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Computation of Fundamental Periods for Moment Frames Using a Hand-Calculated Approach

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ABSTRACT: The second method (known as Method B), specified in the 1997 Uniform Building Code Static Force Procedure, is a rational and accurate approach for finding the fundamental period of a frame. The formula used in Method B, however, is not a popular formula because it involves time-consuming computations of frame deflections which usually require the use of computer software. A hand-calculated approach for the computation of frame deflections using a calculator rather than a computer is suggested in this paper in order to turn Method B into a practical method for determining the fundamental periods of low-rise moment frames. The general stiffness matrix of a three-story, three-bay frame presented in this paper is intended to be used as an aid to compute the deflections for any moment frame within three stories in height and within three bays in width. Examples shown in this paper illustrate the step by step procedure for the computation of the fundamental periods of low-rise moment frames using the proposed hand-calculated approach. These examples also demonstrate that the results obtained from the proposed hand-calculated static approach agree with that obtained from the dynamic analysis.

Keywords: bending moments, concrete beams, concrete columns, degrees of freedom, dynamic analysis, lateral forces, steel frames, and stiffness.

1 INTRODUCTION

In the Equivalent Lateral Force Procedure as specified in the ASCE/SEI 7-05 [1], the approximate fundamental period (T_a) of a structure in the direction being considered can be determined using the following equation:

$$T_a = C_t h_n^x \tag{1}$$

where hn = the height above the base to the highest level of the structure; Ct = 0.0724 for steel moment-resisting frames, 0.0466 for reinforced concrete moment-resisting frames; and x = 0.8 for steel moment-resisting frames, 0.9 for reinforced concrete moment-resisting frames.

In the Static Force Procedure as specified in the Uniform Building Code [2], there are two methods for determining the fundamental period (T). The first (known as Method A) is an approximate method using the following formula:

$$T = C_t \left(h_n\right)^{3/4} \tag{2}$$

where C_t = 0.0853 for steel moment-resisting frames and 0.0731 for reinforced concrete moment-resisting frames.

The second method (known as Method B) is based on the structural properties and deformational characteristics of the resisting elements and is a more rational approach. In this approach, the fundamental period T can be computed using the following formula:

$$T = 2\pi\sqrt{\left(\sum_{i=1}^{n} w_i \delta_i^2\right) \div \left(g\sum_{i=1}^{n} f_i \delta_i\right)} \tag{3}$$

where w_i = the portion of the total seismic dead load located at or assigned to level i; δ_i = the horizontal displacement at level i relative to the base due to applied lateral forces; g = the acceleration due to gravity; and f_i = the lateral force at level i.

Although Eq. (3) is a fairly accurate formula for the computation of the fundamental period of a frame, it is not commonly used by structural engineers because the applied lateral force and the horizontal displacement at each level of the frame are required. In order to make Eq. (3) a practical formula which can be used by structural engineers, a hand-calculated approach for the computation of fundamental periods for low-rise moment frames is presented in this paper. This approach uses the Vertical-Distribution-of-Seismic-Forces formula as shown in ASCE/SEI 7-05 to assign the distribution of lateral forces over the height of the frame:

$$F_x = \left(\frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} \right) (V) \tag{4}$$

where F_x = the lateral force induced at level x of the frame; w_i, w_x = the portion of the total gravity load of the structure assigned to level i or x, respectively; h_i, h_x = the height from the base to level i or x, respectively; k = a distribution exponent related to the frame period, k = 1 for a frame having a period of 0.5 second or less (note that since this paper is dealing with low-rise frames, k = 1 is assumed for all the examples presented; also note that this assumption shall be made only for the computation of the fundamental periods of the frames) ; and V = the total design lateral force or shear at the base of the frame.

This hand-calculated approach also uses the general stiffness matrices presented later in this paper to compute the horizontal displacement at each level of a frame.

2 EXAMPLE FOR THE COMPUTATION OF THE FUNDAMENTAL PERIOD OF A FRAME WITH RIGID BEAMS

The following example demonstrates the accuracy of the approach using Eqs. (3) and (4) for the computation of the fundamental period of a frame with rigid beams.

Example: Compute the fundamental period of the three-story frame shown in Fig. 1. Assume that the beams are rigid (i.e., the flexural rigidity = ∞ for each beam). The column sizes [3] are shown in the figure. The moment of inertia about the x-axis is I_x = 8.91 (10^8) mm^4 for the W14×176 columns and is I_x = 5.16 (10^8) mm^4 for the W12×136 columns. The modulus of elasticity is E = 2.00 (10^5) MPa for all columns. The weight of each floor = 890 kN (200 kips). Neglect the shear and axial deformations for each column and beam.

Approach A (using Eqs. [3] and [4]):

1. Compute the vertical distribution of seismic forces. Assume the total design lateral force V = 100 kN. The lateral force induced at each level thus can be computed as shown in Table 1 using Eq. (4). Note that k =1 (k is a distribution exponent related to the frame period) has been assumed in Eq. (4) as mentioned early in this paper.

Table 1. Computation of the vertical distribution of seismic forces

Level x	h_x(m)	w_x(kN)	$h_x w_x$	$\frac{h_x w_x}{\sum h_i w_i}$	F_x (kN)
3	11.89	890	10,582	0.4816	48.16
2	8.23	890	7325	0.3333	33.33
1	4.57	890	4067	0.1851	18.51
			∑ = 21,974		∑ =100.00

Compute the horizontal displacement of each floor. With the lateral force induced at each level determined, the relative horizontal displacement between each adjacent level of the frame can then be computed using the displacement formula shown in Fig. 2.

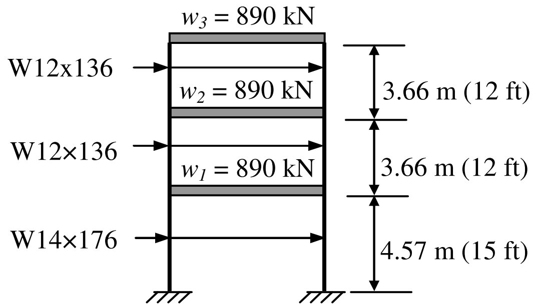


Figure 1. Three-story frame with rigid beams

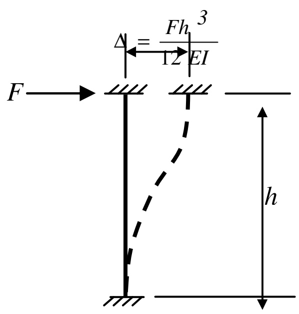


Figure 2. Flexural deformation of a column between rigid beams

The relative horizontal displacement between the 1st floor and the ground floor is

$$\Delta_1 = \frac{(F_1+F_2+F_3)h_1^3}{12EI_x} = \frac{(100\times10^3)(4570)^3}{12(2\times10^5)(2\times8.91\times10^8)}$$
$$= 2.23\,\text{mm}$$

The relative horizontal displacement between the 2nd floor and the 1st floor is

$$\Delta_2 = \frac{(F_2+F_3)h_2^3}{12EI_x} = \frac{(81.49\times10^3)(3660)^3}{12(2\times10^5)(2\times5.16\times10^8)} = 1.61\,\text{mm}$$

The relative horizontal displacement between the roof and the 2nd floor is

$$\Delta_3 = \frac{F_3h_3^3}{12EI_x} = \frac{(48.16\times10^3)(3660)^3}{12(2\times10^5)(2\times5.16\times10^8)} = 0.95\,\text{mm}$$

Therefore, the total horizontal displacement at the 2nd floor is $\delta_2 = \Delta_1 + \Delta_2 = 3.84$ mm. Also, the total horizontal displacement at the roof (3rd level) is $\delta_3 = \Delta_1 + \Delta_2 + \Delta_3 = 4.79$ mm.

Compute the fundamental period of the frame. Table 2 shows the computations of $w_i\delta_i^2$ and $f_i\,\delta_i$ using the results obtained from Steps 1 and 2. The fundamental period of the frame in turn can be determined using Eq. (3) and the results from Table 2:

$$T = 2\pi\sqrt{\left(\sum_{i=1}^{n} w_i\delta_i^2\right) \div \left(g\sum_{i=1}^{n} f_i\delta_i\right)}$$

$$= 2\pi\sqrt{\frac{37{,}970\,\text{kN}\cdot\text{mm}^2}{\frac{9810\,\text{mm}}{\text{sec}^2}(400\,\text{kN}\cdot\text{mm})}} = 0.618\,\text{sec}$$

Table 2. Computation of $w_i\delta_i^2$ and $f_i\,\delta_i$

Level i	w_i	f_i	δ_i	$w_i\delta_i^2$	$f_i\delta_i$
	kN	kN	mm	kN·mm²	kN·mm
3	890	48.16	4.79	20,420	230.7
2	890	33.33	3.84	13,124	128.0
1	890	18.51	2.23	4426	41.3
				Σ = 37,970	Σ = 400.0

Approach B (using the dynamic analysis):

The following demonstrates the computation of the fundamental period of the frame using the dynamic analysis.

The equation of motion for free vibration of a multiple-degree-of-freedom structural system is

$$([K]-\omega^2[M])\{v\} = \{0\}$$

where $[K]$ = the stiffness matrix of the structural system; $[M]$ = the mass matrix of the structural system; $\{v\}$ = the displacement vector of the structural system; and ω = angular frequency.
Referring to Fig. 1, the total combined stiffness of the two columns in the 1st story is

$$k_1 = 2\left(\frac{12EI_x}{h_1^3}\right) = 2\left(\frac{12(2\times10^5)(8.91\times10^8)}{(4570)^3}\right) = 44.8\,\text{kN/mm}$$

$$= 44.8\times10^6\,\text{N/m}$$

The total combined stiffness of the two columns in the 2nd and 3rd stories is

$$k_2 = k_3 = 2\left(\frac{12(2\times10^5)(5.16\times10^8)}{(3660)^3}\right) = 50.5\,\text{kN/mm}$$

$$= 50.5\times10^6\,\text{N/m}$$

The mass of the roof and the floors is

$$m_1 = m_2 = m_3 = \frac{W}{g} = \frac{890{,}000\,\text{N}}{9.81\,\text{m/sec}^2} = 90{,}700\,\text{kg}$$

Therefore,

$$[K]-\omega^2[M]$$

$$= \begin{bmatrix} k_1+k_2 & -k_2 & \\ -k_2 & k_2+k_3 & -k_3 \\ & -k_3 & k_3 \end{bmatrix} - \omega^2 \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix}$$

$$= \begin{bmatrix} 95.3\times10^6 - 90{,}700\omega^2 & -50.5\times10^6 & \\ -50.5\times10^6 & 101\times10^6 - 90{,}700\omega^2 & -50.5\times10^6 \\ & -50.5\times10^6 & 50.5\times10^6 - 90{,}700\omega^2 \end{bmatrix}$$

Setting the determinant equation to zero, that is, $\det([K]-\omega^2[M]) = \{0\}$, results in the 1st modal frequency $\omega_1 =$ 10.16 rad/sec, the 2nd modal frequency $\omega_2 =$ 28.83 rad/sec, and the 3rd modal frequency $\omega_3 =$ 42.28 rad/sec. The fundamental period, T_1, of the frame can then be determined to be

$$T_1 = \frac{2\pi}{\omega_1} = 0.618\,\text{sec}$$

The above results show that the fundamental period obtained from the dynamic analysis agrees with that obtained from Eqs. (3) and (4) for the three-story frame with rigid beams as shown in Fig. 1.

3. GENERAL STIFFNESS MATRICES FOR MOMENT FRAMES WITH FLEXURAL BEAMS

As shown in the previous example, using Eq. (3) involves the computation of the horizontal displacements of the frame under consideration. The computation of the horizontal displacements for a moment frame with flexural beams, however, is very time consuming. A general stiffness matrix is therefore introduced in this paper in order to simplify the computation of the horizontal displacements for moment frames with flexural beams.

The general stiffness matrix of a three-story, three-bay moment frame with fixed column bases as shown in Fig. 3 can be constructed using the following procedure:

Referring to Fig. 3, the basic slope deflection equation for a beam *i-j* in the frame is

$$M_{i-j} = \frac{2E_{i-j}I_{i-j}}{L_{i-j}}\left(2\theta_i + \theta_j\right)$$

where $M_{i\text{-}j}$ = the moment at the end "*i*" of the beam *i-j*; $E_{i\text{-}j}$ = the modulus of elasticity of the beam *i-j*; $I_{i\text{-}j}$ = the moment of inertia of the beam *i-j*; $L_{i\text{-}j}$ = the length of the beam *i-j*; θ_i = the rotation at the end "*i*" of the beam *i-j*; and θ_j = the rotation at the end "*j*" of the beam *i-j*.

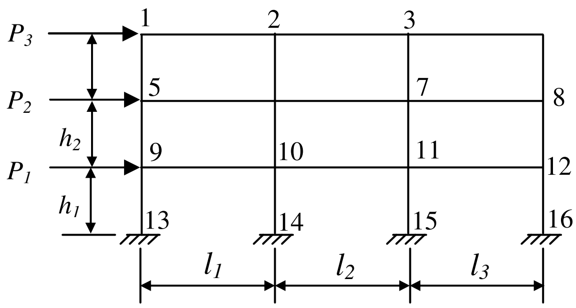


Figure 3. Laterally loaded three-story, three-bay frame

Setting $k_{i-j} = \frac{2E_{i-j}I_{i-j}}{L_{i-j}}$, one has the end moment equation, $M_{i-j} = k_{i-j}\left(2\theta_i + \theta_j\right)$, at the end "*i*" of the beam *i-j*. Also, referring to Fig. 3, the basic slope deflection equation for a column *i-j* in the frame is

$$M_{i-j} = \frac{2E_{i-j}I_{i-j}}{L_{i-j}}\left(2\theta_i + \theta_j - 3R_x\right)$$

where $M_{i\text{-}j}$ = the moment at the end "*i*" of the column *i-j*; $E_{i\text{-}j}$ = the modulus of elasticity of the column *i-j*; $I_{i\text{-}j}$ = the moment of inertia of the column *i-j*; $L_{i\text{-}j}$ = the length of the column *i-j*; θ_i = the rotation at the end "*i*" of the column *i-j*; θ_j = the rotation at the end "*j*" of the column *i-j*; and $R_x = \Delta_x / L_{i\text{-}j}$, where Δ_x = the relative deflection between the ends of the column *i-j* in the x^{th} story of the frame.

Setting $k_{i-j} = \frac{2E_{i-j}I_{i-j}}{L_{i-j}}$, one has the end moment equation, $M_{i-j} = k_{i-j}\left(2\theta_i + \theta_j - 3R_x\right)$, at the end "*i*" of the column *i-j*.

Since the summation of the end moments at each joint equals zero, one has the following equation at joint "1"

$$\sum M_{J1} = M_{1-2} + M_{1-5} = k_{1-2}\left(2\theta_1 + \theta_2\right) + K_{1-5}\left(2\theta_1 + \theta_5 - 3R_3\right) = 0$$

From which, the equation at joint "1" is

$$2\left(k_{1-2} + k_{1-5}\right)\theta_1 + k_{1-2}\theta_2 + k_{1-5}\theta_5 + 3k_{1-5}\left(-R_3\right) = 0$$

Similarly, one has the following equations at joints "2" through "12"

$\sum M_{J2} = 0$;

$$k_{1-2}\theta_1 + 2\left(k_{1-2} + k_{2-3} + k_{2-6}\right)\theta_2 + k_{2-3}\theta_3 + k_{2-6}\theta_6 + 3k_{2-6}\left(-R_3\right) = 0$$

$\sum M_{J3} = 0$;

$$k_{2-3}\theta_2 + 2\left(k_{2-3} + k_{3-4} + k_{3-7}\right)\theta_3 + k_{3-4}\theta_4 + k_{3-7}\theta_7 + 3k_{3-7}\left(-R_3\right) = 0$$

$\sum M_{J4} = 0$;

$$k_{3-4}\theta_3 + 2\left(k_{3-4} + k_{4-8}\right)\theta_4 + k_{4-8}\theta_8 + 3k_{4-8}\left(-R_3\right) = 0$$

$\sum M_{J5} = 0$;

$$k_{1-5}\theta_1 + 2\left(k_{5-6} + k_{1-5} + k_{5-9}\right)\theta_5 + k_{5-6}\theta_6 + k_{5-9}\theta_9 + 3k_{1-5}\left(-R_3\right) + 3k_{5-9}\left(-R_2\right) = 0$$

$\sum M_{J6} = 0$;

$$k_{2-6}\theta_2 + k_{5-6}\theta_5 + 2\left(k_{5-6} + k_{6-7} + k_{2-6} + k_{6-10}\right)\theta_6 + k_{6-7}\theta_7 + k_{6-10}\theta_{10} + 3k_{2-6}\left(-R_3\right) + 3k_{6-10}\left(-R_2\right) = 0$$

$\sum M_{J7} = 0$;

$$k_{3-7}\theta_3+k_{6-7}\theta_6+2(k_{6-7}+k_{7-8}+k_{3-7}+k_{7-11})\theta_7+k_{7-8}\theta_8$$
$$+k_{7-11}\theta_{11}+3k_{3-7}(-R_3)+3k_{7-11}(-R_2)=0$$

$\sum M_{J8}=0$;
$$k_{4-8}\theta_4+k_{7-8}\theta_7+2(k_{7-8}+k_{4-8}+k_{8-12})\theta_8+k_{8-12}\theta_{12}$$
$$+3k_{4-8}(-R_3)+3k_{8-12}(-R_2)=0$$

$\sum M_{J9}=0$;
$$k_{5-9}\theta_5+2(k_{9-10}+k_{5-9}+k_{9-13})\theta_9+k_{9-10}\theta_{10}$$
$$+3k_{5-9}(-R_2)+3k_{9-13}(-R_1)=0$$

$\sum M_{J10}=0$;
$$k_{6-10}\theta_6+k_{9-10}\theta_9+2(k_{9-10}+k_{10-11}+k_{6-10}+k_{10-14})\theta_{10}$$
$$+k_{10-11}\theta_{11}+3k_{6-10}(-R_2)+3k_{10-14}(-R_1)=0$$

$\sum M_{J11}=0$;
$$k_{7-11}\theta_7+k_{10-11}\theta_{10}+2(k_{10-11}+k_{11-12}+k_{7-11}+k_{11-15})\theta_{11}$$
$$+k_{11-12}\theta_{12}+3k_{7-11}(-R_2)+3k_{11-15}(-R_1)=0$$

$\sum M_{J12}=0$;
$$k_{8-12}\theta_8+k_{11-12}\theta_{11}+2(k_{11-12}+k_{8-12}+k_{12-16})\theta_{12}$$
$$+3k_{8-12}(-R_2)+3k_{12-16}(-R_1)=0$$

Since the summation of the end moments of the columns in the same story equals the total shear forces in that story times the story height, one has the following equations for the 3rd, 2nd, and 1st stories, respectively

$$M_{1-5}+M_{5-1}+M_{2-6}+M_{6-2}+M_{3-7}+M_{7-3}+M_{4-8}$$
$$+M_{8-4}=P_3h_3,$$
$$M_{5-9}+M_{9-5}+M_{6-10}+M_{10-6}+M_{7-11}+M_{11-7}$$
$$+M_{8-12}+M_{12-8}=(P_2+P_3)h_2, \quad \text{and}$$

$$M_{9-13}+M_{13-9}+M_{10-14}+M_{14-10}+M_{11-15}+M_{15-11}$$
$$+M_{12-16}+M_{16-12}=(P_1+P_2+P_3)h_1.$$

From the above equations one has

$$3k_{1-5}\theta_1+3k_{2-6}\theta_2+3k_{3-7}\theta_3+3k_{4-8}\theta_4+3k_{1-5}\theta_5$$
$$+3k_{2-6}\theta_6+3k_{3-7}\theta_7+3k_{4-8}\theta_8$$
$$+6(k_{1-5}+k_{2-6}+k_{3-7}+k_{4-8})(-R_3)$$
$$=P_3h_3,$$

$$3k_{5-9}\theta_5+3k_{6-10}\theta_6+3k_{7-11}\theta_7+3k_{8-12}\theta_8+3k_{5-9}\theta_9$$
$$+3k_{6-10}\theta_{10}+3k_{7-11}\theta_{11}+3k_{8-12}\theta_{12}$$
$$+6(k_{5-9}+k_{6-10}+k_{7-11}+k_{8-12})(-R_2)$$
$$=(P_2+P_3)h_2, \quad \text{and}$$

$$3k_{9-13}\theta_9+3k_{10-14}\theta_{10}+3k_{11-15}\theta_{11}+3k_{12-16}\theta_{12}$$
$$+6(k_{9-13}+k_{10-14}+k_{11-15}+k_{12-16})(-R_1)$$
$$=(P_1+P_2+P_3)h_1.$$

The equations developed in Steps 2 and 3 shown above are then summarized in the matrix format as shown in Eq. (5). Note that Σk_{Ji} shown in Eq. (5) represents the summation of the k values of the members connected at the joint "i," while Σk_{Cx} represents the summation of the k values of the columns in the x^{th} story.

4. EXAMPLES FOR THE COMPUTATION OF THE FUNDAMENTAL PERIOD OF MOMENT FRAMES WITH FLEXURAL BEAMS

The following examples demonstrate the computations of fundamental periods of moment frames with flexural beams using two approaches; one is the proposed approach (Eqs. [3], [4], and [5]) and the other one is the traditional dynamic analysis.

Example 1: Compute the fundamental period of the steel moment frame shown in Fig. 4. The beam and column sizes are shown in the figure. The bases of the columns are fixed. The moment of inertia about the x-axis is I_x = 1.66 (10^9) mm^4 for the W30×99 beam and is I_x = 1.25 (10^9) mm^4 for the W14×233 columns. The modulus of elasticity is E = 2.00 (10^5) MPa for all members. The weight of the roof = 267 kN (60 kips). The height and width of the frame are 4.57 m and 8.53 m, respectively. Neglect the shear and axial deformations for all members as well as the weight of the columns.

Approach A (using Eqs. [3] and [5]):

1. Assume the total design lateral force V = 10 kN. Since this is a one-story frame, the lateral force induced at the roof level is 10 kN.

$$\begin{array}{ccccccccccccccc} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} & \theta_{12} & -R_3 & -R_2 & -R_1 \end{array}$$

$$\begin{bmatrix}
2\sum k_{j1} & k_{1-2} & & & k_{1-5} & & & & & & & & 3k_{1-5} & & \\
k_{1-2} & 2\sum k_{J2} & k_{2-3} & & & k_{2-6} & & & & & & & 3\,k_{2-6} & & \\
 & k_{2-3} & 2\sum k_{J3} & k_{3-4} & & & k_{3-7} & & & & & & 3\,k_{3-7} & & \\
 & & k_{3-4} & 2\sum k_{J4} & & & & k_{4-8} & & & & & 3\,k_{4-8} & & \\
k_{1-5} & & & & 2\sum k_{J5} & k_{5-6} & & & k_{5-9} & & & & 3\,k_{1-5} & 3\,k_{5-9} & \\
 & k_{2-6} & & & k_{5-6} & 2\sum k_{J6} & k_{6-7} & & & k_{6-10} & & & 3\,k_{2-6} & 3\,k_{6-10} & \\
 & & k_{3-7} & & & k_{6-7} & 2\sum k_{J7} & k_{7-8} & & & k_{7-11} & & 3\,k_{3-7} & 3\,k_{7-11} & \\
 & & & k_{4-8} & & & k_{7-8} & 2\sum k_{J8} & & & & k_{8-12} & 3\,k_{4-8} & 3\,k_{8-12} & \\
 & & & & k_{5-9} & & & & 2\sum k_{J9} & k_{9-10} & & & & 3\,k_{5-9} & 3\,k_{9-13} \\
 & & & & & k_{6-10} & & & k_{9-10} & 2\sum k_{J10} & k_{10-11} & & & 3\,k_{6-10} & 3\,k_{10-14} \\
 & & & & & & k_{7-11} & & & k_{10-11} & 2\sum k_{J11} & k_{11-12} & & 3\,k_{7-11} & 3\,k_{11-15} \\
 & & & & & & & k_{8-12} & & & k_{11-12} & 2\sum k_{J12} & & 3\,k_{8-12} & 3\,k_{12-16} \\
3\,k_{1-5} & 3\,k_{2-6} & 3k_{3-7} & 3k_{4-8} & 3k_{1-5} & 3k_{2-6} & 3k_{3-7} & 3k_{4-8} & & & & & 6\sum k_{C3} & & \\
 & & & & 3k_{5-9} & 3k_{6-10} & 3k_{7-11} & 3k_{8-12} & 3k_{5-9} & 3k_{6-10} & 3k_{7-11} & 3k_{8-12} & & 6\sum k_{C2} & \\
 & & & & & & & & 3k_{9-13} & 3k_{10-14} & 3k_{11-15} & 3k_{12-16} & & & 6\sum k_{C1}
\end{bmatrix}
\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \\ \theta_{12} \\ -R_3 \\ -R_2 \\ -R_1 \end{bmatrix}
=
\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ P_3h_3 \\ (P_2+P_3)h_2 \\ (P_1+P_2+P_3)h_1 \end{bmatrix} \quad (5)$$

2. Since the one-story, one-bay frame shown in Fig. 4 is a partial structure of the three-story, three-bay frame shown in Fig. 3, the frame shown in Fig. 4 can be extracted from the lower left corner of the frame shown in Fig. 3. The four joints of the one-story, one-bay frame therefore are assigned to be joints "9", "10", "13", and "14", respectively, and the height and width of the frame are assigned to be "h_1" and "l_1", respectively. Also, the lateral force applied to the frame is "P_1" as shown in Fig. 4.

The stiffness matrix equation (Eq. [6]) of the single-story, single-bay moment frame, therefore, can be formed directly from the general stiffness matrix equation (Eq. [5]) of the three-story, three-bay moment frame using the following procedure:

Step 1: Draw three horizontal lines and three vertical lines through θ_9, θ_{10}, and $-R_1$, respectively, on Eq. (5) as shown in Fig. 5.

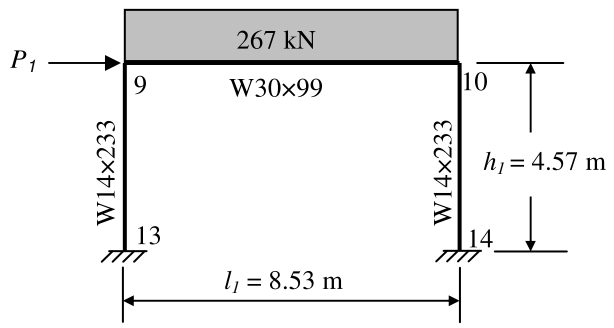


Figure 4. One-story, one-bay steel moment frame with a flexural beam

Step 2: Utilize the stiffness items at the intersections of the lines shown in Fig. 5 to form the stiffness matrix equation of the single-story, single-bay frame shown below (note that $P_2 = P_3 = 0$ for a single-story frame):

$$\begin{matrix} \theta_9 & \theta_{10} & -R_1 \end{matrix}$$
$$\begin{bmatrix} 2\sum k_{J9} & k_{9-10} & 3k_{9-13} \\ k_{9-10} & 2\sum k_{J10} & 3k_{10-14} \\ 3k_{9-13} & 3k_{10-14} & 6\sum k_{C1} \end{bmatrix} \begin{bmatrix} \theta_9 \\ \theta_{10} \\ -R_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P_1 h_1 \end{bmatrix} \quad (6)$$

Note that $k_{i-j} = \dfrac{2E_{i-j}I_{i-j}}{L_{i-j}}$, as mentioned early in this paper. The stiffness items shown in Eq. (6), therefore, can be computed to be

$$2\sum k_{J9} = 2\left(k_{9-10} + k_{9-13}\right)$$

$$= 2\left[\frac{2\left(2\times10^8\right)\left(1.66\times10^{-3}\right)}{8.53} + \frac{2\left(2\times10^8\right)\left(1.25\times10^{-3}\right)}{4.57}\right]$$

$$= 374{,}504\,\text{kN}\cdot\text{m}$$

$2\sum k_{J10} = 2\left(k_{9-10} + k_{10-14}\right) = 374{,}504\ \text{kN}\cdot\text{m}$,

$6\sum k_{C1} = 6\left(k_{9-13} + k_{10-14}\right) = 1{,}312{,}910\ \text{kN}\cdot\text{m}$, and

$$P_1 h_1 = 10\text{kN}\left(4.57\text{m}\right) = 45.7\text{kN}\cdot\text{m}$$

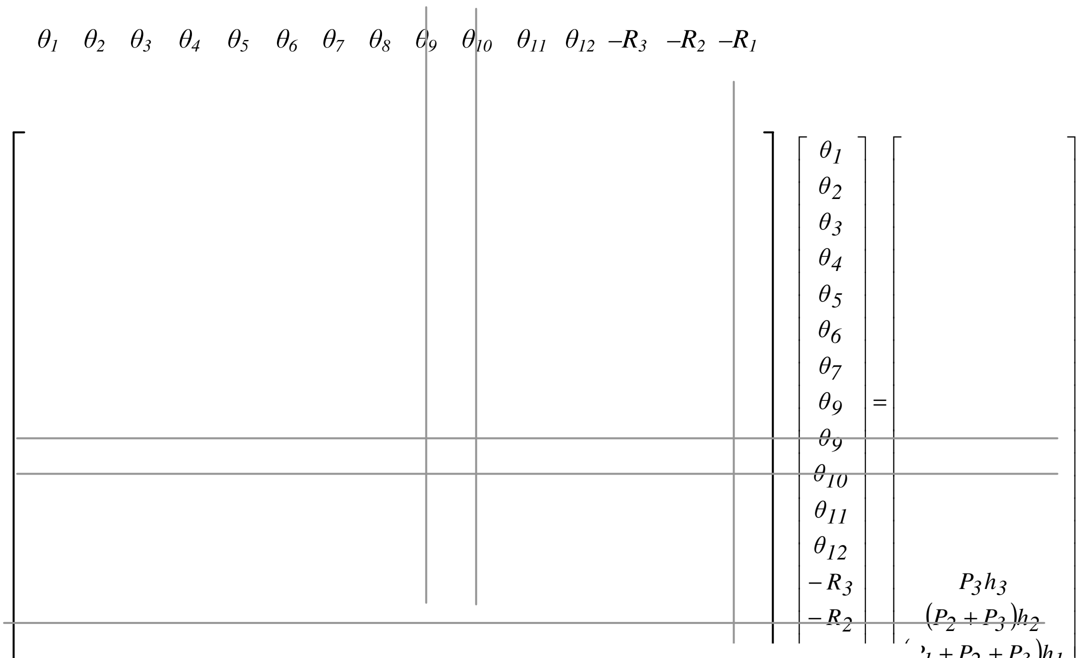


Figure 5. Formation of the stiffness matrix equation for Example 1

Substituting the above values into Eq. (6), one has

$$\begin{bmatrix} 374{,}504 & 77{,}843 & 328{,}227 \\ 77{,}843 & 374{,}504 & 328{,}227 \\ 328{,}227 & 328{,}227 & 1{,}312{,}910 \end{bmatrix} (\text{kN}\cdot\text{m}) \begin{bmatrix} \theta_9 \\ \theta_{10} \\ -R_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 45.7 \end{bmatrix} (\text{kN}\cdot\text{m})$$

The above matrix equation can be represented by $[\text{K}]\{\text{U}\} = \{\text{F}\}$, where $[\text{K}]$ = the stiffness matrix, $\{\text{U}\}$ = the displacement vector, and $\{\text{F}\}$ = the force vector. From $\{\text{U}\} = [\text{K}]^{-1}\{\text{F}\}$, one has

$$R_1 = (-)5.46\times10^{-5}$$

Therefore, the relative horizontal displacement between the roof and the ground floor is

$$\Delta_1 = R_1 \times h_1 = (5.46\times10^{-5})(4.57\text{m}) = 0.25\text{mm}$$

3. From Eq. (3), the fundamental period of the one-story, one-bay frame is

$$T = 2\pi\sqrt{\frac{w\delta^2}{gf\delta}} = 2\pi\sqrt{\frac{w\delta}{gf}}$$

$$= 2\pi\sqrt{\frac{(267)(0.25)\,\text{kN}\cdot\text{mm}}{\frac{9810\,\text{mm}}{\text{sec}^2}(10\,\text{kN})}} = 0.164\,\text{sec}$$

Approach B (using the dynamic analysis):

The system shown in Fig. 4 has three degrees of freedom; they are: one lateral displacement (U_1) and two joint rotations (U_2 and U_3) as shown in Fig. 6. The degrees of freedom caused by joint rotations can be eliminated by using the static condensation method [4] to simplify the dynamic analysis of the moment frame. The following demonstrates the computation of the fundamental period of the frame shown in Fig. 4 using the static condensation method.

1. The total stiffness matrix **K** is composed of stiffness matrices $\mathbf{k_{dd}}$, $\mathbf{k_{dr}}$, $\mathbf{k_{rd}}$, and $\mathbf{k_{rr}}$ and is denoted as

$$\mathbf{K} = \left[\begin{array}{c|c} \mathbf{k_{dd}} & \mathbf{k_{dr}} \\ \hline \mathbf{k_{rd}} & \mathbf{k_{rr}} \end{array}\right]$$

where $\mathbf{k_{dd}}$ = the displacement stiffness of the columns caused by the deflections of the columns; $\mathbf{k_{dr}}$ = the displacement stiffness of the columns caused by the rotations of the joints; $\mathbf{k_{rd}}$ = the rotation stiffness of the joints caused by the deflections of the columns; and $\mathbf{k_{rr}}$ = the rotation stiffness of the joints caused by the rotations of the joints.

The stiffness coefficients for joint translation and joint rotation as shown in Fig. 7 can be used to determine the value of each k_{ij} presented in Fig. 8:

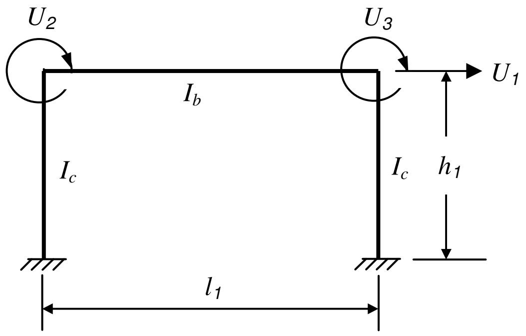
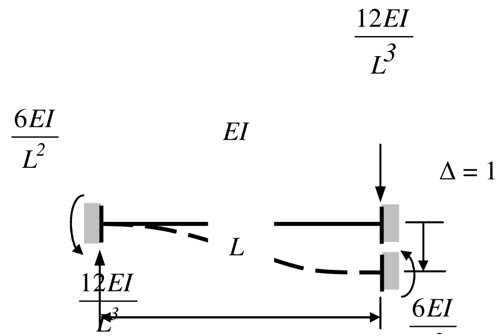
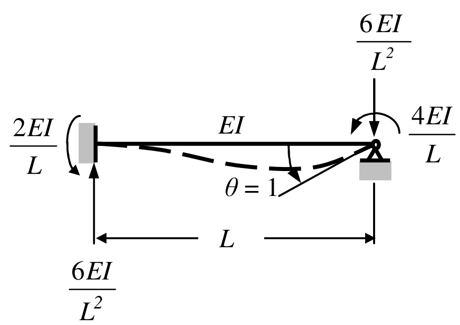


Figure 6. Three-degrees-of-freedom system

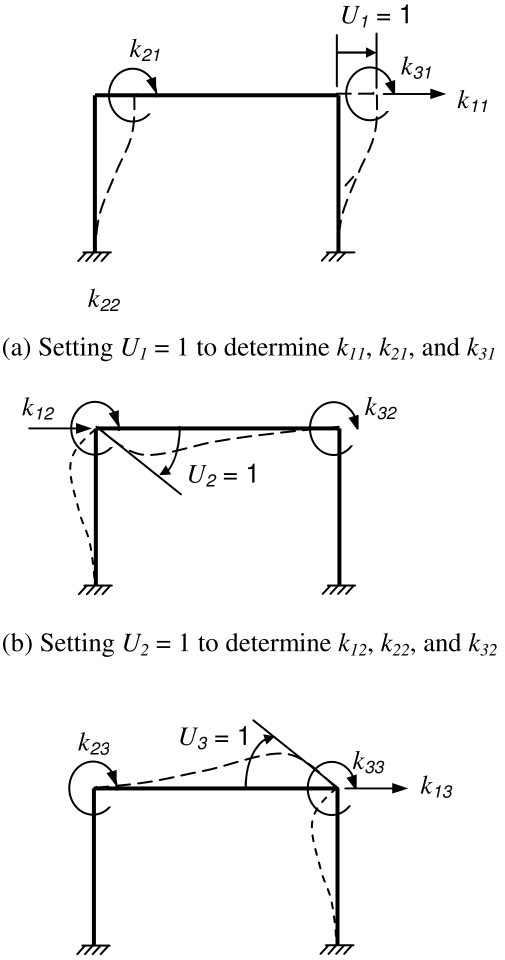


(a) Stiffness coefficients for joint translation



(b) Stiffness coefficients for joint rotation

Figure 7. Stiffness coefficients for joint translation and ioint rotation



(a) Setting $U_1 = 1$ to determine k_{11}, k_{21}, and k_{31}

(b) Setting $U_2 = 1$ to determine k_{12}, k_{22}, and k_{32}

(c) Setting $U_3 = 1$ to determine k_{13}, k_{23}, and k_{33}

Figure 8. Setting $U_j = 1$ to determine k_{ij} for a single-story, single-bay frame

Setting $U_1 = 1$ and $U_2 = U_3 = 0$ as shown in Fig. 8(a), one has

$$k_{11} = 2\left(\frac{12EI_c}{h_1^3}\right) = 62{,}864\,\frac{\text{kN}}{\text{m}}$$

$$k_{21} = k_{31} = \frac{6EI_c}{h_1^2} = 71{,}822\,kN$$

Setting $U_2 = 1$ and $U_1 = U_3 = 0$ as shown in Fig. 8(b), one has

$$k_{12} = \frac{6EI_c}{h_1^2} = 71{,}822\,\text{kN}$$

$$k_{22} = \frac{4EI_b}{l_1} + \frac{4EI_c}{h_1} = 374{,}504\,\text{kN}\cdot\text{m}$$

$$k_{32} = \frac{2EI_b}{l_1} = 77{,}843\,\text{kN}\cdot\text{m}$$

Setting $U_3 = 1$ and $U_1 = U_2 = 0$ as shown in Fig. 8(c), one has

$$k_{13} = \frac{6EI_c}{h_1^2} = 71{,}822\,\text{kN}$$

$$k_{23} = \frac{2EI_b}{l_1} = 77{,}843\,\text{kN}\cdot\text{m}$$

$$k_{33} = \frac{4EI_b}{l_1} + \frac{4EI_c}{h_1} = 374{,}504\,\text{kN}\cdot\text{m}$$

The total stiffness matrix **K**, therefore, is

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 62{,}864 & 71{,}822 & 71{,}822 \\ 71{,}822 & 374{,}504 & 77{,}843 \\ 71{,}822 & 77{,}843 & 374{,}504 \end{bmatrix} = \begin{bmatrix} \mathbf{k_{dd}} & \mathbf{k_{dr}} \\ \mathbf{k_{rd}} & \mathbf{k_{rr}} \end{bmatrix}$$

2. The condensed stiffness matrix $\mathbf{K_c}$ can be obtained from the following equation [4]

$$\mathbf{K_c} = \mathbf{k_{dd}} - \mathbf{k_{rd}^T}\mathbf{k_{rr}^{-1}}\mathbf{k_{rd}} \qquad (7)$$

Therefore, one has

$$\mathbf{K}_c = 62{,}864 - \begin{bmatrix} 71{,}822 & 71{,}822 \end{bmatrix} \begin{bmatrix} 2.7908\times10^{-6} & -5.8008\times10^{-7} \\ -5.8008\times10^{-7} & 2.7908\times10^{-6} \end{bmatrix} \begin{bmatrix} 71{,}822 \\ 71{,}822 \end{bmatrix}$$

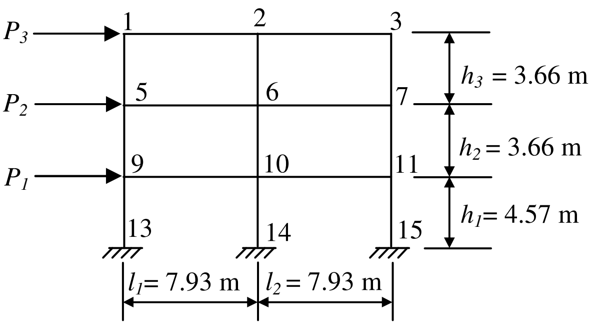
$$= 40{,}056\,\frac{\text{kN}}{\text{m}}$$

3. The static condensation matrix $\mathbf{K_c}$ eliminated the joint rotations, U_2 and U_3. The three-degrees-of-freedom system as shown in Fig. 6, therefore, has been reduced to a single-degree-of-freedom system. The natural period of the structural system, therefore, is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{267\,\text{kN}}{\frac{9.81\,\text{m}}{\text{sec}^2}\left(40{,}056\,\frac{\text{kN}}{\text{m}}\right)}} = 0.164\,\text{sec}$$

The above results show that the fundamental period obtained from the dynamic analysis agrees with that obtained from Eqs. (3) and (5) for the one-story, one-bay moment frame with a flexural beam as shown in Fig. 4.

Example 2: Compute the fundamental period of the three-story, two-bay reinforced concrete moment frame shown in Fig. 9. The exterior column size is 559 mm × 559 mm (22" × 22"), the interior column size is 660 mm × 660 mm (26" × 26"), and the beam size is 457 mm × 559 mm (18" × 22"). The modulus of elasticity E = 2.48 (10^4) MPa for all members. The total weight of the roof is 801 kN (180 kips) and the total weight of each floor is 934 kN (210 kips). Neglect the shear and axial deformations for all members as well as the weight of the columns.



Notes: Exterior columns: 559 mm × 559 mm
Interior columns: 660 mm × 660 mm
All beams: 457 mm × 559 mm

Figure 9. Three-story, two-bay reinforced concrete frame with flexural beams

2. Compute the horizontal displacement of each level of the frame. Considering the factors (0.35 for beams and 0.7 for columns as specified in Section 10.11.1 of ACI 318-05 [5]) for the computation of the moments of inertia of reinforced concrete beams and columns, one has

Approach A (using Eqs. [3], [4], and [5]):

1.Compute the vertical distribution of seismic forces. Assume the total design lateral force V = 100 kN. From Eq. (4) one has F_1 = 19.87 kN, F_2 = 35.79 kN, and F_3 = 44.34 kN. Note that F_i = the lateral force induced at level i of the frame. Also note that k =1 (k is a distribution exponent related to the frame period) has been assumed in Eq. (4) as mentioned early in this paper.

$$I_b = 0.35\frac{457(559)^3}{12} = 2.33\times(10^9)\text{mm}^4$$

$$I_{ce} = 0.7\frac{559(559)^3}{12} = 5.70\times(10^9)\text{mm}^4$$

$$I_{ci} = 0.7\frac{660(660)^3}{12} = 11.07\times(10^9)\text{mm}^4$$

where I_b = the moment of inertia of the beams; I_{ce} = the moment of inertia of the exterior columns; and I_{ci} = the moment of inertia of the interior columns.

Since the three-story, two-bay moment frame shown in Fig. 9 is a partial structure of the three-story, three-bay frame shown in Fig. 3, the stiffness matrix equation of the three-story, two-bay moment frame can be formed directly from the general stiffness matrix equation (Eq. [5]) of the three-story, three-bay moment frame using the procedure shown in the previous example (Example 1). Thus, from Eq. (5) one has Eq. (8) as shown below:

Note that $k_{i-j} = \frac{2E_{i-j}I_{i-j}}{L_{i-j}}$, as mentioned early in this paper. Therefore, one has

$$\begin{bmatrix}
2\sum k_{J1} & k_{1-2} & & k_{1-5} & & & & & & 3k_{1-5} & & \\
k_{1-2} & 2\sum k_{J2} & k_{2-3} & & k_{2-6} & & & & & 3k_{2-6} & & \\
\theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_6 & \theta_7 & \theta_9 & \theta_{10} & \theta_{11} & -R_3 & -R_2 & -R_1 \\
 & k_{2-6} & & k_{5-6} & 2\sum k_{J6} & k_{6-7} & & k_{6-10} & & 3k_{2-6} & 3k_{6-10} & \\
 & & k_{3-7} & & k_{6-7} & 2\sum k_{J7} & & & k_{7-11} & 3k_{3-7} & 3k_{7-11} & \\
 & & & k_{5-9} & & & 2\sum k_{J9} & k_{9-10} & & & 3k_{5-9} & 3k_{9-13} \\
 & & & & k_{6-10} & & k_{9-10} & 2\sum k_{J10} & k_{10-11} & & 3k_{6-10} & 3k_{10-14} \\
 & & & & & k_{7-11} & & k_{10-11} & 2\sum k_{J11} & & 3k_{7-11} & 3k_{11-15} \\
3k_{1-5} & 3k_{2-6} & 3k_{3-7} & 3k_{1-5} & 3k_{2-6} & 3k_{3-7} & & & & 6\sum k_{C3} & & \\
 & & & 3k_{5-9} & 3k_{6-10} & 3k_{7-11} & 3k_{5-9} & 3k_{6-10} & 3k_{7-11} & & 6\sum k_{C2} & \\
 & & & & & & 3k_{9-13} & 3k_{10-14} & 3k_{11-15} & & & 6\sum k_{C1}
\end{bmatrix}
\begin{bmatrix} \theta_1 \\ \theta_2 \\ \\ \theta_6 \\ \theta_7 \\ \theta_9 \\ \theta_{10} \\ \theta_{11} \\ -R_3 \\ -R_2 \\ -R_1 \end{bmatrix}
=
\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ P_3h_3 \\ (P_1+P_3)h_2 \\ (P_1+P_2+P_3)h_1 \end{bmatrix} \quad (8)$$

$$k_{1-2} = k_{2-3} = k_{5-6} = k_{6-7} = k_{9-10} = k_{10-11}$$
$$= \frac{2 \times 2.48(10^7) \times 2.33(10^{-3})}{7.93} = 14{,}574 \text{kN} \cdot \text{m}$$

$$k_{1-5} = k_{5-9} = k_{3-7} = k_{7-11}$$
$$= \frac{2 \times 2.48(10^7) \times 5.70(10^{-3})}{3.66} = 77{,}246 \text{kN} \cdot \text{m}$$

$$k_{9-13} = k_{11-15} = \frac{2 \times 2.48(10^7) \times 5.70(10^{-3})}{4.57}$$
$$= 61{,}864 \text{kN} \cdot \text{m}$$

$$k_{2-6} = k_{6-10} = \frac{2 \times 2.48(10^7) \times 11.07(10^{-3})}{3.66}$$
$$= 150{,}020 \text{kN} \cdot \text{m}$$

$$k_{10-14} = \frac{2 \times 2.48(10^7) \times 11.07(10^{-3})}{4.57} = 120{,}147 \text{kN} \cdot \text{m}$$

The stiffness of each item presented in Eq. (8), therefore, can be determined using the $k_{i\text{-}j}$ values determined above. For example:

$$2\sum k_{J1} = 2(k_{1-2} + k_{1-5}) = 183{,}640 \text{kN} \cdot \text{m};$$

$$2\sum k_{J2} = 2(k_{1-2} + k_{2-3} + k_{2-6}) = 358{,}336 \text{kN} \cdot \text{m};$$

$$6\sum k_{C2} = 6(k_{5-9} + k_{6-10} + k_{7-11}) = 1{,}827{,}072 \text{kN} \cdot \text{m};$$

and

$$6\sum k_{C1} = 6(k_{9-13} + k_{10-14} + k_{11-15}) = 1{,}463{,}250 \text{kN} \cdot \text{m}$$

Also, from Step 1 one has

$$P_3 h_3 = (44.34)(3.66) = 162.28 \text{kN} \cdot \text{m}$$

$$(P_2 + P_3) h_2 = (35.79 + 44.34)(3.66) = 293.28 \text{kN} \cdot \text{m}$$

$$(P_1 + P_2 + P_3) h_1 = (19.87 + 35.79 + 44.34)(4.57)$$
$$= 457.00 \text{kN} \cdot \text{m}$$

Substituting the above values into Eq. (8), results in

R_3 = (-) 0.0010191;

R_2 = (-) 0.0013149; and

R_1 = (-) 0.0009165.

The relative horizontal displacement between each adjacent level of the frame, therefore, can be computed to be:

$$\Delta_1 = R_1 \times h_1 = (0.0009165)(4.57\text{m}) = 4.19\text{mm}$$

$$\Delta_2 = R_2 \times h_2 = (0.0013149)(3.66\text{m}) = 4.81\text{mm}$$

$$\Delta_3 = R_3 \times h_3 = (0.0010191)(3.66\text{m}) = 3.73\text{mm}$$

where Δ_1 = the relative horizontal displacement between the 1st floor and the ground floor; Δ_2 = the relative horizontal displacement between the 2nd floor and the 1st floor; and Δ_3 = the relative horizontal displacement between the roof and the 2nd floor.

Therefore, the total horizontal displacement at the 2nd floor is $\delta_2 = \Delta_1 + \Delta_2$ = 9.00 mm. Also, the total horizontal displacement at the roof (3rd level) is $\delta_3 = \Delta_1 + \Delta_2 + \Delta_3$ = 12.73 mm.

3. Compute the fundamental period of the frame. From the lateral forces determined in Step 1 and from the horizontal displacements determined in Step 2, the fundamental period of the frame can be determined using Eq. (3):

$$T = 2\pi \sqrt{\left(\sum_{i=1}^{n} w_i \delta_i^2\right) \div \left(g \sum_{i=1}^{n} f_i \delta_i\right)}$$

$$= 2\pi \sqrt{\frac{(934\text{kN})(4.19\text{mm})^2 + (934\text{kN})(9.00\text{mm})^2 + (801\text{kN})(12.73\text{mm})^2}{\frac{9810\text{mm}}{\text{sec}^2}\left[(19.87\text{kN})(4.19\text{mm}) + (35.79\text{kN})(9.00\text{mm}) + (44.34\text{kN})(12.73\text{mm})\right]}}$$

= 0.959 sec

Approach B (using the dynamic analysis):

Since it is a long process to perform the dynamic analysis of the three-story, two-bay frame, only the results of the modal frequencies (obtained from the dynamic analysis) of the frame are shown below:

The 1st modal frequency ω_1 = 6.55 rad/sec, the 2nd modal frequency ω_2 = 24.62 rad/sec, and the 3rd modal frequency ω_3 = 54.46 rad/sec.

The fundamental period, T_1, of the frame, therefore, is

$$T_1 = \frac{2\pi}{\omega_1} = 0.959 \text{sec}$$

The above result shows that the fundamental period obtained from the dynamic analysis agrees with that obtained from the proposed approach (using

Eqs. [3], [4], and [5]) for the three-story, two-bay reinforced concrete moment frame shown in Fig. 9.

The approximate fundamental period of the three-story, two-bay frame given in this example can be determined using Eq. (1) (an approximate method specified in the Equivalent Lateral Force Procedure, ASCE/SEI 7-05):

$$T_a = C_t h_n^x = 0.0466(11.89)^{0.9} = 0.433\text{sec}$$

The approximate fundamental period of the frame can also be determined using Eq. (2) (the first method [known as Method A] specified in the 1997 UBC Static Force Procedure):

$$T = C_t (h_n)^{3/4} = 0.0731(11.89)^{3/4} = 0.468\text{sec}$$

The above results indicate that the approximate fundamental period of the frame obtained either from Eq. (1) or from Eq. (2) is quite different from that (0.959 sec) obtained either from the dynamic analysis or from the proposed hand-calculated static approach (using Eqs. [3], [4], and [5]).

4. RESULTS AND DISCUSSION

The results obtained from Example 2 in Section 4 reveal that the approximate fundamental periods obtained using either Eq. (1) (an approximate method specified in the Equivalent Lateral Force Procedure, ASCE/SEI 7-05) or using Eq. (2) (Method A specified in the 1997 UBC Static Force Procedure), are too rough to be used for the final design of a frame and should be used during the preliminary design stage only. Furthermore, the proposed hand-calculated static approach (using Eqs. [3], [4], and [5]), can be considered as an accurate approach for determining the fundamental period of the frame.

The hand-calculated static approach, therefore, is proposed herein to be used along with the Equivalent Lateral Force Procedure (specified in the ASCE/SEI 7-05) to perform both the preliminary design and final design of low-rise moment frames. The proposed approach is especially recommended for the design of reinforced concrete moment frames for the following reasons:

Once the beam and column sizes of a reinforced concrete structure have been determined by an architect, they usually remain unchanged during the structural design process. Therefore, once the stiffness matrix (which can be formed directly from the general stiffness matrix shown in Eq. [5]) is constructed for the preliminary design of a frame, it usually can be reused for the final design of the frame.

The computation of the moments of inertia of reinforced concrete beams and columns shown in Example 2 in Section 4 demonstrates that the proposed hand-calculated approach is able to take the effects of cracked sections of reinforced concrete elements into consideration for the computation of story drifts as specified in Section 12.7.3 of the ASCE/SEI 7-05.

5. SUMMARY AND CONCLUSIONS

Although the Equivalent Lateral Force Procedure as specified in the ASCE/SEI 7-05 and the Static Force Procedure as specified in the 1997 UBC each have an approximate method to be used for determining the fundamental periods of moment frames, the approximate fundamental periods derived from these two procedures are usually too rough to be used for the final design of the frames. Accurate fundamental periods of moment frames are traditionally obtained using the dynamic analysis, which usually requires the use of computer software.

A hand-calculated static approach, therefore, is proposed in this paper to serve as a convenient tool that can be used along with the ASCE/SEI 7-05 Equivalent Lateral Force Procedure to perform both the preliminary design and the final design of low-rise moment frames. Examples presented in this paper have proved that the fundamental periods obtained from the proposed hand-calculated static approach agree with that obtained from the dynamic analysis. The proposed hand-calculated approach, therefore, can be considered as an accurate approach for determining the fundamental periods of moment frames.

The following are the advantages of using the proposed approach to design a moment frame: (1) the proposed approach can be carried out by using hand calculations only (without the use of computer computations); (2) once the fundamental period of a frame is determined (using the proposed procedure and an assumed seismic base shear), it can be used to determine the actual seismic base shear. The actual vertical distribution of seismic forces, the story drifts, and joint rotations of the frame in turn can be determined using Eqs. (4) and (5).; (3) once the actual story drifts and joint rotations of a frame are determined, the final bending moment at the ends of each element of the frame can then be determined using the basic slope deflection equations discussed earlier in this paper.; (4) the proposed approach is able to take the effects of cracked sections of reinforced concrete elements into consideration for the computation of story drifts as specified in Section 12.7.3 of the ASCE/SEI 7-05; (5) the stiffness matrix of any moment frame within three stories in height and within three bays in width can be formed directly from the general stiffness matrix presented in this paper; and (6) the fundamental period of the frame obtained from the proposed hand-calculated static approach can be used to verify the accuracy of that obtained from the dynamic analysis using computer software.

The following are the limitations of the proposed approach presented in this paper: (1) the frame to be designed is limited to three stories in height and three bays in width; and (2) the columns in the same story shall have the same height and the beams in the same bay shall have the same length.

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NOTATION

C_t = numerical coefficient;
$E_{i\text{-}j}$ = modulus of elasticity of the beam (or column) *i-j*;
f_i = lateral force at level *i*;
F_x = lateral force induced at level *x* of the frame;
{F} = force vector;
g = acceleration due to gravity;
h_i, h_x = height from the base to level *i* or *x*, respectively;
h_n = height above the base to the highest level of the structure;
I_b = moment of inertia of the beams;
I_{ce} = moment of inertia of the exterior columns;
I_{ci} = moment of inertia of the interior columns;
$I_{i\text{-}j}$ = moment of inertia of the beam (or column) *i-j*;
I_x = moment of inertia about the x-axis;
K = total stiffness matrix
[*K*] = stiffness matrix of the structural system;
k = distribution exponent related to the frame period;
$\mathbf{K_c}$ = condensed stiffness matrix;
$\mathbf{k_{dd}}$ = displacement stiffness of the columns caused by the deflections of the columns;
$\mathbf{k_{dr}}$ = displacement stiffness of the columns caused by the rotations of the joints;
$\mathbf{k_{rd}}$ = rotation stiffness of the joints caused by the deflections of the columns;
$\mathbf{k_{rr}}$ = rotation stiffness of the joints caused by the rotations of the joints;
$L_{i\text{-}j}$ = length of the beam (or column) *i-j*;
$M_{i\text{-}j}$ = moment at the end "*i*" of the beam (or column) *i-j*;
[*M*] = mass matrix of the structural system;
T = fundamental period;
T_a = approximate fundamental period;
{U} = displacement vector;
V = total design lateral force or shear at the base of the frame;
{*v*} = displacement vector of the structural system;
w_i = portion of the total seismic dead load located at or assigned to level *i*; portion of the total gravity load of the structure assigned to level *i*;
w_x = portion of the total gravity load of the structure assigned to level *x*;
Δ = relative horizontal displacement between each adjacent level of a frame;
Δ_x = relative deflection between the ends of a column in the x^{th} story of the frame;
δ_i = horizontal displacement at level *i* relative to the base due to applied lateral forces;
θ_i = rotation at the end "*i*" of the beam (or column) *i-j*;
θ_j = rotation at the end "*j*" of the beam (or column) *i-j*; and
ω = angular frequency.