A New Analytical Method for Stochastic Response of Structure-Damper System

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ABSTRACT: Fundamental principles from structural dynamics, pseudo excitation method and perturbation techniques are used to develop a new fast stochastic method for seismic analysis of the combined structure-damper system. In the approach, the mathematical equation of structure-damper system is expressed in the perturbation form, based on which the inverse operation of the matrices is avoided. Moreover, the new method also does not need the solution of any complex eigenvalue problem, in contrast to other methods found in the literature. Finally, the computation efficiency of the method is examined, and numerical comparisons with exact results are carried out to verify the accuracy of the proposed method. In all cases examined, the approach presented here shows excellent agreement with the exact results.

KEYWORDS: structure-damper system, non-proportional damping, perturbation techniques, stochastic analysis, pseudo excitation method.

1 INTRODUCTION

The adoption of supplemental damping elements in civil structures has been introduced in the relatively recent past as an innovative technology for reducing the level of vibration, which is usually related to wind and earthquake actions. Because of the non-proportional damping characteristic of the combined structure-damper system, the decoupled seismic analysis can not be performed by the undamped real-valued modal matrix. Alternatively, the structure-damper system can be analyzed directly by considering the combined system as a single dynamic unit, by determining the complex eigenproperties of the combined system via state-space approach firstly proposed by Foss (1958), and then by employing a modal analysis. However, the calculation of complex eigenvalues problem is cumbersome and time-consuming, and thus attempts to overcome the computational difficulties of the approach have been carried out by Lou et al. (2003), Karen et al. (2005), Fernando et al. (2006). Besides, it may sometimes be necessary to consider a number of design alternatives of the damping system and then, such an analysis would involve the evaluation of the dynamic properties of the combined structure-damper system several times. In order to avoid a great deal of repeated calculation of the complex eigenvalue problems, several measures have been studied. Based on the pseudo force method studied by Claret et al. (1991), Lin et al. (2003), an iterative procedure for computing the transfer function matrix of a non-classically damped system has been developed by Jandid et al. (1993), Zavoni et al. (2006). The iterative methods have more advantages than the complex modal superposition methods in terms of speed, and it retains the advantages of the real-valued modal superposition methods.

This paper presents a new stochastic method for the seismic analysis of combined structure-damper system. The pseudo excitation method proposed by Lin (1992) is introduced for its high computation efficiency. Besides, based on the perturbation techniques, the inverse operation of matrices in the pseudo excitation method for non-proportionally damped system is avoided. Meanwhile, the stochastic response analysis of the combined structure-damper system is free of the determination of complex eigenvalues and eigenvectors, and the matrix of power spectral density (PSD) function is expressed in a matrix sequence form. In the end, numerical comparisons with exact results are carried out to examine the accuracy of the proposed method. The results prove the accuracy of the new approach.

2 EQUATIONS OF MOTION

Consider an n-degree-of-freedom discrete structure subjected to base acceleration $\ddot{u}_g(t)$. Attached to this structure is the damping system consisting of m dampers. The equations of motion for the combined structure-damper system can be written as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + (\boldsymbol{C} + \boldsymbol{C}_d)\dot{\boldsymbol{U}} + (\boldsymbol{K} + \boldsymbol{K}_d)\boldsymbol{U} = -\boldsymbol{M}\boldsymbol{E}\ddot{u}_g(t) \quad (1)$$

where, **M, C** and **K** are the mass, damping and stiffness matrices of the structure, respectively. $\boldsymbol{U}$=[u_1,…,u_n] is the relative displacement vector of the structure with respect to the base. A dot over $\boldsymbol{U}$ represents a derivative with respect to time. $\boldsymbol{E}$ means the conventional influence vector representing rigid-body displacements of the structure for a unit translation of the base. $\boldsymbol{C}_d$ and $\boldsymbol{K}_d$ denote the additional damping and stiffness matrices provided by the damping system, which can be obtained by assembling the damping and stiffness coefficients of m dampers. Suppose that the force of the damper is proportional to the relative motion between the supporting points, c_{di} and k_{di} are the damping and stiffness coefficients of the ith damper respectively, the force can be obtained by:

$$\begin{aligned}\boldsymbol{f}_{di} &= \boldsymbol{C}_{di}\dot{\boldsymbol{V}}_{di} + \boldsymbol{K}_{di}\boldsymbol{V}_{di} \\ &= \begin{bmatrix} c_{di} & -c_{di} \\ -c_{di} & c_{di} \end{bmatrix}\begin{bmatrix} \dot{v}_{di,1} \\ \dot{v}_{di,2} \end{bmatrix} + \begin{bmatrix} k_{di} & -k_{di} \\ -k_{di} & k_{di} \end{bmatrix}\begin{bmatrix} v_{di,1} \\ v_{di,2} \end{bmatrix}\end{aligned} \quad (2)$$

in which, $\boldsymbol{C}_{di}$ and $\boldsymbol{K}_{di}$ are the damping and stiffness matrices of the ith damper, $\boldsymbol{V}_{di}$ is the displacement vector of the ith damper's supporting points in the local coordinate system, and $v_{di,1}$ and $v_{di,2}$ correspond to the component coordinates of $\boldsymbol{V}_{di}$. A transformation matrix can be easily constructed to relate $\boldsymbol{V}_{di}$ to $\boldsymbol{U}$. Denoting this transformation matrix by $\boldsymbol{Z}_{di}$, the following equation is obtained as:

$$\boldsymbol{V}_{di} = \boldsymbol{Z}_{di}\boldsymbol{U} \quad (3)$$

Substituting the above expression in equation (2), and then pre-multiplied by the transpose of $\boldsymbol{Z}_{di}$, yielding:

$$\boldsymbol{Z}_{di}^T\boldsymbol{f}_{di} = \boldsymbol{Z}_{di}^T\boldsymbol{C}_{di}\boldsymbol{Z}_{di}\dot{\boldsymbol{U}} + \boldsymbol{Z}_{di}^T\boldsymbol{K}_{di}\boldsymbol{Z}_{di}\boldsymbol{U} \quad (4)$$

in which, the superscript T indicates the transpose operation. The equations of motion of the damping system can then be obtained by the summation of equation (4), leading to

$$\sum_{i=1}^{m}\boldsymbol{Z}_{di}^T\boldsymbol{f}_{di} = \sum_{i=1}^{m}\boldsymbol{Z}_{di}^T\boldsymbol{C}_{di}\boldsymbol{Z}_{di}\dot{\boldsymbol{U}} + \sum_{i=1}^{m}\boldsymbol{Z}_{di}^T\boldsymbol{K}_{di}\boldsymbol{Z}_{di}\boldsymbol{U} \quad (5)$$

Comparing equations (1)-(5) leads to the following relationships:

$$\boldsymbol{C}_d = \sum_{i=1}^{m}\boldsymbol{Z}_{d_i}^T\boldsymbol{C}_{d_i}\boldsymbol{Z}_{d_i} \quad (6a)$$

$$\boldsymbol{K}_d = \sum_{i=1}^{m}\boldsymbol{Z}_{d_i}^T\boldsymbol{K}_{d_i}\boldsymbol{Z}_{d_i} \quad (6b)$$

Equations (6a) and (6b) complete the process of combining the property matrices of the dampers into property matrices of structure-damper system.

3 FORMULATION OF PERTURBATION EQUATIONS

Let $\boldsymbol{\Phi}$=[$\boldsymbol{\varphi}_1$,…,$\boldsymbol{\varphi}_n$] denote the normalized modal matrix of the structure, which is defined by:

$$\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{\Omega} = \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} \quad (7)$$

in which the matrix $\boldsymbol{\Omega}$ is the diagonal matrix listing the natural radian frequencies of the structure. For large structure cases, the first few modes are defined as :

$$\boldsymbol{\Phi}_q = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_{n_q}], n_q << n \quad (8)$$

where n_q<<n is the number of modes obtained up to cutoff frequency.

In order to decouple equation (1), the following transformation is usually adopted:

$$\boldsymbol{U} = \boldsymbol{\Phi}_q\boldsymbol{q} \quad (9)$$

where, $\boldsymbol{q} \in R^{n_q \times n_q}$ is the vector of generalized coordinates. Thus, we can write the differential equations of motion in the modal subspace as follows:

$$\boldsymbol{I}\ddot{\boldsymbol{q}} + \boldsymbol{C}^*\dot{\boldsymbol{q}} + \boldsymbol{K}^*\boldsymbol{q} = -\boldsymbol{\Phi}_q{}^T\boldsymbol{M}\boldsymbol{E}\ddot{u}_g(t) \quad (10)$$

where, $\boldsymbol{I} \in R^{n_q \times n_q}$ is the identity matrix. $\boldsymbol{C}^* \in R^{n_q \times n_q}$ and $\boldsymbol{K}^* \in R^{n_q \times n_q}$ are generalized damping and stiffness matrices which can be written as follows:

$$\boldsymbol{C}^* = \mathrm{diag}[2\omega_1\zeta_1, \cdots, 2\omega_{n_q}\zeta_{n_q}] + \boldsymbol{\Phi}_q{}^T\boldsymbol{C}_d\boldsymbol{\Phi}_q \quad (11a)$$

$$\boldsymbol{K}^* = \mathrm{diag}[\omega_1^2, \cdots, \omega_{n_q}^2] + \boldsymbol{\Phi}_q{}^T\boldsymbol{K}_d\boldsymbol{\Phi}_q \quad (11a)$$

where, diag[ ] denotes a diagonal matrix with elements in the argument; ω_i and ζ_i are the frequency and damping ratio for the ith mode of the structure.

Matrices $\boldsymbol{C}^*$ and $\boldsymbol{K}^*$ can be separated as follows:

$$\boldsymbol{C}^* = \bar{\boldsymbol{C}} + \tilde{\boldsymbol{C}}, \boldsymbol{K}^* = \bar{\boldsymbol{K}} + \tilde{\boldsymbol{K}} \quad (12)$$

where, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{K}}$ are the diagonal matrices containing the main diagonal elements of $\boldsymbol{C}^*$ and $\boldsymbol{K}^*$. Matrices $\tilde{\boldsymbol{C}}$ and $\tilde{\boldsymbol{K}}$ contains all of the elements outside the main diagonal of $\boldsymbol{C}^*$ and $\boldsymbol{K}^*$. The elements in the main diagonal of matrices $\tilde{\boldsymbol{C}}$ and $\tilde{\boldsymbol{K}}$ are zero.

Generally, it can be considered that $\tilde{\boldsymbol{C}}$ and $\tilde{\boldsymbol{K}}$ are little perturbation quantities with respect to the diagonal matrices $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{K}}$, and a parameter ε is introduced to keep track of the orders of different quantities involved in the derivation. Accordingly, the damping and stiffness matrices corresponding to the dampers can be written:

$$\tilde{\boldsymbol{C}} = \varepsilon \Delta \bar{\boldsymbol{C}}\ ,\ \tilde{\boldsymbol{K}} = \varepsilon \Delta \bar{\boldsymbol{K}} \qquad (13)$$

Then, the generalized ordinates can be expressed in terms of the perturbation parameter ε:

$$\boldsymbol{q} = \sum_{j=0}^{\infty} \varepsilon^j \boldsymbol{q}^{(j)} = \boldsymbol{q}^{(0)} + \varepsilon \boldsymbol{q}^{(1)} + \varepsilon^2 \boldsymbol{q}^{(2)} + \varepsilon^3 \boldsymbol{q}^{(3)} + \cdots \quad (14)$$

Thus, equation (10) can be given in a general form based on the perturbation techniques:

$$\boldsymbol{I}\ddot{\boldsymbol{q}}^{(j)} + \bar{\boldsymbol{C}}\dot{\boldsymbol{q}}^{(j)} + \bar{\boldsymbol{K}}\boldsymbol{q}^{(j)} = \bar{\boldsymbol{F}}^{(j)} \qquad (15a)$$

$$\bar{\boldsymbol{F}}^{(0)} = -\boldsymbol{\Phi}_q^{\ T}\boldsymbol{ME}\ddot{u}_g(t) \qquad (15b)$$

$$\bar{\boldsymbol{F}}^{(j+1)} = -\Delta\bar{\boldsymbol{C}}\dot{\boldsymbol{q}}^{(j)} - \Delta\bar{\boldsymbol{K}}\boldsymbol{q}^{(j)}\ ,\ j = 0,1,2,\cdots \quad (15c)$$

In equation (15), the coupling term is eliminated based on the real-valued modal superposition method and the perturbation techniques. Then, the displacement of the structure can be easily obtained according to equations (9) and (14)

$$\boldsymbol{U} = \boldsymbol{\Phi}_q \boldsymbol{q} = \boldsymbol{\Phi}_q \sum_{j=0}^{\infty} \varepsilon^j \boldsymbol{q}^{(j)} \qquad (16)$$

4 STOCHASTIC RESPONSE ANALYSIS

The input of ground motion is considered to be a zero mean Gaussian process, and the power spectral density (PSD) function are denoted as $S_{\ddot{u}_g}$. Because of the computational complexity of traditional random vibration theory, the pseudo excitation method is adopted. The ground motion is assumed to be a pseudo harmonic excitation:

$$\ddot{u}_g(t) = \sqrt{S_{\ddot{u}_g}}\, e^{r\omega t}, r = \sqrt{-1} \qquad (17)$$

Define:

$$\boldsymbol{H}_q = \mathrm{diag}[h_1, \cdots, h_{n_q}] \qquad (18a)$$

$$h_i = \left(\bar{\omega}_i^2 - \omega^2 + 2\bar{\varsigma}_i\bar{\omega}_i r\omega\right)^{-1}, i = 1,2,\cdots,n_q \quad (18b)$$

where, $\bar{\omega}_i^2$ and $2\bar{\varsigma}_i\bar{\omega}_i$ correspond to the diagonal elements of the matrices $\bar{K}$ and $\bar{C}$.

For the 0th order perturbation solution of equation (15) one can take

$$\boldsymbol{q}^{(0)} = \boldsymbol{q}_\omega^{(0)} e^{r\omega t} \qquad (19a)$$

$$\boldsymbol{q}_\omega^{(0)} = \boldsymbol{H}_q \boldsymbol{\gamma}_\omega^{(0)} \qquad (19b)$$

$$\boldsymbol{\gamma}_\omega^{(0)} = -\boldsymbol{\Phi}_q^{\ T}\boldsymbol{ME}\sqrt{S_{\ddot{u}_g}} \qquad (19c)$$

Then, for the jth (j=1,2,…,s) order solution of equation (15) it will have

$$\boldsymbol{q}^{(j)} = \boldsymbol{q}_\omega^{(j)} e^{r\omega t} \qquad (20a)$$

$$\boldsymbol{q}_\omega^{(j)} = \boldsymbol{H}_q \boldsymbol{\gamma}_\omega^{(j)} \qquad (20b)$$

$$\boldsymbol{\gamma}_\omega^{(j)} = -(r\omega\Delta\bar{\boldsymbol{C}} + \Delta\bar{\boldsymbol{K}})\boldsymbol{q}_\omega^{(j-1)} \qquad (20c)$$

Using the pseudo response of structure in equation (19) and equation (20), the generalized coordinate vector of jth order (j=1,2,…,s) can be obtained. Accordingly, $\boldsymbol{q}$ and $\boldsymbol{U}$ are given by equations (14) and (16). The calculation stops While the following condition is satisfied in each frequency:

$$Err = \frac{\left|\varepsilon^s \boldsymbol{q}_\omega^{(s)}\right|}{\sum_{j=0}^{s} \varepsilon^j \boldsymbol{q}_\omega^{(j)}} < Tol \qquad (21)$$

where Tol is the tolerant limit for stopping the iteration, which can be specified according to practical engineering. Then, the displacement PSD of the structure can be obtained by:

$$\boldsymbol{S}_U = \boldsymbol{U}^* \boldsymbol{U}^T \qquad (22)$$

where, the superscript * indicates the complex conjugate.

The diagonal elements of PSD matrix $\boldsymbol{S_U}$ correspond to auto-PSD of displacements of all degree of freedom of the structure. By using $\boldsymbol{S_U}$(k,k) to denote the PSD of the kth degree of freedom displacement of the structure, the variance can be given by the integration over frequency domain:

$$\sigma_{U_k}^2 = \int_{-\infty}^{\infty} \boldsymbol{S}_U(k,k) d\omega \qquad (23)$$

Before closing this section, it is useful to generalize equation (22) for response quantities other than nodal displacement. It is well known that a displacement-related response quantity $z(t)$ such as an internal force or stress, can be expressed in terms of the vector of nodal relative displacement, $\boldsymbol{U}$

$$z(t) = \boldsymbol{\Gamma U} \qquad (24)$$

where, $\boldsymbol{\Gamma}$ is an vector of constants. For the internal force in a member, for example, $\boldsymbol{\Gamma}$ is given in terms of the elements of the stiffness matrix of the member. Thus, the PSD of the response quantity $z(t)$ can be calculated from:

$$S_z = (\boldsymbol{\Gamma U})^* (\boldsymbol{\Gamma U})^T = \boldsymbol{\Gamma U}^* \boldsymbol{U}^T \boldsymbol{\Gamma}^T = \boldsymbol{\Gamma} S_U \boldsymbol{\Gamma}^T \quad (25)$$

5 GENERAL METHOD FOR STRUCTURE-DAMPER SYSTEM

Utilizing the pseudo excitation method, many approaches have been proposed for the stochastic response analysis of non-proportionally damped system. The complex mode superposition method has been introduced in the pseudo excitation method by Xu et al. (2001), Wang (2008), which is similar to the real-valued mode but more applicable. However, it may require a lot of repeated calculation of complex eigenvalue problems for the structure-damper system, which is inefficient in computation. Thereby, Lin proposed a seismic analysis approach of non-proportionally damped system [12] , which can avoid the repeated calculation of complex eigenvalues and eigenvectors. However, it needs plenty of inverse operation of matrices, which would lead to computation difficulties in practical calculation especially for ill-conditioned matrices. According to the approach, the pseudo generalized coordinate vector of the system is assumed to be written as follows:

$$\boldsymbol{q} = (\boldsymbol{q}_{real} + r\boldsymbol{q}_{imag})e^{r\omega t}, r = \sqrt{-1} \qquad (26)$$

where, the subscripts real and imag represent the real part and imaginary part of the complex vector $\boldsymbol{q}$, respectively. The equation (10) can be expressed as：

$$\boldsymbol{P}\boldsymbol{q}_{real} + \boldsymbol{Q}\boldsymbol{q}_{imag} = \boldsymbol{F}_{real} \qquad (27a)$$

$$-\boldsymbol{Q}\boldsymbol{q}_{real} + \boldsymbol{P}\boldsymbol{q}_{imag} = \boldsymbol{F}_{imag} \qquad (27b)$$

where,

$$\boldsymbol{P} = \boldsymbol{K}^* - \omega^2\boldsymbol{I}, \boldsymbol{Q} = -\omega\boldsymbol{C}^* \qquad (28a)$$

$$\boldsymbol{F}_{real} = -\boldsymbol{\Phi}_q^T\boldsymbol{ME}\sqrt{S_{\ddot{u}_g}}, \boldsymbol{F}_{\mathrm{imag}} = 0 \qquad (28b)$$

Utilizing equations (27) and (28), the generalized coordinate vector $\boldsymbol{q}$ can be obtained.

$$\begin{aligned}\boldsymbol{q} &= \boldsymbol{q}_{real} + r\boldsymbol{q}_{imag} \\ &= (\boldsymbol{P} + \boldsymbol{Q}\boldsymbol{P}^{-1}\boldsymbol{Q})^{-1}\boldsymbol{F}_{real} + r\boldsymbol{P}^{-1}\boldsymbol{Q}(\boldsymbol{P} + \boldsymbol{Q}\boldsymbol{P}^{-1}\boldsymbol{Q})^{-1}\boldsymbol{F}_{real}\end{aligned} \qquad (29)$$

Then, the pseudo displacement vector $\boldsymbol{U}$ of the structure is produced by equation (9), and accordingly the power spectral density function matrix of the structural displacement is given by equation (22). The method is efficient in computation approximately requiring $O(n^2)$ operations. However, the inverse operation in equation (29) may lead to computation difficulties. From this point, the new method proposed in this paper is more applicable by using the perturbation equations instead of the matrix inverse operations.

6 COMPUTATION COST

In large structures, the number of the reserved modes n_q is far less than the number of degrees of freedom n, so the calculation related to the number n_q is little enough to be neglected. Meanwhile, the cost independent of the frequency calculation can also be ignored in the stochastic analysis. Table 1 lists the main calculation flow chart and the detailed cost of the new method, in which N_{iter} and N_{freq} denotes the iterative times and the number of frequency points of the PSD respectively. The detailed cost of the new method is n^2N_{freq}, that is the computation efficiency of the new improved method is approximately the same with the method proposed by Lin [10]. At the same time, the inverse operations of matrices are avoided.

Table 1 The calculation flow chart and detailed cost of the new method for stochastic analysis of structure-damper system

Step	Task	Cost
1	$\boldsymbol{\gamma}^{(0)} = -\boldsymbol{\Phi}_q^T\boldsymbol{ME}$	nn_q
	for $1:N_{freq}$	
2	$\boldsymbol{\gamma}_\omega^{(0)} = \boldsymbol{\gamma}^{(0)}\sqrt{S_{\ddot{u}_g}}$	N_{freq}
3	$\boldsymbol{q}_\omega^{(0)} = \boldsymbol{H}_q\boldsymbol{\gamma}_\omega^{(0)}$	n_qN_{freq}
	for $1:N_{\mathrm{iter}}$	
4.1	$\boldsymbol{\gamma}_\omega^{(j)} = -(r\omega\Delta\bar{\boldsymbol{C}} + \Delta\bar{\boldsymbol{K}})\boldsymbol{q}_\omega^{(j-1)}$	$n_q^2N_{\mathrm{iter}}N_{freq}$
4.2	$\boldsymbol{q}_\omega^{(j)} = \boldsymbol{H}_q\boldsymbol{\gamma}_\omega^{(j)}$	$n_qN_{\mathrm{iter}}N_{freq}$
	end	
5	$\boldsymbol{U} = \boldsymbol{\Phi}_q\sum_{j=0}^{N_{iter}}\varepsilon^j\boldsymbol{q}^{(j)}$	nn_qN_{freq}
6	$\boldsymbol{S}_U = \boldsymbol{U}^*\boldsymbol{U}^T$	n^2N_{freq}
	end	
Main cost		n^2N_{freq}

7 NUMERICAL EXAMPLES

Numerical results are presented to demonstrate the accuracy of the new method. As shown in Figure 1, a 20-storey structure with distributed added dampers is used as numerical example for testing the new method. All of the storeys have mass equal to 2×10^6 *kg* and lateral stiffness equal to 5×10^6 *kN/m*. The Rayleigh damping is adopted, and a 5% damping ratio is considered for 1st and 5th mode of the structure. The modal properties of the structure are listed in Table 2. The first 6 modes are selected for the seismic analysis. In the structure, the viscous dampers are selected for the vibration reduction, and the corresponding configurations and positions of the viscous damper are given in Table 3.

The stochastic model of ground motion, the Kanai-Tajimi model, is adopted here, which can be described as:

$$S_{\ddot{u}_g} = \frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\zeta_g\omega_g\omega)^2}S_0 \qquad (30)$$

in which, ω_g=13.96 rad/s, ζ_g=0.72, which correspond to the site type 1 and classification of the earthquake 3 as been defined in GB50011-2001 of the code in china. The seismic intensity is assumed to be S_0=0.06.

The accuracy of the proposed method is examined, and the perturbation results of the displacement variances of the 1st, 10th and 20th floors are shown in Table 4. The solutions corresponding to different perturbation order are listed in order to explain the trend the approximate solution to the exact solution. In the all examined cases, the proposed method shows excellent agreement with the exact results calculated based on the general method proposed by Lin (2004).

The results prove the accuracy of the proposed method. Meanwhile, it can also be seen only a few order of perturbation is required for the given examples. However, it also should be mentioned is that the perturbation process may be unshrinking for large non-proportional damping cases, in which the proposed method based on perturbation equation is invalidated.

Table 2 Modal properties of the structure

Mode	1	2	3	4	5	6	7	8	9	10
Frequency (HZ)	0.61	1.83	3.03	4.22	5.38	6.51	7.60	8.65	9.65	10.59
Damping ratio	0.05	0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.08	0.09
Sum of mass ratio	0.83	0.92	0.95	0.97	0.98	0.98	0.99	0.99	0.99	0.99

Table 3 Configurations of damping coefficients of the viscous dampers in the structure ($\times 10^5$ kN·s/m)

Configuration	Ground-Floor 1	Floor 1-2	Floor 2-3	Floor 10-11	Floor 11-12	Floor 12-13
1	4.0	4.0	4.0	2.0	2.0	2.0
2	6.0	6.0	6.0	2.0	2.0	2.0
3	8.0	8.0	8.0	4.0	4.0	4.0
4	10.0	10.0	10.0	4.0	4.0	4.0

Table 4 Perturbation results of the displacement variances of the 1st, 10th and 20th floors of the structure

Configuration	Floor	Exact results	Perturbation times								Error %
			1	2	3	4	5	6	7	8	
1	1	0.59	0.61	0.60	0.58	0.60	0.60	–	–	–	1.7
	10	5.40	5.37	5.35	5.38	5.41	5.40	5.40	–	–	0.6
	20	7.81	7.71	7.72	7.79	7.83	7.81	7.81	–	–	0.0
2	1	0.53	0.55	0.54	0.54	–	–	–	–	–	1.9
	10	4.95	4.87	4.84	4.90	4.98	4.94	4.94	–	–	0.2
	20	7.22	6.98	7.01	7.17	7.27	7.21	7.22	7.22	–	0.0
3	1	0.47	0.48	0.48	–	–	–	–	–	–	2.1
	10	4.51	4.36	4.33	4.45	4.57	4.48	4.51	4.51	–	0.0
	20	6.57	6.22	6.26	6.52	6.65	6.54	6.59	6.58	6.58	0.2
4	1	0.43	0.43	0.42	0.42	–	–	–	–	–	2.3
	10	4.29	4.08	4.03	4.21	4.40	4.24	4.30	4.30	–	0.2
	20	6.33	5.81	5.86	6.26	6.46	6.25	6.37	6.34	6.34	0.2

8 CONCLUSIONS

A new stochastic method for seismic analysis of structure-damper system has been presented. The accuracy and validity of the proposed approach are proven both mathematically and numerically and the results show excellent agreement with the exact results. In contrast to other methods found in the literature, the two main advantages of the new method are:

1: The determination of complex eigenvalues and eigenvetors of non-proportionally damped system is avoided, by which the computation efficiency can be dramatically improved, especially for the structure-damper system which requires repeated calculation of eigenvalue problems.

2: The inverse operation of the matrices, which may lead to computation difficulties for ill conditioned matrices, is avoided based on the perturbation techniques. Meanwhile, no additional computation cost is introduced. The new method is efficient and accurate for stochastic analysis of seismic response of structure-damper system.

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