approach I presented in Example 1		SAP2000			
			Case 1(a): Pinned condition		Case 1(b): Fixed condition	
	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)
1	161.3	3.6	231.8	0	196.4	1.7
2	124.5	3.6	-17.0	0	80.8	1.8
3	89.0	3.6	89.8	0	89.8	1.8
4	53.4	3.6	196.6	0	98.8	1.8
5	17.9	3.6	-55.0	0	-19.6	1.7

Note: the negative sign indicates that the axial force is in tension.

Table 1 indicates that the traditional statical method Approach I, which uses batter piles to resist a partial lateral force, may result in significant errors in computing the pile axial forces for either assuming a pinned or fixed condition at the ends of the free-standing pile length.

Pile axial and shear forces obtained by using the traditional statical method Approach II presented in Examples 2 and 3 (in which the residual horizontal force H' is eliminated) and by using SAP2000 are tabulated in Tables 2 and 3.

Table 2. Pile axial and shear forces obtained by using the traditional statical method Approach II presented in Example 2 and by using SAP2000

Pile	Approach II presented in Example 2		SAP2000			
			Case 2(a): Pinned condition		Case 2(b): Fixed condition	
	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)
1	165.0	0	160.2	0	161.7	0.2
2	124.5	0	127.8	0	121.1	0.1
3	89.0	0	92.2	0	92.2	0.1
4	53.4	0	56.6	0	63.3	0.1
5	18.3	0	13.4	0	11.9	0.1

Table 3. Pile axial and shear forces obtained by using the traditional statical method Approach II presented in Example 3 and by using SAP2000

Pile	Approach II presented in Example 3		SAP2000			
			Case 3(a): Pinned condition		Case 3(b): Fixed condition	
	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)	Axial force (kN)	Shear (kN)
1	161.3	0	161.0	0	156.9	0.1
2	125.5	0	125.1	0	130.6	0.1
3	89.0	0	90.6	0	90.6	0.1
4	53.8	0	53.4	0	47.9	0.1
5	17.9	0	17.5	0	21.5	0.1

Tables 2 and 3 indicate that the results obtained from the traditional statical method Approach II, which uses batter piles to resist the complete lateral force, are quite close to that obtained by using SAP2000 for both pinned and fixed conditions at the ends of the free-standing pile length.

The results obtained from SAP2000 presented in Tables 1 and 2 are further illustrated as the following:

The results of the Case 1(a), shown in Table 1, are the sum of the results obtained from the vertical force P, the moment M, and the horizontal force H, respectively, as shown in Fig. 7. Note that the unit of the pile axial forces is kN.

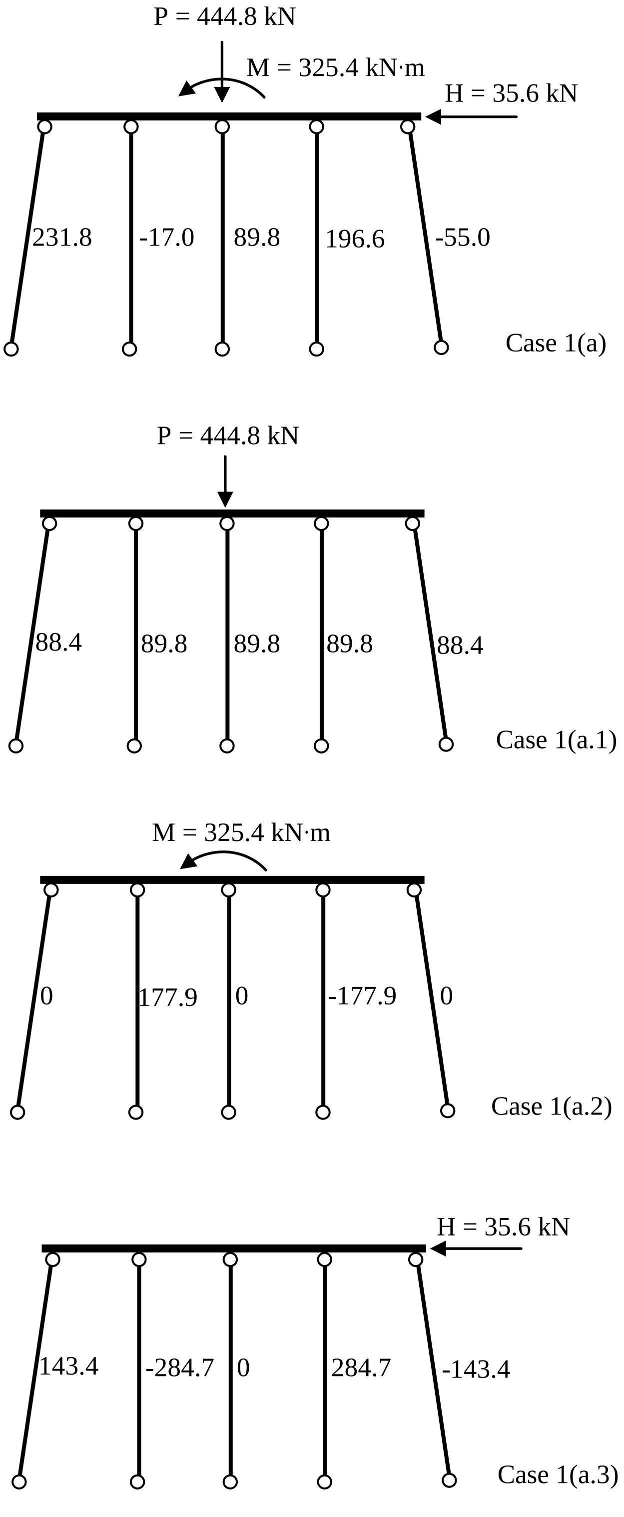

Figure 7. Results of Case 1(a)

The results of the Case 1(b), shown in Table 1, are the sum of the results obtained from the vertical force P, the moment M, and the horizontal force H, respectively, as shown in Fig. 8. Note that the unit of the pile axial and shear forces is kN.

The results of the Case 2(a), shown in Table 2, are the sum of the results obtained from the vertical force P, the moment M, and the horizontal force H, respectively, as shown in Fig. 9. Note that the unit of the pile axial forces is kN.

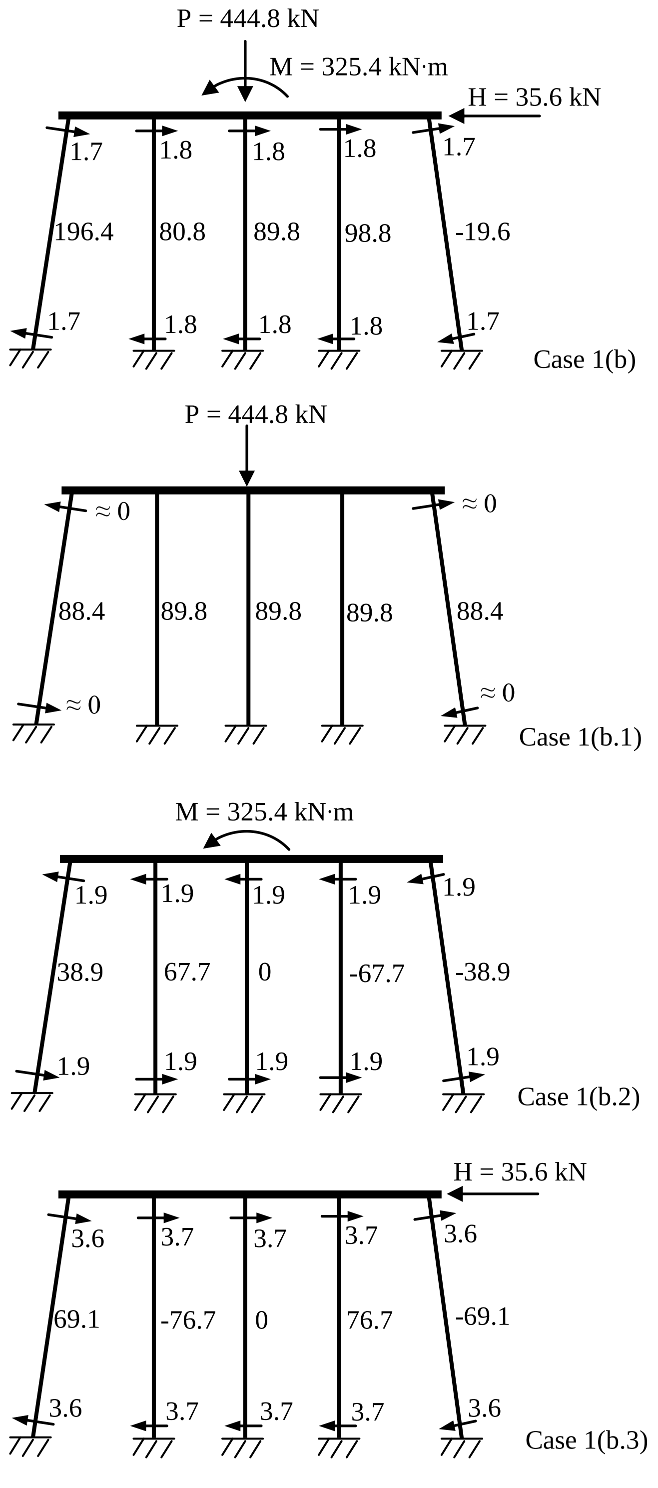

Figure 8. Results of Case 1(b)

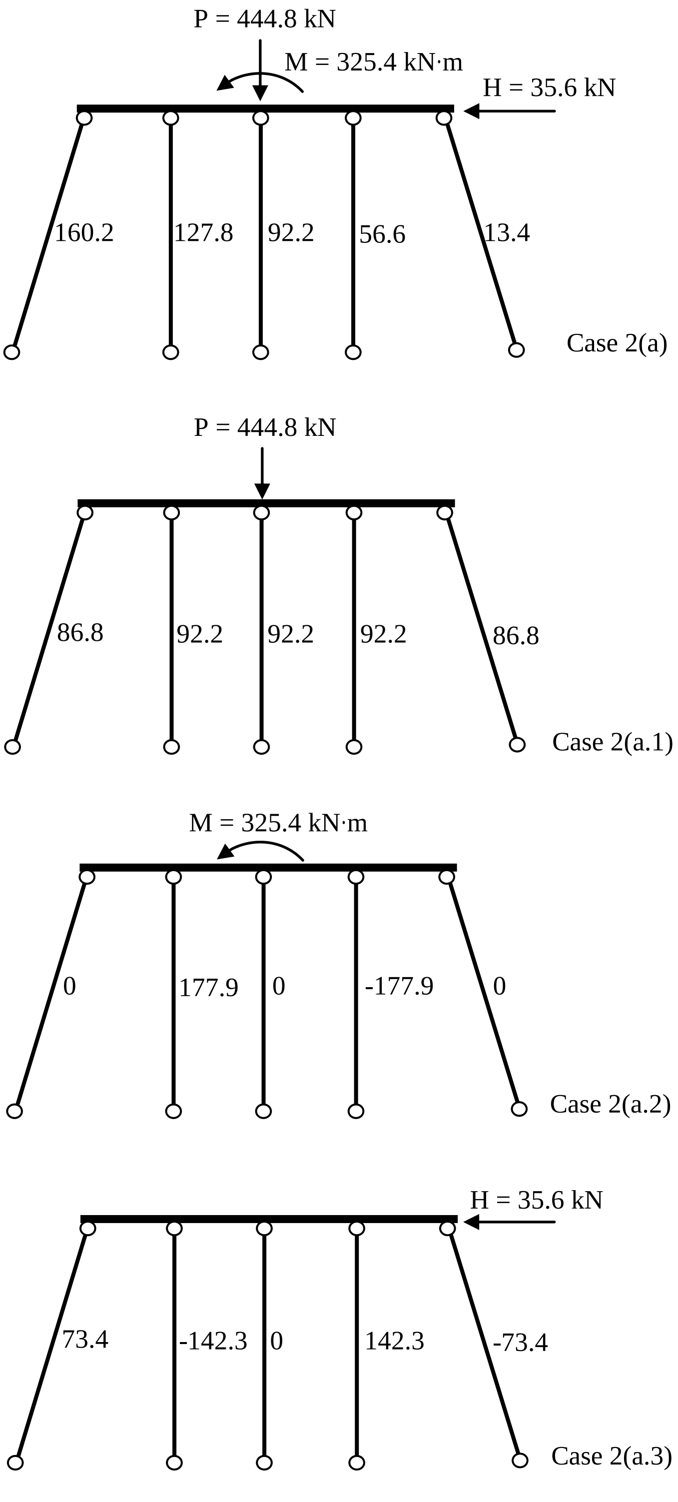

Figure 9. Results of Case 2(a)

The results of the Case 2(b), shown in Table 2, are the sum of the results obtained from the vertical force P, the moment M, and the horizontal force H, respectively, as shown in Fig. 10. Note that the unit of the pile axial and shear forces is kN.

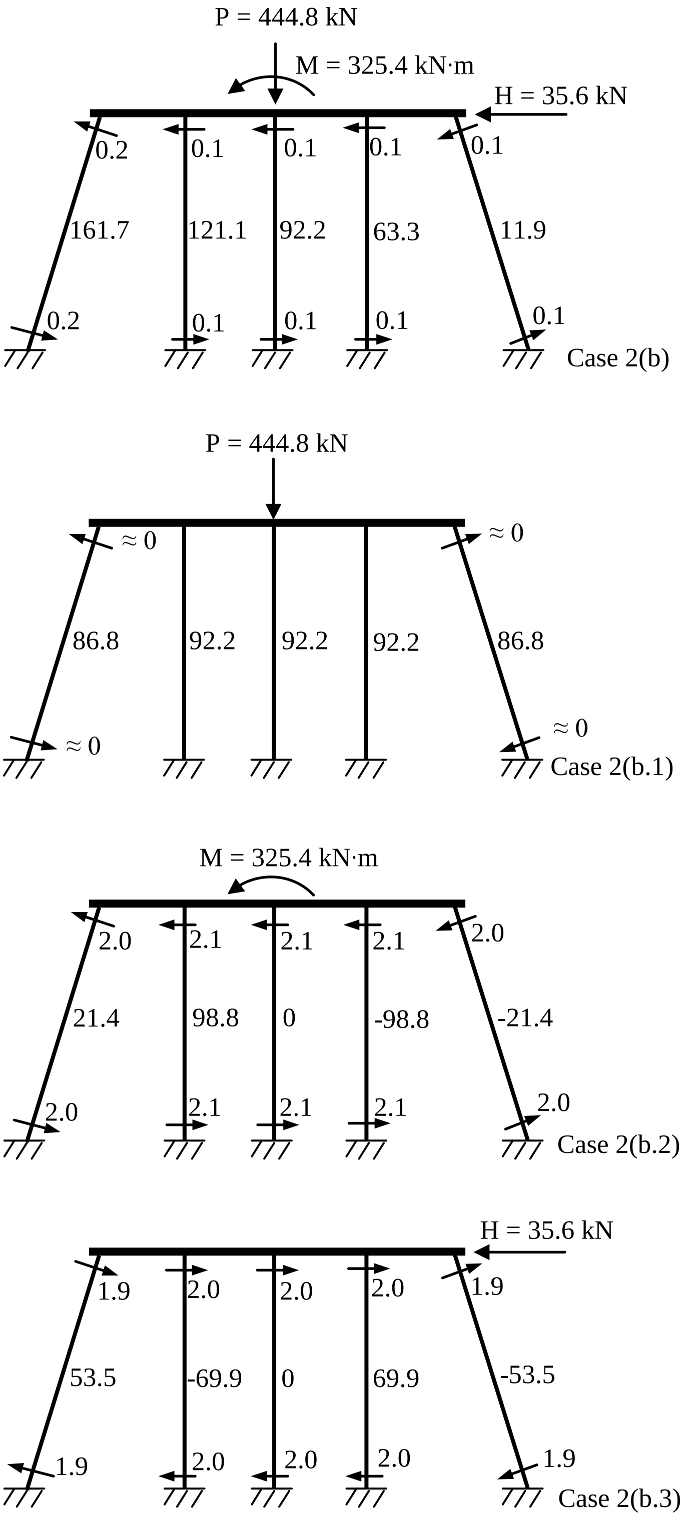

Figure 10. Results of Case 2(b)

Furthermore, since the structural systems shown in Figs. 7 and 9 are statically determinate, the pile axial forces presented in these figures computed by using SAP2000 can be validated by using a simple hand-calculated approach. The following example demonstrates the computation of the pile axial forces shown in Fig. 9 Case 2(a.3) by using a hand-calculated approach:

Referring to Fig. 11, one has

The sum of the moments about the point o, $\sum M_o = 0$.

Thus, $(V_2 + V_4)$ (0.914 m) – 35.6 kN (7.31 m) = 0.

From which and due to the symmetry of the structural system, one has

$V_2 = V_4 = 142.3$ kN.

Note that the axial force V_2 in Pile 2 is in tension while the axial force V_4 in Pile 4 is in compression.

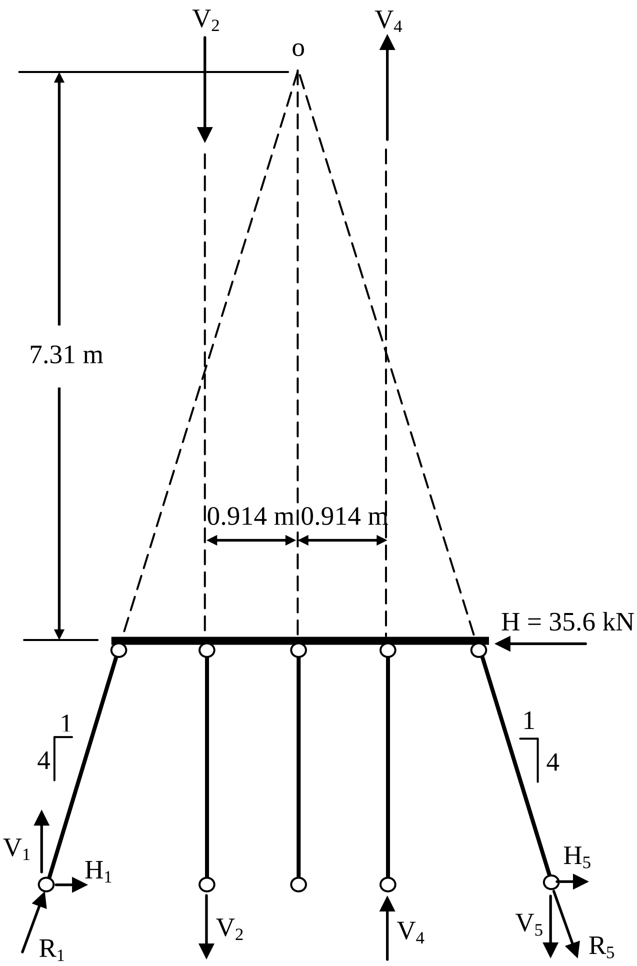

Figure 11. Computation of pile forces for Case 2(a.3)

Also, the sum of the horizontal forces, $\sum H = 0$.

Thus, $H_1 + H_5 - 35.6$ kN = 0.

From which and due to the symmetry of the structural system, one has

$H_1 = H_5 = 17.8$ kN.

From which, one has

$$R_1 = R_5 = (17.8 \text{ kN})\left(\frac{\sqrt{(4)^2 + (1)^2}}{1}\right) = 73.4 \text{ kN}.$$

Note that the axial force R_1 in Pile 1 is in compression while the axial force R_5 in Pile 5 is in tension.

The above results agree with that shown in Fig. 9 Case 2(a.3).

4 CONCLUSIONS

The traditional statical method has two approaches for the analysis of a loaded pile group containing batter piles: Approach I uses batter piles to resist a partial lateral load while Approach II uses batter piles to resist the complete lateral load. Based on the study presented in this paper, Approach I may result in significant errors in computing the pile axial forces. Therefore, the accuracy of Approach I is questionable. The study in this paper also concludes that the results obtained from the traditional statical method Approach II are quite close to the "exact" results obtained by using either computer software or a hand calculation. Therefore, Approach II is quite an accurate approximate approach for the analyses of pile groups containing batter piles.

The following are the assumptions used for each pile group in this study: (1) the pile cap is rigid and the piles are elastic; (2) all piles in the pile group have equal vertical equivalent free-standing lengths; (3) the top and bottom of the equivalent free-standing pile length are either all pinned or all fixed; and (4) the pile group is symmetric.

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NOTATION

- H = horizontal force
- h = the vertical equivalent free-standing length of a pile
- H' = residual horizontal force
- M = overturning moment
- n = the total number of piles
- P = vertical load
- R_i = the axial force in a batter pile
- s = center-to-center spacing of piles
- V_i = the axial force in a vertical pile or the vertical component of the axial force in a batter pile
- x = the distance of a pile to the neutral axis of a pile group