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Bending-Axis Effects on Load-Moment (P-M) Interaction Diagrams for Circular Concrete Columns Using a Limited Number of Longitudinal Reinforcing Bars

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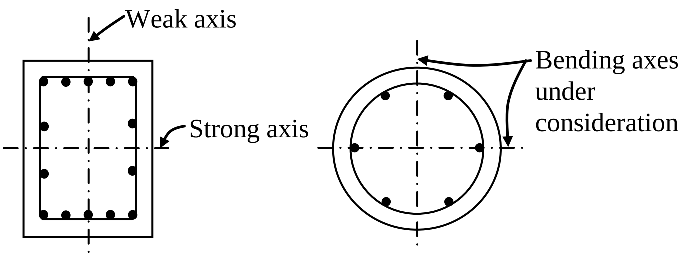
ABSTRACT: A rectangular reinforced concrete column usually has a strong axis and a weak axis. Bending about the strong axis results in a larger bending strength; on the other hand, bending about the weak axis results in a smaller bending strength. Since longitudinal reinforcing bars are usually uniformly distributed around the perimeters of circular columns, there is no strong or weak axis being defined for circular reinforced concrete columns. As a result, bending-axis effects on circular concrete columns using a large number of longitudinal reinforcing bars have traditionally been neglected. However, considerable bending-axis effects on circular concrete columns using a limited number of longitudinal reinforcing bars may exist and should not be neglected. An example of a circular reinforced concrete column using six longitudinal reinforcing bars is presented in this paper to demonstrate the bending-axis effects on the nominal axial compression-bending moment strength (P_n-M_n) interaction diagram of the column. A final P_n-M_n diagram that considers the bending-axis effects is also presented in this paper.

1 INTRODUCTION

Load-moment (P-M) strength interaction diagrams (ACI 1997) have been commonly used for the design of reinforced concrete columns. For a rectangular reinforced concrete section, as shown in Fig. 1(a), the P-M interaction diagram resulting from using the strong axis is different from that resulting from using the weak axis. Since there is no strong or weak axis being defined for a circular reinforced concrete section, it has been assumed that a circular reinforced concrete section will only result in one P-M diagram; it does not matter which bending axis in the section is used. In fact, similar to a rectangular section, the two different bending axes in the circular reinforced concrete section, as shown in Fig. 1(b), may also result in two quite different P-M diagrams. Sections 2, 3, and 4 in this paper demonstrate the development of the nominal axial compression-bending moment strength (P_n-M_n) interaction diagrams for a circular concrete column using 6 longitudinal reinforcing bars.



(a) Rectangular section (b) Circular section

Figure 1. Cross sections of reinforced concrete columns

2 PROPERTIES OF A CIRCULAR COMPRESSION BLOCK

When a circular reinforced concrete column is eccentrically loaded, the area of the Whitney compression block (Whitney 1942) in the cross section of the column can be computed by using either the geometric approach or the trigonometric integrals approach as shown below.

1. Geometric approach: Referring to Fig. 2, the area of a circular compression block can be obtained by subtracting the area of a triangular segment from the area of a circular segment.

The area of the circular segment enclosed by the radius OA and OB and the arc AB can be computed as

$$\left(\frac{h}{2}\right)^2 \pi \times \left(\frac{2\alpha_{rad}}{2\pi}\right) = \alpha_{rad}\left(\frac{h^2}{4}\right)$$

The area of the triangular segment enclosed by the radius OA and OB and the chord AB can be computed as

$$\left[2\left(\frac{h}{2}\right)\sin\alpha\right]\left[\left(\frac{h}{2}\right)\cos\alpha\right]\left(\frac{1}{2}\right) = \frac{h^2}{4}(\sin\alpha\cos\alpha)$$

Therefore, the area of the circular compression block is

$$A = \alpha_{rad}\left(\frac{h^2}{4}\right) - \frac{h^2}{4}(\sin\alpha\cos\alpha) = \frac{h^2}{2}\left(\frac{\alpha_{rad}}{2} - \frac{1}{4}\sin 2\alpha\right) \quad (1)$$

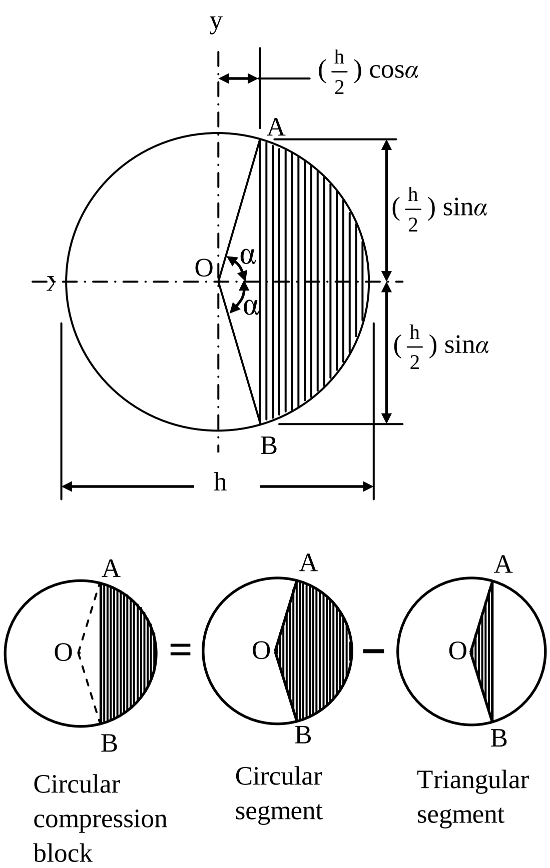


Figure 2. Circular compression block for bending about y-axis (geometric approach)

2. Trigonometric integrals approach: Referring to Fig. 3, the area of the circular compression block is

$$A = \int_0^{\alpha} 2\left(\frac{h}{2}\sin\theta\right)\left(\frac{h}{2}\sin\theta\right)d\theta$$
$$= \frac{h^2}{2}\int_0^{\alpha}\sin^2\theta\, d\theta$$
$$= \frac{h^2}{2}\left(\frac{\alpha_{rad}}{2} - \frac{1}{4}\sin 2\alpha\right)$$

Also, the location of the centroid of the circular compression block is

$$\overline{X} = \frac{\int_0^{\alpha} 2\left(\frac{h}{2}\sin\theta\right)\left(\frac{h}{2}\cos\theta\right)\left(\frac{h}{2}\sin\theta\right)d\theta}{A}$$

$$= \frac{\frac{h^3}{4}\left(\frac{\sin^3\alpha}{3}\right)}{A} \quad (2)$$

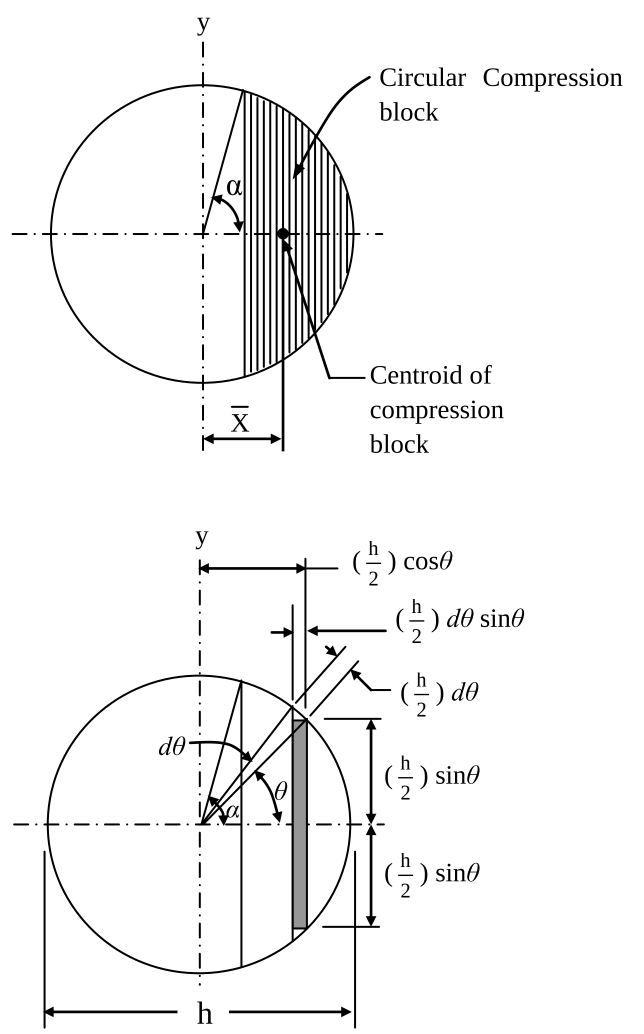


Figure 3. Circular compression block for bending about y-axis (trigonometric integrals approach)

3 EXAMPLES OF THE COMPUTATION OF NOMINAL AXIAL COMPRESSION STRENGTH AND NOMINAL MOMENT STRENGTH

The following two examples demonstrate the computation of the nominal axial compression strength (P_n) and the nominal moment strength (M_n) for a circular spiral column reinforced with six (6) longitudinal bars. The locations of the six longitudinal bars in both examples are symmetric about the bending axis (the y-axis). In Example 1, the bending axis goes through the centroid of the six bars but does not go through any of them (Fig. 4). In Example 2, the bending axis goes through two of the six bars, as well as the centroid of the six bars (Fig. 5). The statics method (Wang et al. 2007) is used for the computation of the P_n and the M_n values for these two examples.

Example 1: Determine the nominal axial compression strength (P_n) and the nominal moment strength (M_n) for a balanced strain condition on the section of a circular spiral column reinforced with 6-#10 (ASTM A615 bars (ASTM 2001)) longitudinal bars for bending about the y-axis as shown in Fig. 4. Given: The compressive strength of concrete, $f_c' =$ 20.7 MPa (3 ksi); the yield stress of steel, f_y = 414 MPa (60 ksi); and the modulus of elasticity of steel, E_s = 200,000 MPa (29,000 ksi). Note that the diameter of a #10 ASTM A615 bar is 32.3 mm (1.270") and the area of the bar, A_s = 8.19 cm^2 (1.27 in.^{2}); the diameter of a #3 ASTM A615 bar is 9.5 mm (0.375"). Also note that a balanced strain condition exists at a cross section when the strain of the tension steel reaches ε_y (the strain corresponding to f_y; i.e. $\varepsilon_y = f_y/E_s$) just as the maximum strain of the concrete in compression reaches its assumed ultimate strain of 0.003 (ACI 2008).

1. Determine the location of the neutral axis:
Referring to Fig. 4, the distance from extreme compression fibers to extreme tension steel, d_t = 444.2 mm. The yield strain of the steel, $\varepsilon_y = (f_y/E_s)$ = (414 MPa / 200,000 MPa) = 0.00207. The distance from the extreme compression fibers to the neutral axis for a balanced strain condition can be determined as

$$x_b = \frac{0.003}{0.003 + \varepsilon_y}\left(d_t\right) = 263 \text{ mm}$$

The depth of the Whitney equivalent rectangular stress distribution (Whitney 1942) in concrete for the balanced strain condition can be determined as

$$a_b = \beta_1 x_b = 0.85\ (263\text{ mm}) = 223.5\text{ mm}$$

Note that the factor β_1 is 0.85 for $f_c' \leq$ 27.6 MPa (4000 psi) (ACI 2008).

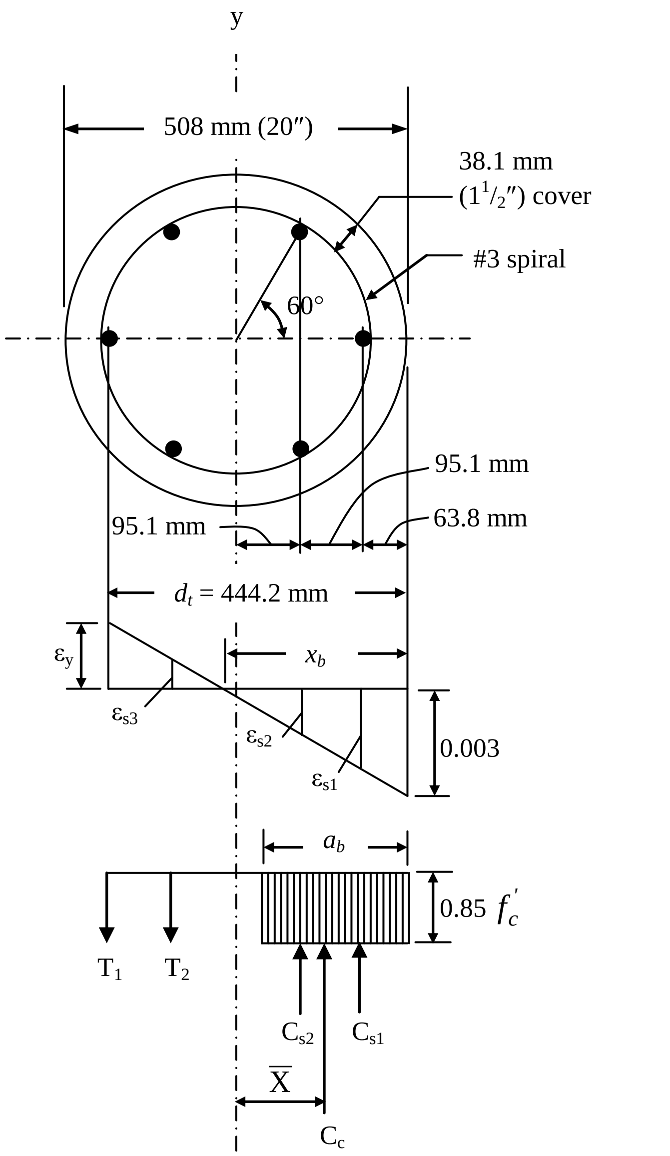


Figure 4. Balanced strain condition of Example 1

2. Calculate the properties of the circular compression block:
Referring to Figs. 3 & 4,

$$\alpha = \cos^{-1}\left(\frac{\frac{h}{2} - a_b}{\frac{h}{2}}\right) = \cos^{-1}\left(\frac{254\text{mm} - 223.5\text{mm}}{254\text{mm}}\right)$$

$$= 83.1° = 1.45 \text{ rad.}$$

The area of the circular compression block can be computed using Eq. (1):

$$A = \frac{(508)^2}{2}\left(\frac{1.45}{2} - \frac{1}{4}\sin 166.2°\right) = 858.5\,\text{cm}^2$$

The location of the centroid of the circular compression block can be determined using Eq. (2):

$$\overline{X} = \frac{\frac{(508)^3}{4}\left(\frac{\sin^3 83.1°}{3}\right)}{85850} = 124.5\,\text{mm}$$

3. Calculate the compressive force in concrete in the circular compression block:

$$C_c = 0.85\, f_c^{'}\, A = 0.85(0.0207)(85850) = 1510 \text{ kN}$$

4. Calculate the strains and forces in the tension and the compression steel:
Referring to Fig. 4, the strains of the steel are

$\varepsilon_{s1} = 0.00227$ ($> \varepsilon_y$)

$\varepsilon_{s2} = 0.00119$ ($< \varepsilon_y$), and

$\varepsilon_{s3} = 0.000984$ ($< \varepsilon_y$)

The forces in the steel are

$T_1 = A_s f_y = 819(0.414) = 339$ kN

$T_2 = A_s\, \varepsilon_{s3}\, E_s$
$= 2(819)(0.000984)(200) = 322$ kN

$C_{s1} = A_s\,(f_y - 0.85\, f_c^{'})$
$= 819[0.414 - 0.85(0.0207)] = 325$ kN

$C_{s2} = A_s\,(\varepsilon_{s2}\, E_s - 0.85\, f_c^{'})$
$= 2(819)[(0.00119)(200) - 0.85(0.0207)]$
$= 361$ kN

Note that the value of ($0.85\, f_c^{'}$) is the stress that has already been considered for the computation of the compressive force C_c.

5. Calculate the nominal axial compression strength:

$P_n = C_c + C_{s1} + C_{s2} - T_1 - T_2$
$= 1535$ kN (345 kips)

6. Calculate the nominal moment strength:

$$M_n = [C_c \times \overline{X} + (T_1 + C_{s1})(190.2) + (T_2 + C_{s2})(95.1)]\left(\frac{1}{1000}\right)$$

= 379 kN·m (280 k·ft)

Example 2: Determine the nominal axial compression strength (P_n) and the nominal moment strength (M_n) for a balanced strain condition on the section, as shown in Fig. 5. While the section and material properties of this example are the same as that given in Example 1, the bending axis of this example is perpendicular to that of Example 1.

1. Determine the location of the neutral axis:
Referring to Fig. 5, the distance from extreme compression fibers to extreme tension steel, d_t = 418.8 mm. The distance from the extreme compression fibers to the neutral axis for a balanced strain condition can be determined as

$$x_b = \frac{0.003}{0.003 + \varepsilon_y}(d_t) = 247.8 \text{ mm}$$

The depth of the Whitney equivalent rectangular stress distribution in concrete for the balanced strain condition can be determined as

$$a_b = \beta_1 x_b = 0.85\ (247.8 \text{ mm}) = 210.6 \text{ mm}$$

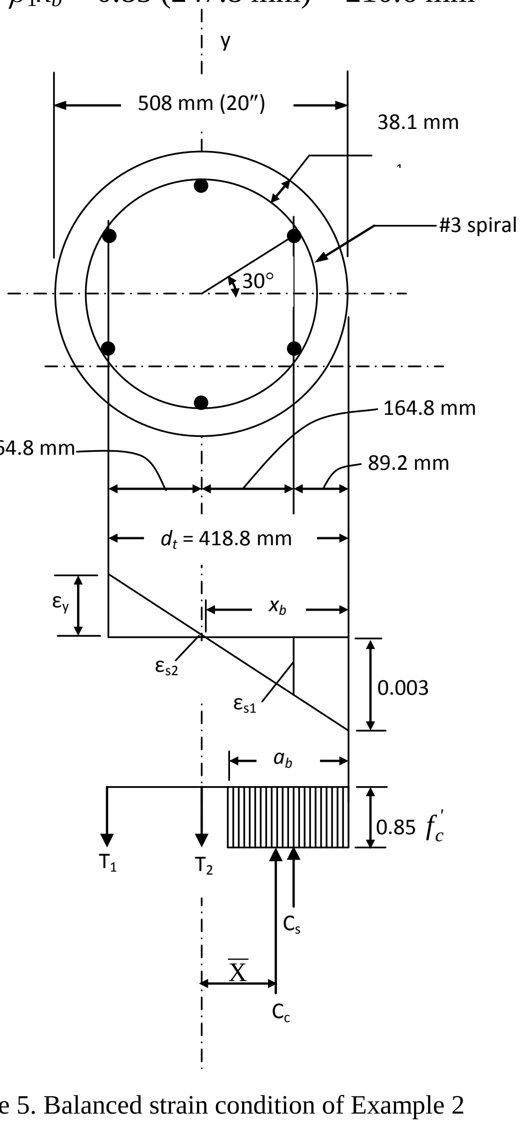


Figure 5. Balanced strain condition of Example 2

2. Calculate the properties of the circular compression block:

Referring to Figs. 3 & 5,

$$\alpha = \cos^{-1}\left(\frac{254\text{mm} - 210.6\text{mm}}{254\text{mm}}\right) = 80.16° = 1.399 \text{ rad.}$$

The area of the circular compression block can be computed using Eq. (1):

$$A = \frac{(508)^2}{2}\left(\frac{1.399}{2} - \frac{1}{4}\sin 160.3°\right) = 794 \text{ cm}^2$$

The location of the centroid of the circular compression block can be determined using Eq. (2):

$$\overline{X} = \frac{\frac{(508)^3}{4}\left(\frac{\sin^3 80.16°}{3}\right)}{79400} = 131.6 \text{ mm}$$

3. Calculate the compressive force in concrete in the circular compression block:

$C_c = 0.85\, f_c^{'}\, A$
$= 0.85(0.0207)(79400) = 1397$ kN

4. Calculate the strains and forces in the tension and the compression steel:

Referring to Fig. 5, the strains of the steel are

$\varepsilon_{s1} = 0.00192$ ($< \varepsilon_y$), and

$\varepsilon_{s2} = 0.0000744$ ($< \varepsilon_y$)

The forces in the steel are

$T_1 = A_s f_y = 2(819)(0.414) = 678$ kN

$T_2 = A_s\, \varepsilon_{s2}\, E_s$
$= 2(819)(0.0000744)\,(200) = 24$ kN

$C_s = A_s\,(\varepsilon_{s1}\, E_s - 0.85\, f_c^{'})$
$= 2(819)[(0.00192)\,(200) - 0.85(0.0207)]$
$= 600$ kN

5. Calculate the nominal axial compression strength:

$P_n = C_c + C_s - T_1 - T_2 = 1295$ kN (291 kips)

6. Calculate the nominal moment strength:

$$M_n = [C_c \times \overline{X} + (C_s + T_1)(164.8)]\left(\frac{1}{1000}\right)$$
$$= 394 \text{ kN·m (291 k·ft)}$$

The results derived from Examples 1 & 2 demonstrate the bending-axis effects on the balanced strain condition for a circular column with longitudinal bars. Due to the bending-axis effects, the M_n value obtained from Example 1 for the balanced strain condition is different from that obtained from Example 2. The M_n value obtained from Example 1 (379 kN·m) is about 96% of that obtained from Example 2 (394 kN·m). Also, the P_n value obtained from Example 2 for the balanced strain condition is different from that obtained from Example 1. The P_n value obtained from Example 2 (1295 kN) is only about 84% of that obtained from Example 1 (1535 kN).

4 NOMINAL AXIAL COMPRESSION-BENDING MOMENT STRENGTH INTERACTION DIAGRAMS

By using the statics method, as shown in Examples 1 & 2, the "P_n" value (the nominal axial compression strength) and the "M_n" value (the nominal bending moment strength) corresponding to each "a" value (the depth of the Whitney equivalent rectangular concrete stress distribution) are determined and are summarized, as shown in Tables 1 & 2. Table 1 is for the bending at axis location I (the location of the bending axis is shown in Example 1). Table 2 is for the bending axis at location II (the location of the bending axis is shown in Example 2).

Table 1. P_n and M_n data for the bending axis at location I (the location of the bending axis is shown in Example 1)

a		P_n		M_n	
mm	inches	kN	kips	kN·m	ft-kips
508.0	20.00	5510	1239	0	0
449.7	17.71	4684	1053	131	97
428.6	16.87	4508	1014	160	118
400.9	15.78	4219	949	198	146
369.0	14.53	3844	864	241	178
332.9	13.11	3396	763	282	208
301.7	11.88	2937	660	315	232
270.7	10.66	2430	546	343	253
239.5	9.43	1857	418	367	271
223.5	8.80	1535	345	379	280
210.1	8.27	1295	291	379	280
196.6	7.74	1037	233	378	279
175.1	6.89	592	133	374	276
142.7	5.62	147	33	329	243
133.7	5.27	0	0	312	230

Table 2. P_n and M_n data for the bending axis at location II (the location of the bending axis is shown in Example 2)

a		P_n		M_n	
mm	inches	kN	kips	kN·m	ft-kips
508.0	20.00	5510	1239	0	0
454.6	17.90	4684	1053	121	89
434.4	17.10	4508	1014	149	110
402.6	15.85	4219	949	191	141
369.7	14.55	3844	864	236	174
334.6	13.18	3396	763	281	207
302.3	11.90	2937	660	318	234
270.1	10.63	2430	546	349	258
236.7	9.32	1857	418	378	279
221.3	8.71	1535	345	388	286
210.6	8.29	1295	291	394	291
194.3	7.65	1037	233	382	282
168.6	6.64	592	133	356	263
146.2	5.76	147	33	328	242
139.6	5.50	0	0	318	234

Based on the data shown in Tables 1 & 2, the P_n-M_n interaction diagrams for the bending axis at locations I & II are constructed and are shown in Fig. 6.

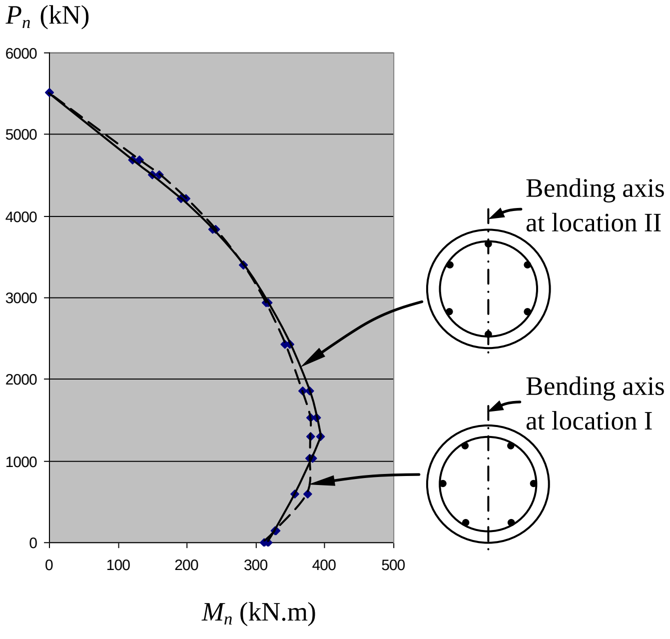


Figure 6. P_n-M_n interaction diagrams for bending axis at locations I & II

5 BENDING-AXIS EFFECTS ON P_n-M_n INTERACTION DIAGRAMS

Fig. 6 presents the two P_n-M_n interaction diagrams for the circular column. One results from the bending axis at location I, while the other results from the bending axis at location II. Due to the bending-axis effects, the two diagrams deviate from each other and also intersect with each other at several points. Four zones are identified between the intersected points, as shown in Fig. 7. Each zone has two envelopes for the P_n-M_n diagram.

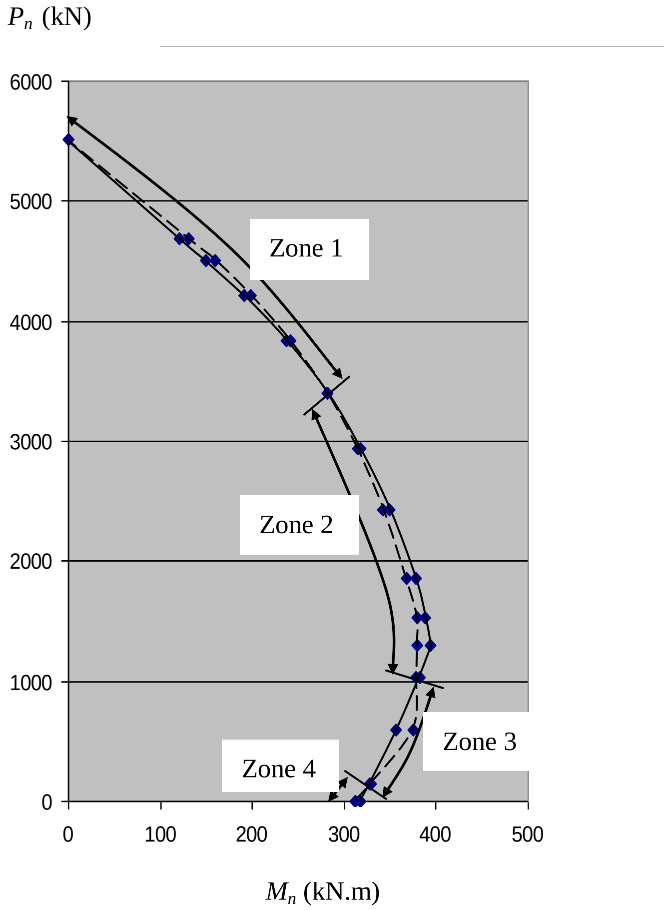


Figure 7. Bending axis effects on P_n-M_n interaction diagrams

6 PROPOSED FINAL P_n-M_n INTERACTION DIAGRAM

Within each zone as shown in Fig. 7, the envelope which goes through the smaller M_n values is the one that controls the bending moment strength for that zone. Referring to Figs. 6 & 7, since the bending axis at location II results in smaller M_n values in zones 1 & 3, it controls the envelopes in these two zones. Also, since the bending axis at location I results in smaller M_n values in zones 2 & 4, it controls the envelopes in these two zones. Fig. 8 presents the proposed final P_n-M_n interaction diagram that takes the bending-axis effects into consideration for the circular column. Note that the final diagram was constructed by using the controlling envelope in each zone.

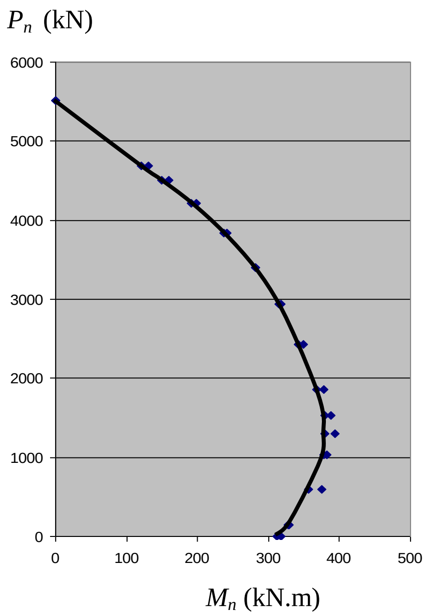


Figure 8. Final P_n-M_n interaction diagram

7 CONCLUSIONS

Load-moment (*P-M*) strength interaction diagrams for rectangular and circular cross sections have been commonly used for the design of reinforced concrete columns. Two *P-M* diagrams resulting from two bending axes (strong and weak axes) are usually used for the design of a rectangular reinforced concrete column. However, since there is only one bending axis being considered for the design of a circular reinforced concrete column, the bending-axis effects on the *P-M* interaction diagrams for circular reinforced concrete columns have traditionally been ignored. As demonstrated in this paper, the bending-axis effects on the *P-M* interaction diagrams for a circular concrete column using a limited number of longitudinal reinforcing bars are considerable and should not be neglected. A recommended P_n-M_n diagram, therefore, is proposed in this paper in order to take the bending-axis effects into consideration.

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NOTATION

A	=	area of a circular compression block
A_s	=	area of reinforcement
a	=	depth of Whitney equivalent rectangular stress distribution in concrete
a_b	=	depth of Whitney equivalent rectangular stress distribution in concrete for a balanced strain condition
C_c	=	compressive force in concrete in a circular compression block
C_s	=	compressive force in longitudinal reinforcement
d_t	=	distance from extreme compression concrete fibers to extreme tension reinforcement
E_s	=	modulus of elasticity of reinforcement
f'_c	=	specified compressive strength of concrete
f_y	=	specified yield strength of reinforcement
M_n	=	nominal moment strength
P_n	=	nominal axial load strength
T	=	tensile force in longitudinal reinforcement
$\overline{X}$	=	location of the centroid of a circular compression block
x_b	=	distance from extreme compression fibers to the neutral axis for a balanced strain condition
β_1	=	the ratio a_b / x_b
ε_s	=	strain in longitudinal reinforcement