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Modelling reinforced concrete structures subjected to impulsive loading using concrete lattice model

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ABSTRACT: A two-dimensional behavioral rate dependent lattice model (RDLM) capable of analyzing reinforced concrete members subjected to impulsive loading is presented. As a result of the impulsive loads reaching their peak intensity in extremely short durations of time, material nonlinearities and strain rate effects play an important role in the analysis. A numerical approach based on an explicit finite element formulation is introduced to solve two-dimensional planar structural problems under impulsive loads. The procedure incorporates equilibrium and compatibility conditions and utilizes realistic rate dependent stress-strain relationships for cracked concrete. The model inherently takes into account some major influencing factors, progressive cracking of concrete in tension, the inelastic response in compression, the yielding of reinforcing steel, and strain rate sensitivity of concrete and steel. Correlation studies between analytical and experimental results on reinforced concrete beams subject to impulsive loading are conducted and are shown to be in good agreement.

Keywords: Concrete, impulsive loading, lattice model, finite element method, high strain-rate

1 INTRODUCTION

The numerical simulation of the real behavior in the nonlinear finite element method lies primarily in the improvements of the constitutive models of materials. At present, a considerable number of models are available for the nonlinear behavior of reinforced concrete under compressive stress, for the long-term behavior of concrete, for simulating crack initiation and propagation, etc. However less attention has been paid to the modeling of strain rate sensitivity of concrete and steel reinforcement under impulsive loading. The analytical prediction of failure modes of reinforced concrete structures under severe impulsive loads is very difficult, due to the complexity of concrete behavior in the high strain-rate domain. As a result of the impulsive loads reaching their peak intensity in extremely short durations of time, material nonlinearities and strain-rate effects play an important role in the analysis. Most current design and analysis methods are based on simplified equivalent static forces or single degree of freedom systems. Although this approach may be suitable for preliminary analysis, it contains inherent inconsistencies. The finite element method provides an effective numerical approach for the modelling of structural problems involving severe impulsive loading. Thus, one of the main aims of this study is to improve the realism of these models, and to account for the strain rate sensitivity to predict the nonlinear structural response of two-dimensional concrete structures subject to impulsive loading.

The rate dependent lattice model (RDLM) for concrete and steel, developed in this study, is based on a strain rate sensitive concrete and reinforcement material model in tension and compression and is suitable for the two-dimensional analysis of RC structures. Major sources of nonlinearities in reinforced concrete structures has been taken into consideration, such as the progressive cracking of concrete in tension, the inelastic response in compression, the yielding of reinforcing steel, and strain-rate sensitivity of concrete and steel reinforcement. The smeared crack approach is employed to model concrete cracking behavior. Steel reinforcement is modelled as a strain rate dependent elasto-viscoplastic material. Material nonlinearities including crack propagation, concrete crushing, post-failure residual strength, as well as the strain rate effects on the response of concrete and steel can be modelled by the proposed explicit code RC-IMPULSIVE developed in this study.

The explicit code is validated using experimental test results from the Woomera blast trial and other test data available from literature. Numerical analysis and a parametric study of reinforced concrete structures subjected to impact and blast loading were also performed using the computer code. The capability of program RC-IMPLUSIVE in predicting accurately the nonlinear dynamic response of concrete structures under impulsive loading has been proven.

2 CONSTITUTIVE MODEL FOR CONCRETE AT HIGH STRAIN-RATES

For concrete structures subjected to blast or impact effects, a response at very high strain-rates is often sought. At these high strain-rates, the strength of concrete can increase significantly. A series of impact tests were carried out by Ngo [1], based on the Split Hopkinson Pressure bar (SHPB) set-up, using large-diameter concrete cylinders to achieve a range of loading rates and hydrostatic pressures. Concrete specimens with different strengths were tested. Based on the results of the experimental program using Hopkinson Bar apparatus, and through a rigorous calibration process, a new strain-rate dependent constitutive model is proposed by the authors. This applies to concrete under a dynamic load, and takes into account the strain-rate effect by incorporating multiplying factors for increases in the peak stress and strain at peak strength. This model is applicable to concrete strengths varying from 32 MPa to 160 MPa with a strain-rate up to 300 s-1. In this model, the rate dependent dynamic peak stress (f'cd) can be expressed as:

$$K_{cd} = \frac{f'_{cd}}{f'_{cs}} = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)^{1.026\alpha} \qquad \text{for } \dot{\varepsilon} \le \dot{\varepsilon}_1$$

$$K_{cd} = \frac{f'_{cd}}{f'_{cs}} = A_1 \ln(\dot{\varepsilon}) - A_2 \qquad \text{for } \dot{\varepsilon} > \dot{\varepsilon}_1$$

where f'_{cs} = static compressive strength (MPa)
$\dot{\varepsilon}$ = strain-rate
$\dot{\varepsilon}_s$ = 3×10^{-5} s^{-1} (quasi-static strain-rate)
α = $1/(20 + f'_{cs}/2)$
$\dot{\varepsilon}_1$ = $0.0022\, f'^{2}_{cs} - 0.1989\, f'_{cs} + 46.137$
A_1 = $-0.0044\, f'_{cs} + 0.9866$
A_2 = $-0.0128\, f'_{cs} + 2.1396$

Fig. 1 shows the DIF curves plotted using the proposed model for three concrete groups: NSC, HSC and RPC. Test results are also plotted to compare with the proposed model. It should be noted that in the above formulae, the turning point strain-rate, $\dot{\varepsilon}_1$, is a function of the static compressive strength f'_{cs}.

3 *RATE DEPENDENT LATTICE CONCRETE MODEL (RDLM)*

The discrete lattice model was initially developed by Niwa et al. [2]. This model has been successfully used to model RC structures under cyclic loading [3]. The discrete lattice model was extended by Tanabe and others [3, 4], so as to be able to model continuum RC structures using a finite element method. In the equivalent continua lattice model (ECLM) [4] a RC structure can be modelled as parallel layers of concrete and reinforcement lattices, as shown in Fig. 2. This model can be expressed using the finite element (FE) formulation, by smearing out concrete and reinforcement lattices into a continuum. The equivalent continua lattice model has been used successfully in modelling RC columns subject to quasi-static and cyclic loadings [5].

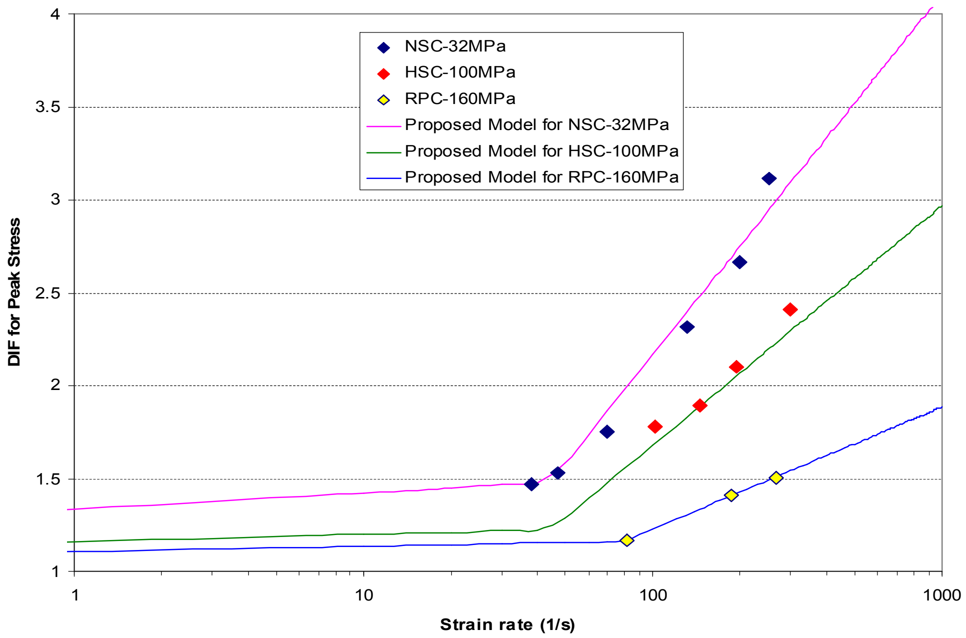

Fig.1. Proposed DIF model for peak stress

In this study, the equivalent continua lattice model (ECLM) was further improved to model RC structures subjected to impulsive loading, such as blasts or impacts. The new model, called Rate Dependent Lattice Model (RDLM), incorporates a strain-rate sensitive lattice model for both the compressive and tensile behavior of concrete. The FE formulation of the proposed model is described in detail by Ngo [1].

4 FINITE ELEMENT FORMULATION

The constitutive equation for the RDLM is comprised of six lattice components. These include compressive and tensile concrete lattices, longitudinal and transverse reinforcement lattices, and two shear lattices, to evaluate the shear transfer on the crack surfaces. The main lattice [Dmain] includes the concrete and reinforcement lattices, which are then combined with the shear lattices [Dshear] to obtain the total stiffness matrix [Dtotal] of the constitutive equation for the model; i.e.

$$[\boldsymbol{D}_{total}] = [\boldsymbol{D}_{main}] + [\boldsymbol{D}_{shear}] \quad (1)$$

Eq. (1) is the general form of the constitutive equation for the RDLM. The stress-strain relationship for the lattices is described below.

4.1 *Main lattice*

The stress-strain matrix of a cracked reinforced concrete element is expressed by assuming a uniformly strained 2D continuum. The local strains in each lattice component are calculated from which the stress vectors in the local coordinate are evaluated using the uniaxial stress-strain relationship of each lattice component. The equivalent stress-strain matrix in

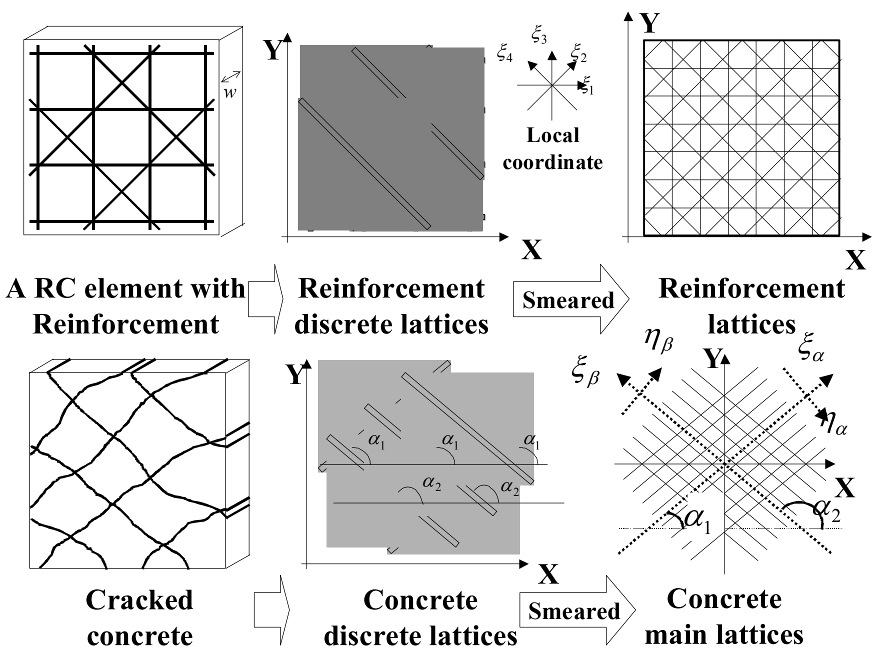

Fig. 2. Lattice model

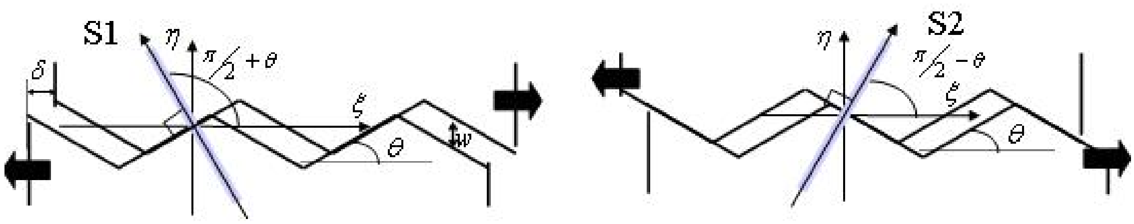

Fig. 3. Shear transfer model

the global coordinate system, which is the general form of the constitutive equation for main lattice, is expressed as:

$$\Delta\{\sigma_g\} = [L_\varepsilon]^T [R_n][L_\varepsilon]\Delta\{\varepsilon_g\} = [D_{main}]\Delta\{\varepsilon_g\} \quad (2)$$

In Eq. (2), the stiffness matrix [Dmain] for the main lattice in the RDLM is written as:

$$[D_{main}] = \begin{bmatrix} \sum_1^n E_i t_i \cos^4\alpha_i & \sum_1^n E_i t_i \cos^2\alpha_i \sin^2\alpha_i & \sum_1^n E_i t_i \cos^3\alpha_i \sin\alpha_i \\ & \sum_1^n E_i t_i \sin^4\alpha_i & \sum_1^n E_i t_i \cos\alpha_i \sin^3\alpha_i \\ symm. & & \sum_1^n E_i t_i \cos^2\alpha_i \sin^2\alpha_i \end{bmatrix} \quad (3)$$

The values of ti are the thickness ratios of each lattice component that are smeared out and the subscript i denotes the number of elements in the Z direction (element width direction). αi is the angle of inclination of each lattice to global coordinate.

4.2 *Shear lattice*

The shear transfer mechanism along the crack surface is shown in Fig. 3. To model the effect aggregate interlock in the RDLM, two shear lattices S1 and S2 are provided. The effect of aggregate interlock is dependent on the relative movement of concrete on two sides of the crack. The aggregate interlock will occur in two ways. On one hand, the interlocking effect, due to the collision of a side of both surfaces as shown in Fig. 3 with acute angle, will be activated for plus shear direction, while the other, the surface of the crack of obtuse angle will be activated for minus shear direction on the other hand. In each of these interlocking effects, two elements of shear lattices S1 and S2 are allocated in perpendicular direction to the surface, which will carry the shear forces along a crack. The dowel effect can be accounted for by changing the inclination angles of the reinforcement lattice components in proportion to the progress of fracture.

The local coordinate defining the crack surface (η, ξ) is shown in Fig. 4. The rigidity of the shear lattices in the local coordinate system is denoted as Dshear,ξ η. To transform this into global coordinatesystem, the shear controlling matrix Ω, and global coordinate transformation matrices T1, T2 are used to define the global shear stiffness matrix Dshear as follows:

$$[D_{shear}] = [T_1][\Omega][D_{shear,\xi\eta}][T_2] \quad (4)$$

5 MATERIAL MODEL FOR LATTICE COMPONENTS

5.1 *Concrete in compression*

A rate dependent compressive concrete model with a pronounced post-peak decay has been developed in this study. Under the bi-axial state of stress, compressive cracked concrete includes the softening of compressive concrete due to the presence of transverse tensile strains. The softening coefficient η is considered to be unity at the uniaxial state of stress. The concrete stress-strain relationship in compression is shown in Fig. 5 and is expressed as follows:

$$f_{cd} = \eta K_{cd} f'_{cs}\left[\frac{2\varepsilon}{\varepsilon_{cd}} - \left(\frac{\varepsilon}{\varepsilon_{cd}}\right)^2\right] \quad \text{for } \varepsilon \le \varepsilon_{cd}$$

$$f_{cd} = \eta\left[K_{cd} f'_{cs} - Z_d(\varepsilon - \varepsilon_{cd})\right] \quad \text{for } \varepsilon > \varepsilon_{cd}$$

where, fcd = compressive dynamic strength under high strain rate (MPa)
ε = principle compressive strain of concrete
εcd = dynamic strain at peak stress = εcs (-0.00002 f'cs + 0.0057) $\dot{\varepsilon}$

ε_{cs} = static strain at peak stress = $\frac{4.26}{\sqrt[4]{f'_{cs}}}\frac{f'_{cs}}{E_{cs}}$

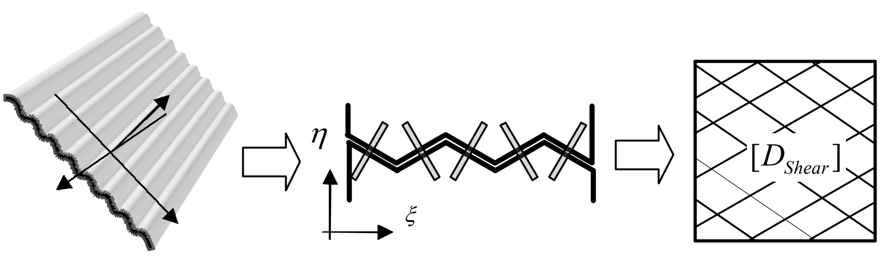

Fig. 4. FE formulation of shear lattices

$$\eta = \frac{1.0}{0.8 - 0.34(\varepsilon_t / \varepsilon_{cd})} \le 1.0$$

In the strain-softening region, it is assumed that the stress reduces linearly to Kdf'cf'res. In this study the residual strength f'res is taken as 0.2. Unloading is modelled to decrease with the initial coefficient. On the other hand, in the reloading path, the stress is assumed to increase linearly to the post maximum stress point in the previous stress history.

5.2 *Concrete in tension: tensile softening*

Concrete is assumed to be an elastic material before cracking. In order to incorporate some ductility in the post-cracking region, a bi-linear tension softening model, as shown in Fig. 6, is adopted. A dynamic increase factor, Ktd, for tensile concrete proposed by Malvar and Ross [6] is adopted to account for strain rate effect. Thus the dynamic tensile strength of concrete is as follows:

$$f_{td} = K_{td} f_{ts}$$

$$K_{td} = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)^{1.016\delta} \quad \text{for } \dot{\varepsilon} \le 1s^{-1}$$

$$K_{td} = \beta\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)^{1/3} \quad \text{for } \dot{\varepsilon} > 1s^{-1}$$

where f_{ts} = static tensile strength of concrete (MPa) $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$ = strain rate up to 160 s^{-1} and quasi-static strain rate respectively, $\log \beta = 6\,\delta - 2$; $\delta = 1/(8 + 8 f'_{cs}/f_{co})$; and f_{co} = 10 MPa

5.3 *Concrete in tension: Tension stiffening*

The tension stiffening effect in the cracked reinforced concrete, as shown in Fig. 7, due to bond effect is also considered to be the rate dependent and is given by:

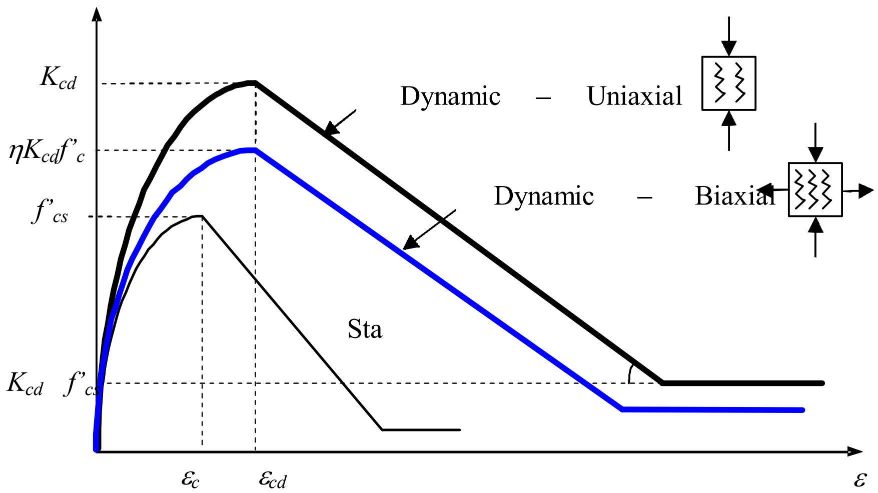

Fig. 5. Concrete stress-strain model under compression

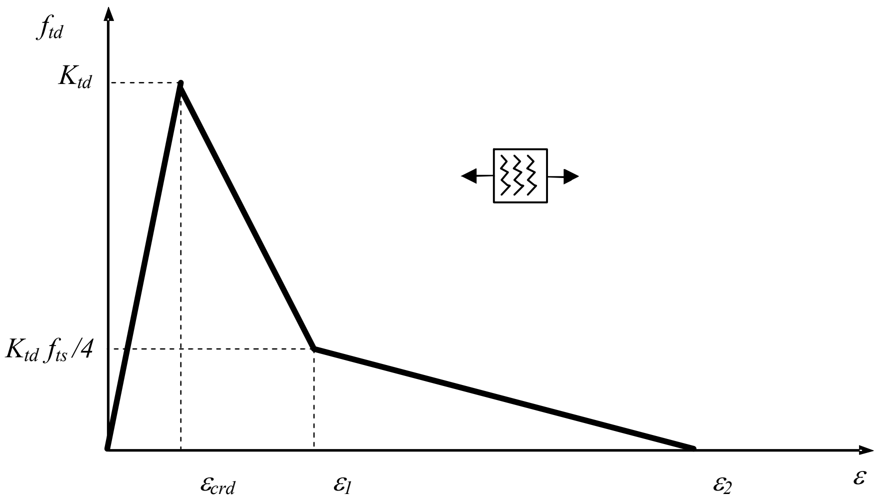

Fig. 6. Concrete stress-strain model under tension

$$f_{td} = K_{td} f_{ts} \left(\frac{\varepsilon_{crd}}{\varepsilon} \right)^C$$

The bond effect coefficient, c, is assumed to be 0.4 in this study. The unloading and reloading paths of the tensile model are similar to the concrete model for compression.

5.4 *Reinforcement model*

The stress-strain relationship of reinforcement is expressed by a bilinear model, representing elasto-viscoplastic behavior with linear isotropic hardening and is assumed to be identical in tension and compression (Fig. 8). In the elastic range, the material behavior is rate independent and linear until the yield stress, which is strain rate dependent, is reached. The rate effects are considered to be equal in both tension and compression. The dynamic increase factor (DIF) for yield stress (fyd) recommended by Malvar [7] is adopted in this study as follows:

$$f_{yd} = f_{ys} \left(\frac{\dot{\varepsilon}}{10^{-4}} \right)^{\alpha}$$

$$\alpha = 0.074 - 0.04 \frac{f_{ys}}{414}$$

where fys = static yield strength of steel (MPa)
fyd = dynamic yield strength of steel (MPa)
$\dot{\varepsilon}$ = strain rate of steel

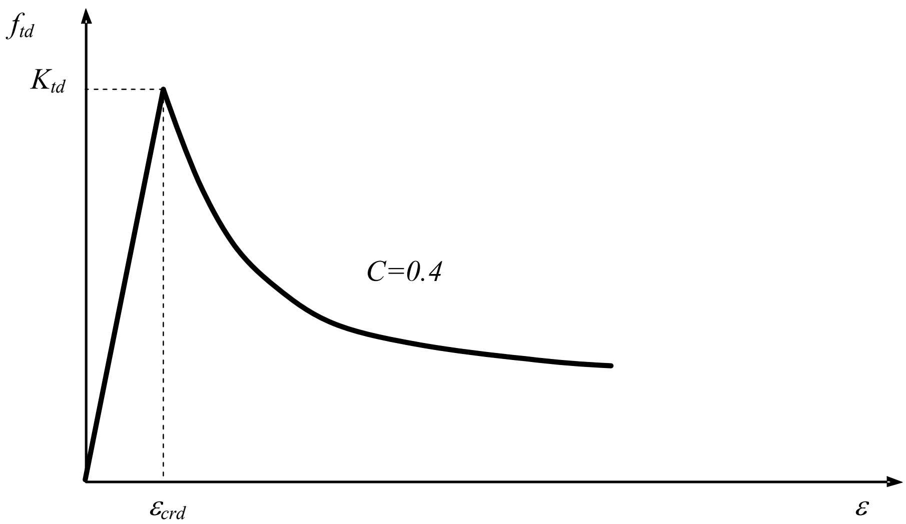

Fig. 7. Tension stiffening model

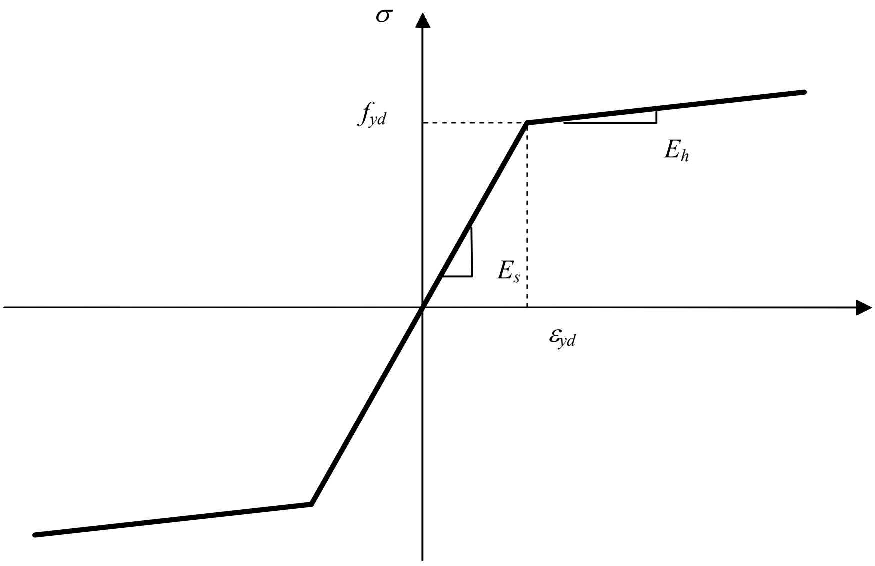

Fig. 8. Reinforcement model

6 COMPUTER IMPLEMENTATION AND CODE VALIDATION

A computer program, named RC-IMPULSIVE, has been developed for the finite element linear and nonlinear transient dynamic analysis of two-dimensional reinforced concrete structures subjected to impulsive loading. The program is written in FORTRAN in modular form, and possesses sufficient flexibility to add new options resulting from further research. For time integration the program uses the central difference scheme. The explicit algorithm was optimized to reduce the computational time (see details in Ngo [1]). The program implements the rate-dependent continuum lattice model described above, combined with a tensile crack-monitoring algorithm. The algorithm, which traces the opening of new cracks as well as the closing and re-opening of existing ones, is activated after total and elastic evaluation for each sampling point at each time step.

In order to validate the computer code RC-IMPULSIVE, analytical results are compared withthe experimental results from the impact test conducted by Agardh et al. [8] and with the testing on Ductal Panels during the Woomera blast trial in May 2004. The results from the blast trial are presented in detail in Ngo [1]. Finite element modeling

Table 1. Material properties of concrete and steel

	Material properties	Agardh et al.	Ductal panels
Concrete	Compressive strength, f'_c	112	164.2 MPa
	Tensile strength, f_t (MPa)	6.7	22.8
	Elastic modulus, E_c (GPa)	45.3	44.1
	Poisson's ratio, υ	0.3	0.3
	Mode I fracture energy, G_f	156	325
Longitudinal reinforcement	Elastic modulus, E_s (GPa)	207	198
	Yield stress, f_y (MPa)	586	1680 (prestressed)
Transverse stirrup	Elastic modulus, E_s (GPa)	207	
	Yield stress, f_y (MPa)	586	

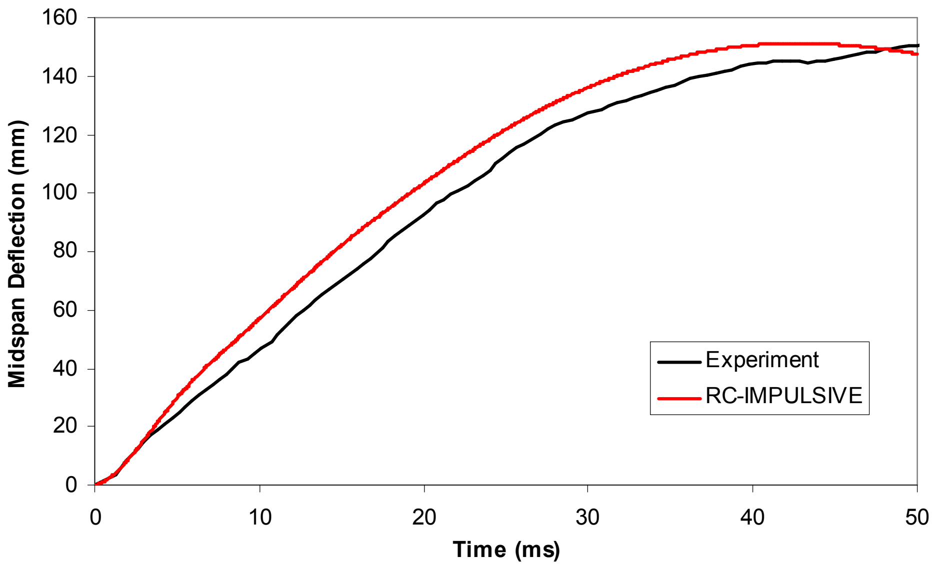

Fig. 9. Comparison of midspan deflections

of the above using RC-IMPULSIVE is presented in this section.

6.1 *Reinforced concrete beams under impact loading by Agardh et al. [1]*

Five high strength concrete beams of 4200mm × 340mm × 170mm dimensions were subjected to a drop weight of 718 kg striking the middle section of the beams with a velocity of 6.7m/s. The main material properties of concrete and steel employed in the analysis are listed in Table 1. The displacements were recorded with a 16mm, high speed video camera with 1500 frames/sec. Reinforcement strains of bottom bars, accelerations at top of the beams and velocities of the drop weight were also recorded. The dynamic analysis is performed using the program RC-IMPULSIVE with a time step of 5 micro seconds and no viscous damping has been considered. The analytical model consists of 595 nodes, 504 four-node elements and 202 truss elements. Fig. 9 shows the time history of the mid-span deflection predicted by RC-IMPULSIVE and the experimental results and is found to be in excellent agreement with the experimental results. The crack pattern predicted by this program also follows the crack pattern observed in the impact test (Fig.10 and Fig. 11).

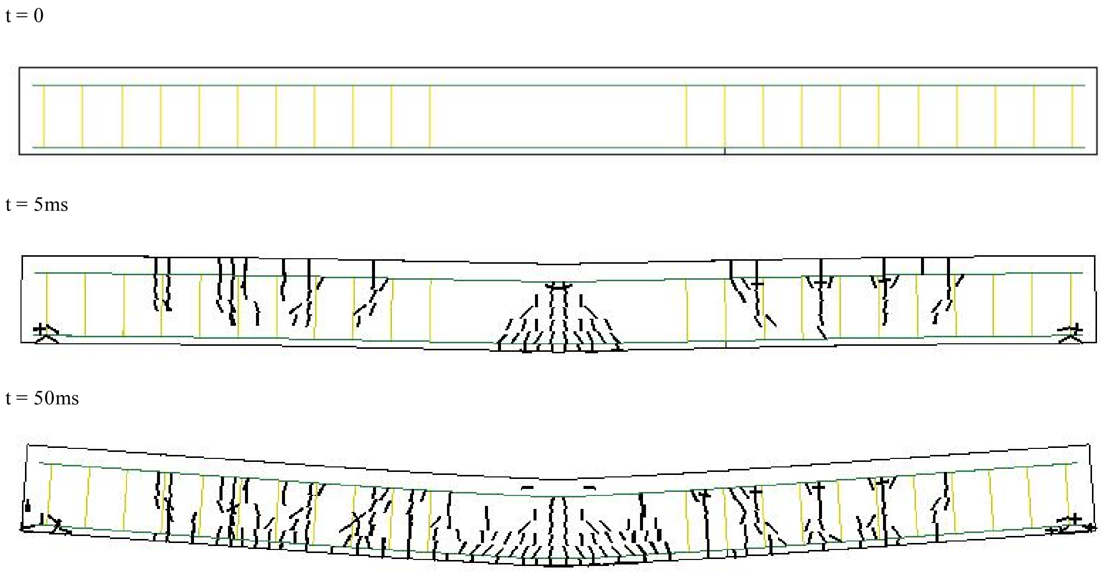

Fig. 10. Crack patterns of the HSC beam predicted by RC-IMPULSIVE

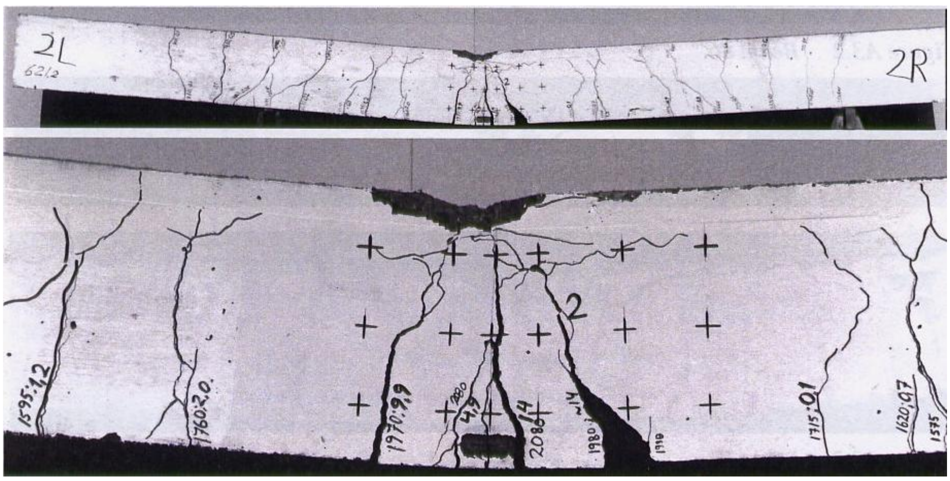

Fig. 11. Crack pattern of the HSC beam after impact

6.2 *Ductal panels subject to blast loading*

A total of three ductal panels were prepared for the blast trial at Woomera, South Australia, in May 2004. Each panel has the dimension of 1000 x 2000 mm. The panel thicknesses are 75 mm and 100 mm. The one-way ductal panels, supported only on two vertical sides, are modelled as simply supported beams with a span of 2m. The panels were prestressed to 20% of the ultimate strength of the tendons. Concrete is modelled using plane stress elements with four nodes. The prestressed tendon is simulated as perfectly bonded truss elements. The FE model includes 205 nodes, 160 four-node elements and 40 truss elements. The prestressed effects are modelled by setting initial tensile stresses for steel elements, as well as initial compressive stresses for concrete elements. The time step length is chosen to be 2.5 micro-seconds. The result for ductal panel 1 has been presented here. The main material properties of concrete and steel employed in the analysis for the panel are listed in Table 1.

Panel 1 is a 100 mm thick prestressed Ductal panel located at 30m stand-off distance. The blast results in an average reflected impulse at the panel surface equal to 3771 kPa.msec with an average reflected pressure of 1513 kPa. The computed displacement-time history and experimental measurements at the mid-span of Ductal Panel 1 are plotted for comparison in Fig. 12 which shows very good agreement. The peak inward deflection of 50.4 mm was recorded which agrees closely with the analytical results, for which the computed peak inward displacement of 48.6 mm is reached in 14.4 msec. Again there is good agreement between the computed peak outward displacement of 35.3 mm reached at time t =37.9 msec, compared with the recorded deflection of 37.0 mm.

The deformed shape of Ductal Panel-1 at differ-

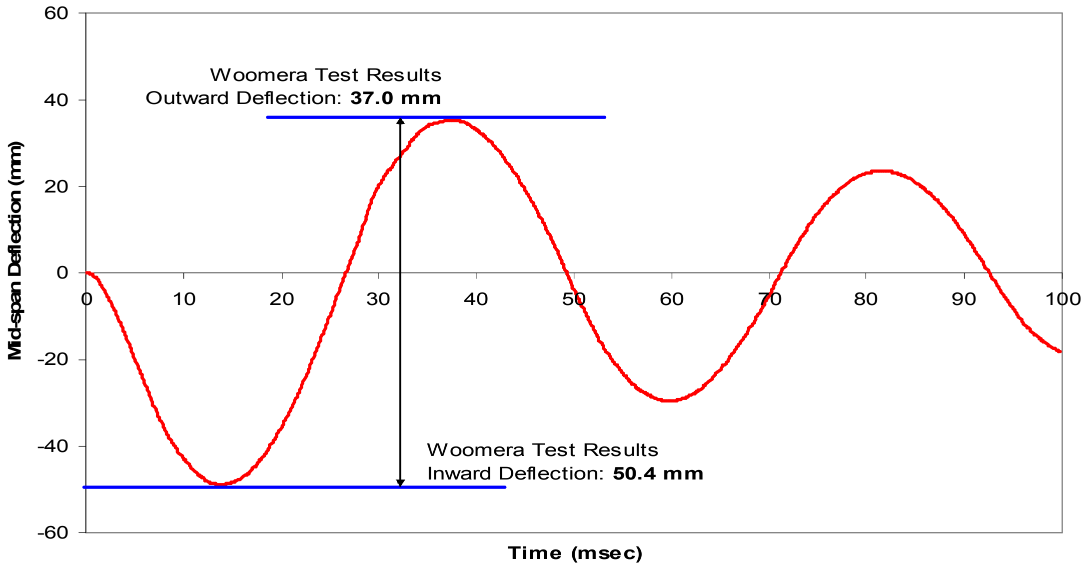

Fig. 12. Mid-span deflection history of 100mm thick Ductal Panel 1 at 30m predicted by RC-IMPULSIVE compared with Woomera test results

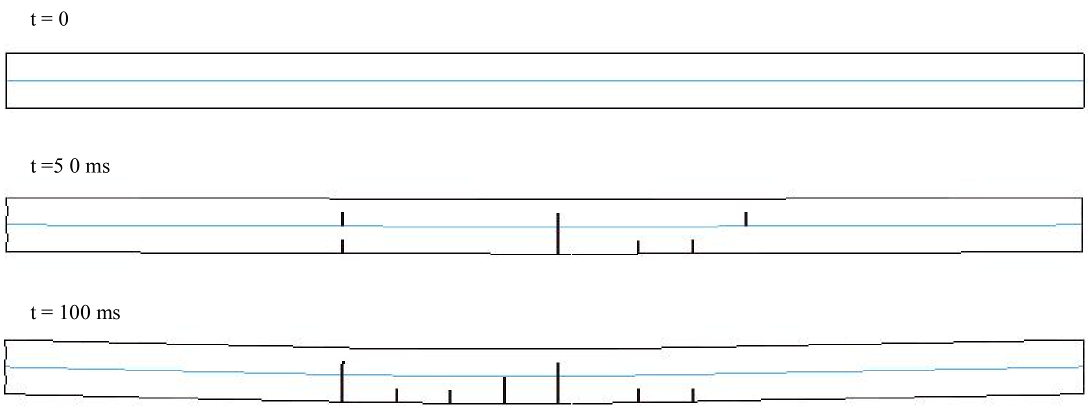

Fig. 13. Crack patterns of Ductal Panel -1 predicted by RC-IMPULSIVE

ent time intervals is plotted in Fig. 13 in which the crack patterns of concrete at various stages in the loading history of the panel are also displayed. These results match the experimentally recorded crack pattern in Fig. 14 quite well on both front and rear faces. It may be further observed that although there is some diffusion in the crack pattern, as in all smeared crack analysis, strain localisation is clearly monitored. Similar to the experimental behavior, only minor cracks are developed and concrete crushing does not occur for this panel.

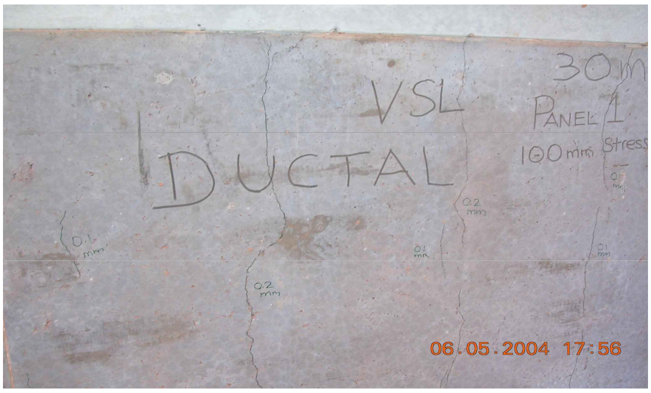

Fig. 14. Crack pattern of Ductal Panel -1

7 CONCLUSIONS

The inclusion of strain rate effects on the proposed rate dependent lattice models for concrete and steel leads to a realistic nonlinear dynamic analysis of reinforced concrete structures under impulsive loading conditions. The computer program, RC-IMPLUSIVE, developed in this study, offers excellent opportunities for linear and nonlinear dynamic analysis of reinforced concrete structures where deformations, stresses, and the progressive fracture in concrete and steel can be traced. The dynamic response of the test panels under blast loading, as predicted by RC-IMPULSIVE, shows that an accurate numerical simulation of the experimental observations is possible using the proposed nonlinear numerical method, despite the highly variable nature of the loading conditions.

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